International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I10P101
Volume 72 Issue 10, 1-8, October 2026
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Original Article

Multiplicative Sombor and Euclidean Reciprocal Index and Coindex of Certain Networks

Akhshaya P S[1], S S Sandhya[2]

[1,2]*Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai, Tamil Nadu, India.*
[Affiliated to Manonmaniam Sundaranar University, Abhishekapatti, Thirunelveli]

[1]*Corresponding Author: akhshayapadmakumar12@gmail.com*

Received: 18 August 2026 Revised: 23 September 2026 Accepted: 11 October 2026 Published: 27 October 2026

Abstract *- Topological indices are mathematical numbers obtained from the molecular graph structure, which characterize properties of the molecules, such as their branched, symmetric, and connected structure. Topological indices have considerable importance in Quantitative Structure-Activity Relationship (QSAR) and quantitative structure-property relationship (QSPR) studies, which allow the prediction of the properties of the molecule without performing laborious experiments. In this paper, we introduce a new topological index called the Multiplicative Euclidean Reciprocal Index (ERI), which is the square root of the sum of reciprocals of squared degrees of adjacent nodes for all edges in the molecular graph.*

Keywords *- Graph, Hexagonal Network, Oxide Network, Rhombus Oxide Network, Silicate Network, Graphene, Prism Network.*

1. Introduction

This paper deals solely with finite, simple, undirected graphs. Let $G = (V, E)$ be such a graph with p vertices and q edges. The standard terminology and notation are based on Harary [3]. Topological indices are numerical graph invariants that encode structural properties and are powerful tools for analyzing molecular behavior, network robustness, and biological interactions. The field traces its origins to the Wiener index (1947), which correlated hydrocarbon structures with physical properties, followed by seminal contributions such as the Randić index (1975) for predicting molecular behaviour and the Zagreb indices (1972) for quantifying branching patterns. A significant advancement came with Gutman Sombor index (2021), which defined a new vertex-degree-based graph invariant, denoted by $SO(G)$ and is defined by

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d(u)^2 + d(v)^2}$$

Where $d(u)$ and $d(v)$ are the degrees of adjacent vertices u and v, respectively [1].

Inspired by the mathematical elegance and practical utility of Gutman's Sombor index (SO), we propose the Euclidean Reciprocal Index (ERI) as a novel advancement in graph-theoretical characterization. The ERI of a graph G is formally defined as the sum over all edges $uv \in E(G)$, of the square roots of the reciprocal squared degrees of adjacent vertices.

$$\mathrm{ERI(G)} = \sum_{\mathrm{uv} \in \mathrm{E(G)}} \sqrt{\frac{1}{\mathrm{d(u)}^2} + \frac{1}{\mathrm{d(v)}^2}}$$

Where $\mathrm{E(G)}$ is the edge set of G, $\mathrm{d(u)}$ and $\mathrm{d(v)}$ denote the degrees of adjacent vertices u and v, respectively.

The Multiplicative approach in the topological index provides a completely distinct point of view from which the structure of molecules and networks can be analyzed; however, it has not been studied extensively. The multiplicative indices generally show an exponential relationship with respect to the size of the network and can complement additive indices, especially for large-scale networks, as the multiplication will be more affected by structural variations. In this study, we examined the multiplicative Sombor and Euclidean Reciprocal indices as well as their coindices in the same set of networks.

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2. Main Results

Definition 1.1. The Multiplicative Euclidean Reciprocal Index ($mERI$) of a graph G is defined as the product over all edges of the Euclidean norm of the reciprocal degree vector.

$$mERI(G) = \prod_{uv \in E(G)} \sqrt{\frac{1}{d(u)^2} + \frac{1}{d(v)^2}}$$

Where E(G) is the edge set of G, d(u) and d(v) denote the degrees of adjacent vertices u and v, respectively.

