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Original Article

A Dynamic MILP Framework for Multi-Stage Food-Grain Supply Chain Optimization: A Case Study of Jharkhand's Public Distribution System

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Abstract *- Food-grain supply chains involve several activities such as procurement, processing, transportation, storage, and distribution. In the Public Distribution System (PDS) of India, these activities are closely connected because procured paddy has to be processed in rice mills before the milled rice can be stored and distributed to different demand locations. This study develops a dynamic Mixed-Integer Linear Programming (MILP) model for a four-stage food-grain supply chain consisting of procurement centres, rice mills, warehouses, and PDS distribution points. The model considers milling recovery, period-wise milling capacity, procurement availability, route activation, warehouse capacity, inventory balance, demand fulfilment, opportunity cost, and shortage penalty. Transportation costs are calculated from inter-node distances using the applicable Jharkhand Government transportation-rate structure.*

The model is tested through computational experiments using the East Singhbhum network and is further examined using observed procurement-stage data and a calibrated downstream representation of the Seraikela-Kharsawan district network. For the East Singhbhum simulation, the total system cost decreases from INR 76,102,850.08 under the baseline allocation to INR 75,105,464.62 with the proposed MILP, giving a reduction of 1.31%. Sensitivity analysis further shows that PDS demand and transportation cost have relatively strong effects on total system cost, while paddy availability, milling capacity, and milling recovery affect both cost and systemic shortage over the tested ranges. For the Seraikela-Kharsawan network, the calibrated validation gives a reduction in total system cost from INR 7,813,080.15 to INR 7,114,689.99, corresponding to 8.94%, while maintaining the prescribed paddy throughput and zero shortage. The Stage-1 transportation calculation was also checked using the Modified Vogel's Approximation Method (VAM), with a difference of 0.84% between the manual and MATLAB-based calculations. The results indicate that the proposed model can be used to evaluate food-grain flows, processing capacity, inventory, transportation expenditure, and PDS demand fulfilment in a multi-stage network.

Keywords *- Rice milling, Public Distribution System, Transportation cost, Inventory management, Supply chain network, Jharkhand.*

1. Introduction

Food-grain supply chains are closely connected with food security because they link procurement, processing, storage, transportation, and final distribution. This is particularly relevant in Jharkhand, where food-security programmes depend on the proper functioning of procurement agencies, storage facilities, transport arrangements, and the Public Distribution System (PDS). The Jharkhand Economic Survey 2022–23 reports activities related to food-grain production and procurement, stocking, storage management, and PDS distribution. At the national level, the Food Corporation of India (FCI) is involved in procurement, storage, movement, and distribution of foodgrains. Thus, food-grain management involves a chain of connected decisions rather than only a transportation problem (Jharkhand Department of Planning and Development 2023; Food Corporation of India 2023) [9, 7].

India's PDS operates over a large geographical area, so decisions on procurement, processing, transportation, storage, inventory, and demand have to be considered together. Mathematical programming has been used for many of these decisions, including food-grain storage, transportation, allocation, and network design (Dantzig 1963; Bazaara et al. 2010; Taha 2017) [5,

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2, 26]. Indian studies have also examined storage and transportation for the PDS, shipment-cost minimization, transportation-allocation, and inventory-transportation planning (Asgari et al. 2013; Tanksale and Jha 2014, 2016; Mogale et al. 2016, 2017, 2018a, 2018b, 2020) [1, 19-23, 27, 28].

The Indian food-grain chain also has a processing stage that cannot be ignored. Paddy procured at procurement centres is sent to rice mills, converted into milled rice, moved to storage facilities, and then supplied to PDS demand locations. The main stages considered in this study are therefore:

Procurement Centres → Rice Mills → Warehouses → PDS Distribution Points.

Different parts of this chain have already been studied. Mogale et al. (2016) [21], for example, considered a two-stage transportation-allocation problem, while Mogale et al. (2017) [22] dealt with multi-period inventory transportation. Mogale et al. (2018b) [23] considered movement and storage decisions through an MINLP model. These studies provide useful foundations, but the physical conversion of paddy into milled rice and its connection with downstream warehousing and PDS distribution are not represented together in the same multi-period MILP formulation.