Definition 1.2. The Multiplicative Euclidean Reciprocal coindex $m\overline{ERI}(G)$ is defined as the product of all non-adjacent vertex pairs of the Euclidean norm of the reciprocal degree vector

$$m\overline{ERI}(G) = \prod_{uv \notin E(G)} \sqrt{\frac{1}{d(u)^2} + \frac{1}{d(v)^2}}$$

Where E(G) is the edge set of G, d(u) and d(v) denote the degrees of non-adjacent vertices u and v, respectively.

Theorem 1.3. For the hexagonal network HX_n ($n \geq 2$), then

$mSO(HX_n) = (2^{11} \times 3^7 \times 5)\sqrt{65}(n-3)(n-2)^2(3n-5)$

$m\overline{SO}(HX_n) = (2^{10} \times 3^5 \times 5^2 \times 7)\sqrt{130}\,(3n-7)(n-1)(n-2)^3(6n^2-27n+32)(3n^2-9n+5)(3n^3-12n^2+10n+3)$

$mERI(HX_n) = \frac{15\sqrt{65}}{2}(n-3)(n-2)^2(3n-5)$

$m\overline{ERI}(HX_n) = \frac{225\sqrt{130}}{64}\,(3n-7)(n-1)(n-2)^3(6n^2-27n+32)(3n^2-9n+5)(3n^3-12n^2+10n+3)$

Proof. Let $u_1, u_2, \ldots, u_n$ be vertices of HX_n. It has $3n^2-3n+1$ vertices and $9n^2-15n+6$ edges. There are 6 vertices of degree 3, $6n-12$ vertices of degree 4, and $3n^2-9n+7$ vertices of degree 6.

Multiplicative Sombor coindex of a hexagonal network is given by

$mSO(HX_n) = 12\sqrt{3^2+4^2} \times 6\sqrt{3^2+6^2} \times 6(n-3)\sqrt{4^2+4^2} \times 12(n-2)\sqrt{4^2+6^2} \times (9n^2-33n+30)\sqrt{6^2+6^2}$

$= (2^{11} \times 3^7 \times 5)\sqrt{65}(n-3)(n-2)^2(3n-5)$

Multiplicative Sombor coindex of a hexagonal network is given by

$m\overline{SO}(HX_n) = \left(\binom{6}{2} - 0\right)\sqrt{3^2+3^2} \times (6(6n-12)-12)\sqrt{3^2+4^2} \times (6(3n^2-9n+7)-6)\sqrt{3^2+6^2} \times \left(\binom{6n-12}{2} - 6(n-3)\right)\sqrt{4^2+4^2} \times \left((6n-12)(3n^2-9n+7) - 12(n-2)\right)\sqrt{4^2+6^2} \times \left(\binom{3n^2-9n+7}{2} - (9n^2-33n+30)\right)\sqrt{6^2+6^2}$

$= (2^{10} \times 3^5 \times 5^2 \times 7)\sqrt{130}\,(3n-7)(n-1)(n-2)^3(6n^2-27n+32)(3n^2-9n+5)(3n^3-12n^2+10n+3)$

The multiplicative Euclidean reciprocal index of a hexagonal network is given by

$mERI(HX_n) = 12\sqrt{\frac{1}{3^2}+\frac{1}{4^2}} \times 6\sqrt{\frac{1}{3^2}+\frac{1}{6^2}} \times 6(n-3)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}} \times 12(n-2)\sqrt{\frac{1}{4^2}+\frac{1}{6^2}} \times (9n^2-33n+30)\sqrt{\frac{1}{6^2}+\frac{1}{6^2}}$

$= \frac{15\sqrt{65}}{2}(n-3)(n-2)^2(3n-5)$

Multiplicative Euclidean Reciprocal coindex of hexagonal network HX_n is given by