The milling stage creates a direct material-balance issue because paddy and milled rice are not equal quantities. Only a part of the paddy entering a mill is obtained as milled rice, and the processing quantity is also limited by the capacity of the mill. Rice-mill planning has been studied in terms of resource use, processing routes, product portfolios, and internal planning (Lim et al. 2013; Lim et al. 2014) [11, 12]. The present model connects this processing stage with procurement and downstream PDS operations by using a milling recovery factor and period-specific milling-capacity constraints.

Transportation cost is another practical issue. In public-sector food-grain movement, the applicable rate can depend on the distance slab rather than on one fixed average rate. The Jharkhand government provides a transportation-rate structure for food-grain logistics. In this study, the distance between two connected nodes is converted into the applicable transportation cost using the government rate structure (Jharkhand State Food & Civil Supplies Corporation 2024) [10]. This makes the cost coefficients directly related to the distances in the network.

Post-Harvest Loss (PHL) is also relevant because losses can occur during handling, transportation, storage, and processing. Chandrasekaran and Ranganathan (2017) [4] examined optimization of an agricultural supply chain with PHL and carbon-emission considerations, while Nath et al. (2024) [24] reviewed technologies and practices for reducing grain post-harvest losses.

A recent study by Mahato et al. (2026) [15] examined food-grain supply-chain optimization using a nonlinear programming model with post-harvest loss, quality deterioration, and sensitivity analysis. The present model does not introduce PHL or quality deterioration as separate decision variables. Instead, the physical conversion from paddy to milled rice is represented through the milling recovery factor and the material-balance constraint. This distinction is important when comparing the present formulation with the authors' earlier work.

The time dimension is also important because procurement, milling, storage, and PDS distribution may take place in different periods. Multi-period inventory and transportation models have been developed for food-grain systems (Tanksale and Jha 2016; Mogale et al. 2017) [28, 22]. In this study, flows and inventory are indexed by planning period. Inventory carried from one period to the next is linked through an inventory-balance equation, while warehouse capacity and period-wise demand are imposed in the corresponding periods.

The objective function includes opportunity cost and shortage penalty in addition to direct logistics costs. Opportunity cost represents the economic effect associated with processing and movement decisions, while the shortage penalty assigns a cost to unmet PDS demand. Du et al. (2022) [6] also considered economic effects beyond direct transportation cost in grain-flow optimization. The objective used here, therefore, combines procurement-to-mill transportation, mill-to-warehouse transportation, warehouse-to-PDS transportation, milling cost, opportunity cost, inventory holding cost, and shortage penalty.

The literature shows that many useful models are available for food-grain transportation, storage, procurement, processing, and inventory planning. However, there is still a need for a formulation that connects the four stages used in this study and at the same time accounts for milling recovery, period-specific milling capacity, distance-based government transportation rates, inventory movement, warehouse capacity, route activation, opportunity cost, and shortage penalty. This study addresses that requirement through a Dynamic MILP formulation.

The main contributions are as follows. First, a four-stage food-grain supply-chain model is developed for procurement centres, rice mills, warehouses, and PDS demand points over multiple periods. Second, milling recovery and period-specific mill capacity are included so that paddy processing is linked with the actual availability of milled rice. Third, transportation costs are obtained from inter-node distances using the applicable Jharkhand government rate structure. Fourth, transportation, milling, opportunity, inventory-holding, and shortage costs are considered together in the objective function. The model is tested through East Singhbhum computational experiments, sensitivity analysis, Seraikela-Kharsawan real-data-driven calibrated validation, and a separate Modified VAM check of the Stage-1 transportation calculation.

The work is part of a continuing study of food-grain supply-chain optimization in Jharkhand. Mahato and Keshri (2025a) [13] developed an LP-based model for food-grain storage and transportation costs, while Mahato and Keshri (2025b) [14] reviewed optimization approaches for grain supply chains. Procurement-centre-to-mill transportation for Seraikela-Kharsawan was examined separately by Mahato and Keshri (2025c) [16]. Mahato et al. (2026) [15] later studied post-harvest loss and quality deterioration through a nonlinear programming model. The present article uses these studies as background but takes a different modelling route: it develops a dynamic MILP that links procurement, milling, warehousing, and PDS distribution in one multi-period network and includes binary route activation, inventory, warehouse capacity, opportunity cost, shortage penalty, and distance-based government transportation costs.

2. Literature Review

2.1. Mathematical Optimization in Food-Grain Supply Chains

Mathematical optimization is widely used in food-grain supply chains for transportation, storage, inventory allocation, procurement, and network design. Linear programming is useful when the decisions and constraints are continuous, while mixed-integer and network-flow models can also represent route or facility decisions (Dantzig 1963; Bazaara et al. 2010; Taha 2017) [5, 2, 26].

For food-grain systems, these methods are useful because procurement, transportation, storage, and distribution decisions affect one another. Asgari et al. (2013) [1] developed a model-based approach for wheat storage and transportation. Maiyar and Thakkar (2015) [17] studied cost minimization for food-grain shipments, and Chakraborty et al. (2020) [3] examined food-grain inventory transportation in India through an MILP model supported by heuristic procedures.

These studies show the usefulness of optimization for food-grain logistics. At the same time, many of them deal with selected stages or particular transportation and inventory decisions instead of following the complete physical flow from procurement and processing to storage and PDS distribution.

2.2. Food-Grain Procurement, Transportation, Storage, and Multi-Stage Network Models

Indian food-grain supply chains have received attention because procurement, storage, and distribution are central to the PDS. Tanksale and Jha (2014) [27] developed a model for food-grain storage and transport in the Indian PDS. Tanksale and Jha (2016) [26] later considered a multi-period, multi-foodgrain inventory transportation problem.

Mogale et al. (2016) [21] studied a two-stage transportation-allocation problem for Indian food grains. Mogale et al. (2017) [22] extended the planning to multiple periods. Mogale et al. (2018a) [20] considered silo location and allocation with dwell time, while Mogale et al. (2018b) [23] used an MINLP formulation for movement and storage decisions. Mogale et al. (2020) [19] examined food-grain procurement through a supply-chain network model.

The above work shows a gradual move from transportation-allocation models to multi-period and network-based formulations. The complete sequence from paddy procurement, rice milling, storage, and PDS distribution is still less commonly represented as one integrated model.

2.3. Processing, Milling Capacity, and Material Conversion

Rice milling creates a physical conversion between the input and output quantities. The quantity of paddy received by a mill is therefore not the same as the quantity of milled rice available for the next stage.

Lim et al. (2013) [11] developed a multi-period mathematical model for the optimal planning of an integrated, resource-efficient rice mill. Their framework considered resource allocation, processing routes, product portfolios, fluctuating product demands, and resource availability within a multi-period optimization framework. Lim et al. (2014) [12] subsequently developed a mathematical framework for synthesizing a sustainable integrated rice-mill complex, considering resource planning, product portfolios, economic performance, and environmental aspects.

These studies show how optimization can be used inside rice-mill operations. Their main concern, however, is mill planning, resource use, process configuration, and sustainability rather than linking the mill with procurement, warehouse inventory, and PDS distribution.

For a food-grain supply-chain model, the milling stage has to be connected with both the upstream paddy flow and the downstream rice flow. A recovery factor and a mill-capacity limit are therefore needed to keep the material balance physically meaningful.

2.4. Post-Harvest Loss, Storage, and Food-Grain Deterioration

Post-harvest loss (PHL) can reduce the quantity of grain available for consumption and can increase the effective cost of supply-chain operations. Losses may occur during harvesting, handling, transport, storage, processing, and other post-harvest activities.

Chandrasekaran and Ranganathan (2017) [4] developed an optimization framework for an Indian agricultural supply chain with PHL and carbon-emission objectives. Nath et al. (2024) [24] reviewed technologies and operational measures for reducing grain post-harvest losses. Mahato et al. (2026) [15] examined PHL and quality deterioration in a nonlinear food-grain supply-chain model, together with sensitivity analysis.

Government evidence also demonstrates the continuing relevance of post-harvest losses in agricultural supply chains. The Ministry of Food Processing Industries, based on the NABCONS study commissioned in 2022, reported post-harvest loss estimates for several agricultural commodities. For cereals, the reported estimate from the NABCONS study was 3.89–5.92%, indicating that post-harvest losses remain relevant to food-grain systems (Ministry of Food Processing Industries 2022) [18].