$m\overline{ERI}(HX_n) = \left(\binom{6}{2} - 0\right)\sqrt{\frac{1}{3^2}+\frac{1}{3^2}} \times (6(6n-12)-12)\sqrt{\frac{1}{3^2}+\frac{1}{4^2}} \times (6(3n^2-9n+7)-6)\sqrt{\frac{1}{3^2}+\frac{1}{6^2}} \times \left(\binom{6n-12}{2} - 6(n-3)\right)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}} \times \left((6n-12)(3n^2-9n+7) - 12(n-2)\right)\sqrt{\frac{1}{4^2}+\frac{1}{6^2}} \times \left(\binom{3n^2-9n+7}{2} - (9n^2-33n+30)\right)\sqrt{\frac{1}{6^2}+\frac{1}{6^2}}$

$= \frac{225\sqrt{130}}{64}(3n-7)(n-1)(n-2)^3(6n^2-27n+32)(3n^2-9n+5)(3n^3-12n^2+10n+3)$

Example 1.4. Figure 1 shows a hexagonal network, HX_6, $mSO(HX_6) = 2^{15}\times 3^8\times 5\times 13\times\sqrt{65}$, $m\overline{SO}(HX_6) = 2^{17}\times 3^7\times 5^3\times 7\times 11\times 31\times 43\times 59.\sqrt{130}$, $mERI(HX_6) = 2^3\times 3^2\times 5\times 13\times\sqrt{65}$ and $m\overline{ERI}(HX_6) = 2\times 3^4\times 5^3\times 11\times 31\times 43\times 59\times\sqrt{130}$

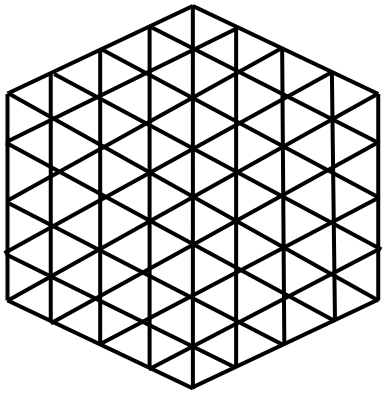


Fig. 1 Hexagonal Network

Theorem 1.5. Let OX_n denote the oxide network of dimension n, then

$mSO(OX_n) = 576n^2(3n-2)\sqrt{10}$
$m\overline{SO}(OX_n) = 2592\sqrt{5}n^3(6n-1)(3n-2)(3n+1)(9n^3-6n^2-4n+3)$
$mERI(OX_n) = \frac{9}{2}n^2(3n-2)\sqrt{10}$
$m\overline{ERI}(OX_n) = \frac{9n^3\sqrt{5}}{16}(6n-1)(3n-2)(3n+1)(81n^3-54n^2-36n+27)$

Proof. Let $u_1, u_2, \dots, u_n$ be vertices of OX_n ,Oxide network has $9n^2+3n$ vertices and$18n^2$ edges. There are $6n$ vertices of degree 2 and $9n^2-3n$ vertices of degree 4.
Multiplicative Sombor index of oxide network is given by
$mSO(OX_n) = 12\text{n}\sqrt{2^2+4^2}\times 6n(3n-2)\sqrt{4^2+4^2}$
$= 576n^2(3n-2)\sqrt{10}$

Multiplicative Sombor coindex of oxide network is given by
$mSO(OX_n) = \left(\binom{6n}{2}-0\right)\sqrt{2^2+2^2}\times(6\text{n}(9\text{n}^2-3n)-12n)\sqrt{2^2+4^2}\times\left(\binom{9n^2-3n}{2}-6n(3n-2)\right)\sqrt{4^2+4^2}$
$= 2592\sqrt{5}n^3(6n-1)(3n-2)(3n+1)(9n^3-6n^2-4n+3)$

The multiplicative Euclidean reciprocal index of the oxide network is given by
$mERI(OX_n) = 12\text{n}\sqrt{\frac{1}{2^2}+\frac{1}{4^2}}\times 6n(3n-2)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$
$= \frac{9}{2}n^2(3n-2)\sqrt{10}$
The multiplicative Euclidean reciprocal coindex of the oxide network is given by
$m\overline{ERI}(OX_n) = \left(\binom{6n}{2}-0\right)\sqrt{\frac{1}{2^2}+\frac{1}{2^2}}\times(6\text{n}(9\text{n}^2-3n)-12n)\sqrt{\frac{1}{2^2}+\frac{1}{4^2}}\times\left(\binom{9n^2-3n}{2}-6n(3n-2)\right)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$
$= \frac{9n^3\sqrt{5}}{16}(6n-1)(3n-2)(3n+1)(81n^3-54n^2-36n+27)$

Example 1.6. Figure 2 shows the oxide network OX_5, $mSO(OX_5) = 2^6\times 3^2\times 5^2\times 13\times\sqrt{10}$, $m\overline{SO}(OX_5) = 2^{10}\times 3^4\times 5^3\times 13\times 29\times 479\times\sqrt{5}$, $mERI(OX_5) = \frac{2925}{2}\sqrt{10}$ and $m\overline{ERI}(OX_5) = 2\times 3^4\times 5^3\times 13\times 29\times 479\times\sqrt{5}$.

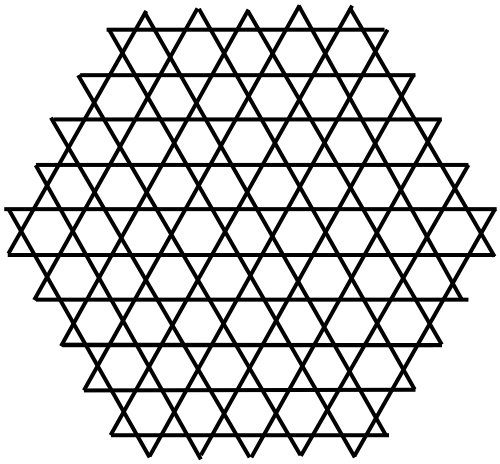


Fig. 2 Oxide Network

Theorem 1.7. The rhombus oxide network ($RHOX_n$) of dimension n has

$mSO(RHOX_n) = 512\sqrt{5}(n-1)(2n-1)(3n-1)$

$m\overline{SO}(RHOX_n) = 128\sqrt{5}(n-1)^2(4n^2-n-1)(3n^2+n-1)(9n^3-3n^2-14n+4)$

$mERI(RHOX_n) = (n-1)(2n-1)(3n-1)\sqrt{5}$

$$m\overline{ERI}(RHOX_n) = \frac{\sqrt{5}}{8}(4n^2-n-1)(3n^3-2n^2-2n+1)(9n^4-12n^3-11n^2+18n-4)$$

Proof. Let $u_1, u_2, \ldots, u_n$ be vertices of OX_n . Oxide network has $3n^2+2n$ vertices and $6n^2$ edges. There are $4n$ vertices of degree 2 and $3n^2-2n$ vertices of degree 4.

Multiplicative Sombor index of rhombus oxide network is given by

$mSO(RHOX_n) = 2\sqrt{2^2+2^2} \times 4(2n-1)\sqrt{2^2+4^2} \times (6n^2-8n+2)\sqrt{4^2+4^2}$

$= 512\sqrt{5}(n-1)(2n-1)(3n-1)$

Multiplicative Sombor coindex of rhombus oxide network is given by

$m\overline{SO}(RHOX_n) = \left(\binom{4n}{2}-2\right)\sqrt{2^2+2^2} \times \left(4n(3n^2-2n)-4(2n-1)\right)\sqrt{2^2+4^2} \times \left(\binom{3n^2-2n}{2}-(6n^2-8n+2)\right)\sqrt{4^2+4^2}$

$= 128\sqrt{5}(n-1)^2(4n^2-n-1)(3n^2+n-1)(9n^3-3n^2-14n+4)$

The multiplicative Euclidean reciprocal index of the rhombus oxide network is given by

$mERI(RHOX_n) = 2\sqrt{\frac{1}{2^2}+\frac{1}{2^2}} \times 4(2n-1)\sqrt{\frac{1}{2^2}+\frac{1}{4^2}} \times (6n^2-8n+2)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$