The literature, therefore, treats PHL as a problem that can occur at more than one stage. In an integrated model, material balances have to remain consistent from procurement to processing, storage, transport, and distribution. In the present work, PHL is kept as an operational background issue rather than a separate decision variable, while paddy-to-rice conversion is controlled through the milling recovery factor.

2.5. Transportation Cost, Distance-Based Rates, and Opportunity Cost

Transportation cost is usually a major part of food-grain logistics expenditure. Many models use a fixed cost coefficient for each origin-destination pair, but public-sector logistics can use rates that change with distance.

Maiyar and Thakkar (2015) [17] examined cost minimization in food-grain shipments, highlighting the importance of transportation allocation in reducing logistics expenditure. In the Indian PDS context, transportation and handling arrangements are also associated with prescribed rates and operational conditions. Such rate structures provide a practical basis for converting physical transportation distances into applicable cost coefficients.

In addition to direct transportation expenditure, opportunity cost can be relevant when alternative grain-flow decisions have different economic consequences. Du et al. (2022) [6] incorporated opportunity cost into a grain-trade optimization framework to evaluate the economic and resource implications of alternative grain-flow configurations. This indicates that food-grain optimization can consider economic consequences beyond direct transportation expenditure.

The model used in this study combines transportation, milling, and opportunity costs and obtains transportation coefficients through the distance-to-cost conversion mechanism used for the Jharkhand network.

2.6. Multi-Period Inventory, Capacity, and Demand Fulfilment

Food-grain operations are dynamic because procurement, processing, storage, and consumption do not necessarily happen at the same time. Inventory may have to be carried from one period to another to maintain supply continuity.

Tanksale and Jha (2016) [28] developed a multi-period multi-foodgrain inventory transportation model for the Indian context, while Mogale et al. (2017) [22] developed a multi-period inventory-transportation model for tactical planning of food-grain supply chains. These studies show the importance of time-indexed transportation and inventory decisions.

Warehouse and silo capacity also play a critical role in food-grain network planning. Mogale et al. (2018a) [20] incorporated dwell time into the grain silo location-allocation problem, while Mogale et al. (2018b) [23] considered storage capacity within an MINLP framework for food-grain movement and storage decisions.

The reviewed work confirms the importance of time-dependent inventory and capacity decisions. There is still scope to represent procurement, milling capacity, warehouse inventory, warehouse capacity, and PDS demand together in one multi-period MILP model.

2.7. Computational and Heuristic Approaches

Exact mathematical programming and heuristic methods are both used in food-grain supply-chain problems. Exact LP and MILP models provide a systematic way to represent several constraints at the same time, while classical optimization references provide the basic theory (Dantzig 1963; Bazaraa et al. 2010; Taha 2017) [5, 2, 26].

For transportation and inventory problems with greater computational complexity, heuristic approaches have also been investigated. Tanksale and Jha (2016) [28] proposed an effective heuristic for a multi-period, multi-foodgrain inventory transportation problem, while Chakraborty et al. (2020) [3] combined a MILP formulation with a heuristic solution approach for food-grain inventory transportation in India.

These studies show that heuristic methods can be useful for transportation and inventory problems, while an exact MILP is suitable when several operational decisions must be handled together. In this article, MILP is used for the integrated four-stage problem and Modified VAM is used separately to check the Stage-1 transportation calculation.

2.8. Recent Developments in Food-Grain Supply-Chain Sustainability and Uncertainty

Recent work has also brought sustainability, environmental effects, reliability, and uncertainty into food-grain optimization. Chandrasekaran and Ranganathan (2017) [4] included PHL and carbon emissions, and Nath et al. (2024) [24] reviewed measures for reducing grain post-harvest losses.

Sun et al. (2024) [25] studied a green and reliable intermodal routing problem for food-grain transportation with carbon taxation, carbon trading, and multi-source uncertainty. Such studies show that food-grain optimization is moving beyond simple transportation-cost minimization.

Even when additional sustainability or uncertainty factors are included, the basic material flow still has to be represented correctly. Procurement availability, milling recovery, processing capacity, storage capacity, inventory, transportation cost, and demand must be consistent. This study, therefore, concentrates on an integrated deterministic MILP for the main operational stages of the Jharkhand PDS network.

2.9. Positioning of the Present Study and Research Gap

The literature covers transportation and storage optimization (Asgari et al. 2013; Tanksale and Jha 2014) [1, 27], shipment-cost minimization (Maiyar and Thakkar 2015) [17], multi-stage and multi-period planning (Tanksale and Jha 2016; Mogale et al. 2016, 2017) [21, 22, 28], silo location-allocation (Mogale et al. 2018a) [20], movement and storage (Mogale et al. 2018b) [23], procurement network modelling (Mogale et al. 2020) [19], and heuristic solution methods (Chakraborty et al. 2020) [3]. Rice-mill studies have also addressed processing, resource allocation, and sustainable mill planning (Lim et al. 2013, 2014) [11, 12]. More recently, PHL and quality deterioration have been studied through nonlinear programming (Mahato et al. 2026) [15].

Nevertheless, a modelling gap remains in the simultaneous integration of the following characteristics within a single food-grain PDS network:

1. procurement-centre-to-rice-mill transportation;
2. rice-milling and material recovery;
3. period-specific milling capacity;
4. mill-to-warehouse transportation;
5. multi-period warehouse inventory dynamics;
6. warehouse capacity and activation;
7. warehouse-to-PDS distribution;
8. distance-dependent government transportation costs;
9. opportunity cost and shortage penalty; and
10. demand fulfilment within a unified MILP framework.

This study addresses the above gap through a Dynamic MILP for the four-stage chain: Procurement Centres → Rice Mills → Warehouses → PDS Distribution Points. The model uses a milling recovery factor, period-wise mill capacity, distance-based

transportation costs, time-indexed inventory balances, warehouse capacity, and demand constraints. The model is then tested through computational experiments, sensitivity analysis, and the Seraikela-Kharsawan real-data-driven calibrated case.

3. Mathematical Formulation

The food-grain network is formulated as a Dynamic Mixed-Integer Linear Programming (MILP) model. Paddy and milled rice move through four connected layers—procurement centres, rice mills, warehouses, and PDS demand points—over a finite number of planning periods. The formulation first defines the sets, parameters, and decision variables. The objective function then combines the main cost components, followed by constraints for supply, milling, inventory, warehouse capacity, route activation, and demand. The milling recovery factor links paddy input with milled-rice output, while the time index allows flows and inventory to change from one period to another.

3.1. Sets, Indices, and Decision Variables

The following sets and indices are considered in the proposed model:

- P: Set of procurement centres, indexed by p.
- M: Set of rice mills, indexed by m.
- W: Set of warehouses, indexed by w.
- D: Set of PDS demand points, indexed by d.
- T: Set of planning periods, indexed by t.

The decision variables are defined as follows:

- $Q_{pm,t}$: Quantity of paddy transported from procurement centre p to rice mill m during period t (Qtl).
- $R_{mw,t}$: Quantity of milled rice transported from rice mill m to warehouse w during period t (Qtl).
- $S_{wd,t}$: Quantity of rice transported from warehouse w to PDS demand point d during period t (Qtl).
- $I_{w,t}$: Inventory held at warehouse w at the end of period t (Qtl).
- $U_{d,t}$: Unmet demand (shortage) at PDS demand point d during period t (Qtl).
- y_{pm}: Binary variable equal to 1 if the procurement-centre p to rice-mill m transportation route is activated, and 0 otherwise.
- z_w: Binary variable equal to 1 if warehouse w is activated for inventory allocation, and 0 otherwise.

3.2 Model Parameters

The principal parameters used in the proposed model are defined as follows:

- $A_{p,t}$: Available quantity of paddy at the procurement centre p during period t.
- Cap_w: Storage capacity of warehouse w.
- $CapM_m$: Maximum paddy-processing capacity of rice mill m
- $D_{d,t}$: Demand at PDS demand point d during period t.
- C_{pm}: Transportation cost per quintal from the procurement centre p to rice mill m.
- C_{mw}: Transportation cost per quintal from rice mill m to warehouse w.
- C_{wd}: Transportation cost per quintal from warehouse w to PDS demand point d.
- MC_m: Milling cost associated with rice mill m.
- H_w: Inventory holding cost per quintal at warehouse w per period.
- OC_m: Opportunity Cost coefficient associated with the processing and movement of grain through the rice mill m.
- π_d: Penalty cost per quintal of unmet demand at PDS demand point d.
- I_w^0: Initial inventory available at the warehouse w at the beginning of the planning horizon (Qtl).