$= (n-1)(2n-1)(3n-1)\sqrt{5}$

The multiplicative Euclidean reciprocal coindex of the rhombus oxide network is given by

$$m\overline{ERI}(RHOX_n) = \left(\binom{4n}{2}-2\right)\sqrt{\frac{1}{2^2}+\frac{1}{2^2}} \times \left(4n(3n^2-2n)-4(2n-1)\right)\sqrt{\frac{1}{2^2}+\frac{1}{4^2}} \times \left(\binom{3n^2-2n}{2}-(6n^2-8n+2)\right)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$$

$$= \frac{\sqrt{5}}{8}(4n^2-n-1)(3n^3-2n^2-2n+1)(9n^4-12n^3-11n^2+18n-4)$$

Example 1.8. Figure 3 shows rhombus oxide network $RHOX_3$, $mSO(RHOX_3) = 2^{13} \times 5 \times \sqrt{5}$, $m\overline{SO}(RHOX_3) = 2^{15} \times 29 \times 89 \times \sqrt{5}$, $mERI(RHOX_3) = 2^4 \times 5 \times \sqrt{5}$ and $m\overline{ERI}(RHOX_3) = 2^5 \times 29 \times 89 \times \sqrt{5}$.

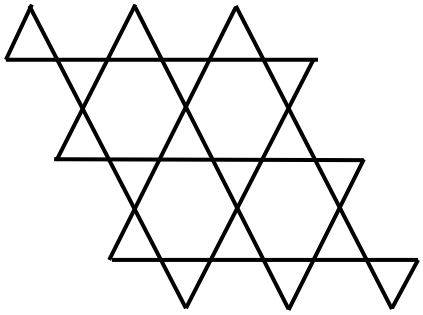


Fig. 3 Rhombus Oxide Network

Theorem 1.9. Let SL_n denote the silicate network of dimension n, then

$mSO(SL_n) = 23328n^3(3n+1)(3n-2)\sqrt{5}$

$m\overline{SO}(SL_n) = 972\sqrt{5}n^3(6n^3+12n^2+5n-3)(9n^3+6n^2-6n-1)(81n^3-54n^2-36n+27)$

$mERI(SL_n) = 4n^3(3n+1)(3n-2)\sqrt{5}$

$m\overline{ERI}(SL_n) = \frac{\sqrt{5}}{2}n^3(6n^3+12n^2+5n-3)(27n^3-18n^2-12n+9)(9n^3+6n^2-6n-1)$

Proof. Silicate network has $15n^2+3n$ vertices and $36n^2$ edges. There are $6n^2+6n$ vertices of degree 3 and $9n^2-3n$ vertices of degree 6.

Multiplicative Sombor index of silicate network is given by

$mSO(SL_n) = 6n\sqrt{3^2+3^2} \times 6n(3n+1)\sqrt{3^2+6^2} \times 6n(3n-2)\sqrt{6^2+6^2}$

$= 23328n^3(3n+1)(3n-2)\sqrt{5}$

Multiplicative Sombor coindex of silicate network is given by

$m\overline{SO}(SL_n) = \left(\binom{6n^2+6n}{2} - 6n\right)\sqrt{3^2+3^2} \times \left((6n^2+6n)(9n^2-3n) - 6n(3n+1)\right)\sqrt{3^2+6^2} \times \left(\binom{9n^2-3n}{2} - 6n(3n-2)\right)\sqrt{6^2+6^2}$

$= 972\sqrt{5}n^3(6n^3+12n^2+5n-3)(9n^3+6n^2-6n-1)(81n^3-54n^2-36n+27)$

Multiplicative Euclidean Reciprocal index of silicate network SL_n is given by

$mERI(SL_n) = 6n\sqrt{\frac{1}{3^2}+\frac{1}{3^2}} \times 6n(3n+1)\sqrt{\frac{1}{3^2}+\frac{1}{6^2}} \times 6n(3n-2)\sqrt{\frac{1}{6^2}+\frac{1}{6^2}}$