$$C_{ij} = f(d_{ij}) = \begin{cases} ₹37.48 \ per\ quintal, if\ \ 0\ km < distance \leq 8\ km \\ ₹43.47 \ per\ quintal, if\ 8\ km < distance \leq 20\ km \\ ₹54.00 \ per\ quintal, if\ 20\ km < distance \leq 40\ km \\ ₹63.47 \ per\ quintal, if\ 40\ km < distance \leq 80\ km \\ ₹73.47 \ per\ quintal, if\ 80\ km < distance \leq 100\ km \\ ₹74.94 \ per\ quintal, if\ 100\ km < distance \end{cases}$$

The distance-based transportation cost is determined using the distance-to-cost converter function, where d_{ij} represents the distance between two connected nodes and C_{ij} represents the applicable government-approved transportation rate corresponding to the relevant distance slab.

3.3. The Integrated Objective Function

The objective is to minimize the total cost of operating the food-grain network over the planning horizon. It includes transportation, milling, opportunity, inventory holding, and shortage-related costs.

The total objective function Z is formulated as:

$$\min Z = \sum_{t\in T}\sum_{p\in P}\sum_{m\in M} C_{pm}Q_{pm,t} + \sum_{t\in T}\sum_{m\in M}\sum_{w\in W}(C_{mw} + MC_m + OC_m)R_{mw,t} + \sum_{t\in T}\sum_{w\in W}\sum_{d\in D} C_{wd}S_{wd,t} + \sum_{t\in T}\sum_{w\in W} H_w I_{w,t} + \sum_{t\in T}\sum_{d\in D}\pi_d U_{d,t} \quad (1)$$

The first term represents procurement-centre-to-mill transportation. The second term covers mill-to-warehouse transportation together with milling and opportunity costs. The third term is warehouse-to-PDS transportation. The fourth term is inventory holding cost, and the last term is the penalty for unmet demand.

The milling recovery factor is a physical conversion parameter, so it is not added as a separate cost term. Its effect enters through the milling material-balance constraint.

3.4. Systemic and Logical Constraints

The following constraints are used to keep the flows feasible and the material balance consistent over the planning horizon.

3.4.1. Procurement, Supply and Route Activation

The total quantity of paddy transported from the procurement centre p during period t cannot exceed its available supply:

$$\sum_{m\in M} Q_{pm,t} \le A_{p,t}, \qquad \forall p \in P, \qquad t \in T \quad (2)$$

The flow is also logically linked to the route-activation variable:

$$Q_{pm,t} \le A_{p,t} y_{pm}, \qquad \forall p \in P, \qquad m \in M, \qquad t \in T \quad (3)$$

Therefore, a procurement-to-mill flow can be positive only when the corresponding route is activated.

3.4.2. Milling Balance and Material Recovery

The total quantity of milled rice transported from the mill mm to warehouses is determined by the quantity of paddy received by the mill and the fixed milling recovery factor:

$$\sum_{w\in W} R_{mw,t} = \eta \sum_{p\in P} Q_{pm,t}, \qquad \forall m \in M, \qquad t \in T \quad (4)$$

where the adopted recovery factor links the paddy received by the mill with the milled rice sent to the warehouses. This keeps the upstream and downstream quantities consistent.

The paddy-processing capacity of each rice mill is restricted by its maximum allowable processing capacity in each planning period:

$$\sum_{p\in P} Q_{pm,t} \le CapM_m, \qquad \forall m \in M, \qquad t \in T \tag{5}$$

3.4. 3. Inventory Flow Balance

Warehouse inventory in each period is linked with opening inventory, incoming milled rice, outgoing PDS supply, and closing inventory:

$$I_{w,t} = I_{w,t-1} + \sum_{m\in M} R_{mw,t} - \sum_{d\in D} S_{wd,t}, \qquad \forall w \in W, \qquad t \in T \tag{6}$$

For the first planning period, the opening inventory is represented by:

$$I_{w,0} = I_w^0 \qquad \forall w \in W \tag{7}$$

This balance equation keeps track of the quantity entering and leaving each warehouse in every period.