$= 4n^3(3n+1)(3n-2)\sqrt{5}$

The multiplicative Euclidean reciprocal coindex of the silicate network is given by

$m\overline{ERI}(SL_n) = \left(\binom{6n^2+6n}{2} - 6n\right)\sqrt{\frac{1}{3^2}+\frac{1}{3^2}} \times \left((6n^2+6n)(9n^2-3n) - 6n(3n+1)\right)\sqrt{\frac{1}{3^2}+\frac{1}{6^2}} \times \left(\binom{9n^2-3n}{2} - 6n(3n-2)\right)\sqrt{\frac{1}{6^2}+\frac{1}{6^2}}$

$= \frac{\sqrt{5}}{2}n^3(6n^3+12n^2+5n-3)(27n^3-18n^2-12n+9)(9n^3+6n^2-6n-1)$

Example 1.10. Figure 4 shows Silicate Network SL_2, $mSO(SL_2) = 2^{10} \times 3^6 \times 7 \times \sqrt{5}$, $m\overline{SO}(SL_2) = 2^5 \times 3^7 \times 43 \times 83 \times 103 \times \sqrt{5}$, $mERI(SL_2) = 2^7 \times 7 \times \sqrt{5}$, and $m\overline{ERI}(SL_2) = 2^2 \times 3 \times 43 \times 83 \times 103 \times \sqrt{5}$.

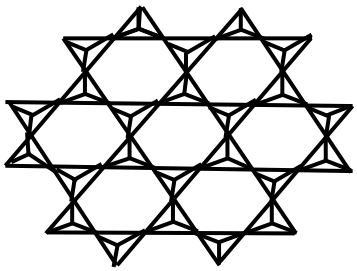


Fig. 4 Silicate Network

Theorem 1.11. Let $G_{p,q}$ denote the graphene network of dimension n has

$mSO(G_{p,q}) = 24\sqrt{13}(p+4)(p+2q-2)(3pq+2p+2q-1)$

$m\overline{SO}(G_{p,q}) = 24\sqrt{13}(2p^2+2q^2+4pq+2p+3q-3)(18p^2q+18pq^2+18pq-19p-20q-16)(2p^2q^2-8pq+2p+q+4)$

$mERI(G_{p,q}) = \frac{\sqrt{13}}{9}(p+4)(p+2q-2)(3pq+2p+2q-1)$

$m\overline{ERI}(G_{p,q}) = \frac{\sqrt{13}}{9}(2\mathrm{p}^2+4\mathrm{pq}+2\mathrm{q}^2+2\mathrm{p}+3\mathrm{q}-3)(18p^2q+18pq^2+18pq-19p-20q-16)(2p^2q^2-8pq+2p+q+4)$

Proof. Graphene network has $2pq+2p+2q$ vertices and $3pq+2p+2q-1$ edges, $p,q>1$. There are $2p+2q+2$ vertices of degree 2 and $2pq-2$ vertices of degree 3.

Multiplicative Sombor index of the graphene network is given by

$$mSO(G_{p,q}) = (\mathrm{p}+4)\sqrt{2^2+2^2}\times(2p+4q-4)\sqrt{2^2+3^2}\times 3pq+2p+2q-1\sqrt{3^2+3^2}$$

$= 24\sqrt{13}(p+4)(p+2q-2)(3pq+2p+2q-1)$

Multiplicative Sombor coindex of the graphene network is given by

$m\overline{SO}(G_{p,q}) = \left(\binom{2p+2q+2}{2}-(\mathrm{p}+4)\right)\sqrt{2^2+2^2}\times\left(9(2p+2q+2)(2pq-2)-(2p+4q-4)\right)\sqrt{2^2+3^2}\times\left(\binom{2pq-2}{2}-(3pq-2p-q-1)\right)\sqrt{3^2+3^2}$

$= 24\sqrt{13}(2p^2+2q^2+4pq+2p+3q-3)(18p^2q+18pq^2+18pq-19p-20q-16)(2p^2q^2-8pq+2p+q+4)$

The multiplicative Euclidean reciprocal index of the graphene network is given by

$mERI(G_{p,q}) = (\mathrm{p}+4)\sqrt{\frac{1}{2^2}+\frac{1}{2^2}}\times(2p+4q-4)\sqrt{\frac{1}{2^2}+\frac{1}{3^2}}\times 3pq+2p+2q-1\sqrt{\frac{1}{3^2}+\frac{1}{3^2}}$

$= \frac{\sqrt{13}}{9}(p+4)(p+2q-2)(3pq+2p+2q-1)$

The multiplicative Euclidean reciprocal coindex of the graphene network is given by

$m\overline{ERI}(G_{p,q}) = \left(\binom{2p+2q+2}{2}-(\mathrm{p}+4)\right)\sqrt{\frac{1}{2^2}+\frac{1}{2^2}}\times\left(9(2p+2q+2)(2pq-2)-(2p+4q-4)\right)\sqrt{\frac{1}{2^2}+\frac{1}{3^2}}\times\left(\binom{2pq-2}{2}-(3pq-2p-q-1)\right)\sqrt{\frac{1}{3^2}+\frac{1}{3^2}}$

$= \frac{\sqrt{13}}{9}(2\mathrm{p}^2+4\mathrm{pq}+2\mathrm{q}^2+2\mathrm{p}+3\mathrm{q}-3)(18p^2q+18pq^2+18pq-19p-20q-16)(2p^2q^2-8pq+2p+q+4)$

Example 1.12. Figure 5 shows a graphene network $G_{4,3}$, $mSO(G_{4,3}) = 2^9\times 3^1\times 7^2\times\sqrt{13}$, $m\overline{SO}(G_{4,3}) = 2^{10}\times 3^3\times 7^1\times 23^1\times 197^1\times\sqrt{13}$, $mERI(G_{4,3}) = 2^6\times 7^2\times\frac{1}{3^2}\times\sqrt{13}$ and $m\overline{ERI}(G_{4,3}) = 2^7\times 7^1\times 23^1\times 197\times\sqrt{13}$

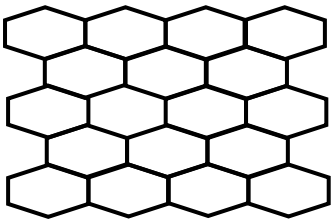


Fig. 5 Graphene Network

Theorem 1.13. The prism network $P_m \times C_n$ with dimension m *and* n has

$mSO(P_m \times C_n) = 480n^3(2m-5)$

$m\overline{SO}(P_m \times C_n) = 240n^3(2n-3)(2n(m-2)-1)[(m-2)(2n(m-2)-1)-(2m-5)]$

$mERI(P_m \times C_n) = \frac{5}{18}n^3(2m-5)$

$m\overline{ERI}(P_m \times C_n) = \frac{5n^3}{72}(2n-3)(2n(m-2)-1)[(m-2)(2n(m-2)-1)-(2m-5)]$

Proof. The Prism network has $2n(m-1)$ vertices and $3n$ edges. There are $2n$ vertices of degree 3 and $2n(m-2)$ vertices of degree 4.

Multiplicative Sombor index of the prism network $P_m \times C_n$ is given by

$mSO(P_m \times C_n) = 2n\sqrt{3^2+3^2} \times 2n\sqrt{3^2+4^2} \times (2mn-5n)\sqrt{4^2+4^2}$

$= 480n^3(2m-5)$

Multiplicative Sombor coindex of the prism network is given by $P_m \times C_n$,

$m\overline{SO}(P_m \times C_n) = \left(\binom{2n}{2} - 2\mathrm{n}\right)\sqrt{3^2+3^2} \times \left((2n)(2n(m-2)) - 2\mathrm{n}\right)\sqrt{3^2+4^2} \times \left(\binom{2n(m-2)}{2} - (2mn-5n)\right)\sqrt{4^2+4^2}$