3.4.4. Warehouse Capacity and Activation

The inventory stored at warehouse w is restricted by its available storage capacity and its operational status:

$$I_{w,t} \le Cap_w z_w, \qquad \forall w \in W, \qquad t \in T \tag{8}$$

If the warehouse is not activated, no inventory can be assigned to it. When it is activated, inventory can be held up to the specified warehouse capacity.

3.4.5. Demand Satisfaction and Non-Negativity

The total quantity supplied to PDS demand point d, together with the unmet demand, must satisfy the corresponding demand:

$$\sum_{w\in W} S_{wd,t} + U_{d,t} = D_{d,t}, \qquad \forall d \in D, \qquad t \in T \tag{9}$$

The non-negativity conditions are:

$$Q_{pm,t}, \qquad R_{mw,t}, \qquad S_{wd,t}, \qquad I_{w,t}, \qquad U_{d,t} \ge 0 \tag{10}$$

The binary variables satisfy:

$$y_{pm} \in \{0,1\}, \qquad \forall p \in P, \qquad m \in M \tag{11}$$

$$z_w \in \{0,1\}, \qquad \forall w \in W \tag{12}$$

Together, these constraints maintain supply feasibility, material conservation, milling recovery, warehouse-capacity limits, demand balance, and the logical activation of routes and warehouses.

3.5. Validation-Specific Throughput Constraint

For the real-data validation of the Seraikela-Kharsawan district network, a common total paddy throughput is imposed on both the baseline and proposed MILP configurations to ensure a consistent basis for comparison:

$$\sum_{p\in P}\sum_{m\in M}\sum_{t\in T} Q_{pmt} = Q^{Target} = 58{,}299.16 \text{ Qtl.} \tag{13}$$

Here, the prescribed throughput is calculated from the required milled-rice quantity and the adopted milling recovery factor. Using the same throughput in the baseline and MILP cases makes the cost comparison consistent because both cases process the same amount of paddy.

This throughput condition is used only for the Seraikela-Kharsawan validation and is not a general constraint of the Dynamic MILP model.

4. Results and Discussion

4.1 Comparative Simulation: Baseline Allocation versus Proposed MILP

The East Singhbhum network was first used to compare a nearest-feasible baseline allocation with the proposed MILP under the same network, supply, milling, warehouse, demand, transportation, and cost assumptions. The network consists of 53 procurement centres, 20 tagged rice mills, 3 representative warehouses, and 11 PDS demand blocks. Annual paddy receipt is 543,011.04 Qtl, and annual CMR receipt is 372,117.85 Qtl. The observed paddy-to-CMR ratio is 0.6853, while the benchmark milling recovery used in the optimization is 0.67. The planning horizon comprises three periods with an initial inventory of 6,201.96 Qtl.

The East Singhbhum case is treated as a computational simulation network; the network quantities and operating parameters are based on the stated study assumptions, while the inter-node distances are generated for simulation using a fixed random seed (20260920).

For the baseline, a nearest-feasible heuristic was implemented in MATLAB. Paddy was allocated from each procurement centre to feasible mills in ascending order of distance, milled rice was assigned to feasible warehouses using the same rule, and warehouse stocks were allocated to PDS demand points according to increasing distance. Capacity constraints were respected at each stage, while unmet demand and closing inventory were recorded. The same distance matrices, government distance-to-cost conversion, and cost components were used for the baseline and MILP, making the baseline a reproducible reference allocation.

The proposed model was solved in MATLAB using intlinprog with the HiGHS solver engine and obtained a 0% optimality gap. The objective value was independently recalculated from the resulting flow variables and agreed with the solver value within numerical tolerance. Supply, milling-capacity, warehouse-capacity, inventory-balance, demand-balance, and binary-logic constraints were also verified after the run.