$= 240n^3(2n-3)(2n(m-2)-1)[(m-2)(2n(m-2)-1)-(2m-5)]$

The multiplicative Euclidean reciprocal index of the prism network is given by

$mERI(P_m \times C_n) = 2\mathrm{n}\sqrt{\frac{1}{3^2}+\frac{1}{3^2}} \times 2\mathrm{n}\sqrt{\frac{1}{3^2}+\frac{1}{4^2}} \times (2mn-5n)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$

$= \frac{5}{18}n^3(2m-5)$

The multiplicative Euclidean reciprocal coindex of the prism network is given by

$$m\overline{ERI}(P_m \times C_n) = \left(\binom{2n}{2} - 2\mathrm{n}\right)\sqrt{\frac{1}{3^2}+\frac{1}{3^2}} \times \left((2n)(2n(m-2)) - 2\mathrm{n}\right)\sqrt{\frac{1}{3^2}+\frac{1}{4^2}} \times \left(\binom{2n(m-2)}{2} - (2mn-5n)\right)\sqrt{\frac{1}{4^2}+\frac{1}{4^2}}$$

$$= \frac{5n^3}{72}(2n-3)(2n(m-2)-1)[(m-2)(2n(m-2)-1)-(2m-5)]$$

Example 1.14. Figure 6 shows a prism network $P_3 \times C_6$, $mSO(P_3 \times C_6) = 2^8 \times 3^4 \times 5$, $m\overline{SO}(P_3 \times C_6) = 2^8 \times 3^6 \times 5^2 \times 11, mERI(P_3 \times C_6) = 60$ *and* $m\overline{ERI}(P_3 \times C_6) = 2 \times 3^3 \times 5^2 \times 11$

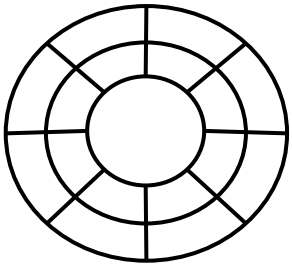


Fig. 6 Prism Network

3. Conclusion

In this paper, we derived closed-form expressions for the Multiplicative Sombor index (mSO), its coindex ($m\bar{S}O$), Multiplicative Euclidean Reciprocal index ($mERI$), and its coindex ($m\bar{E}RI$) for diverse networks. The Multiplicative indices exhibit exponential growth with network size, in stark contrast to the polynomial growth of their additive counterparts. This extreme sensitivity makes multiplicative indices particularly useful for distinguishing large networks and detecting small structural changes, such as defects or edge effects in nanomaterials.

References

[1] Ivan Gutman, "Geometric Approach to Degree–Based Topological Indices: Sombor Indices," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 86, no. 1, pp. 11-16, 2021. [Google Scholar] [Publisher Link]

[2] Ghulam Dustigeer et al., "On Multiplicative Degree-Based Topological Indices for Planar Octahedron Networks," *Main Group Metal Chemistry*, vol. 43, no. 1, pp. 219-228, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[3] Frank Harary, *Graph Theory*, Narosa Publishing House, New Delhi, 1988. [Google Scholar] [Publisher Link]

[4] Batmend Horoldagvaa, and Chunlei Xu, "On Sombor Index of Graphs," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 86, no. 3, pp. 703-713, 2021. [Google Scholar] [Publisher Link]

[5] Sakander Hayat, and Muhammad Imran, "Computation of Topological Indices of Certain Networks," *Applied Mathematics and Computation*, vol. 240, pp. 213-228, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[6] V.R. Kulli and Ivan Gutman, "Computation of Sombor Indices of Certain Networks," *International Journal of Applied Chemistry*, vol. 8, no. 1, pp. 1-5, 2021. [CrossRef] [Google Scholar] [Publisher Link]