Table 1. Comparative simulation results for the East Singhbhum network

Performance Indicator	Baseline Allocation	Proposed MILP	Improvement
Paddy processed (Qtl)	543,011.04	543,011.04	0.00%
Milled rice (Qtl)	363,817.40	363,817.40	0.00%
PDS dispatch (Qtl)	370,019.36	370,019.36	0.00%
Total inventory (Qtl)	4,103.48	4,103.48	0.00%
Systemic shortage (Qtl)	2,098.49	2,098.49	0.00%
Active procurement–mill routes	204	211	—
Warehouse receiving periods	9	9	—
Total system cost (INR)	**76,102,850.08**	**75,105,464.62**	**1.31% reduction**

Note: The improvement in total system cost is calculated relative to the baseline allocation. The number of active routes is a logical outcome of the allocation and is not itself treated as a cost-reduction measure.

Table 1 shows that the total system cost falls from INR 76,102,850.08 for the baseline allocation to INR 75,105,464.62 with the MILP. The saving is INR 997,385.46, or 1.31%. Paddy processed, milled rice produced, PDS dispatch, inventory, and systemic shortage remain unchanged. Hence, for this particular simulation, the savings come mainly from changing the spatial allocation of transportation flows.

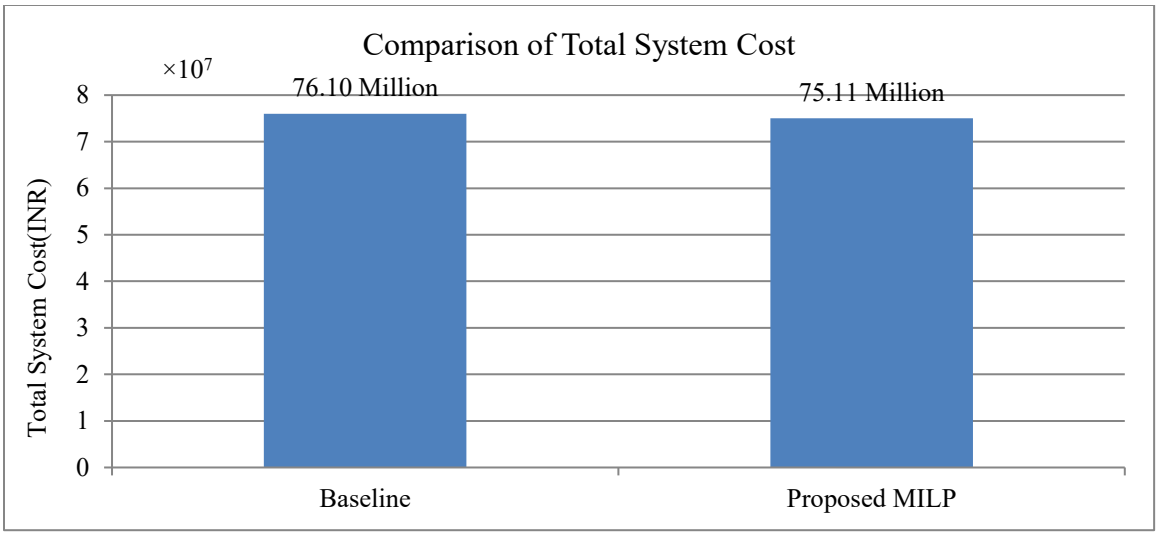


Fig. 1 Comparison of total system cost between the baseline allocation and the proposed optimal MILP for the East Singhbhum network

The cost components in Table 2 show where this saving comes from. The largest reduction is in warehouse-to-PDS transportation, followed by paddy-to-mill and mill-to-warehouse transportation. Milling, opportunity, holding, and shortage-penalty costs remain unchanged because the corresponding quantities and parameter values are the same in both cases.

Table 2. Cost-component comparison between baseline and proposed MILP

Cost Component	Baseline (INR)	Proposed MILP (INR)	Saving (INR)
Paddy-to-mill transportation	23,049,843.79	22,772,738.60	277,105.19
Mill-to-warehouse transportation	18,352,968.89	18,210,189.01	142,779.88
Milling cost	10,914,521.90	10,914,521.90	0.00
Opportunity cost	1,819,086.98	1,819,086.98	0.00
Warehouse-to-PDS transportation	21,431,957.91	20,854,457.53	577,500.38
Holding cost	9,848.34	9,848.34	0.00
Shortage penalty	524,622.26	524,622.26	0.00
Total system cost	**76,102,850.08**	**75,105,464.62**	**997,385.46**

Note: Component totals may differ from the reported system total by INR 0.01 due to rounding.

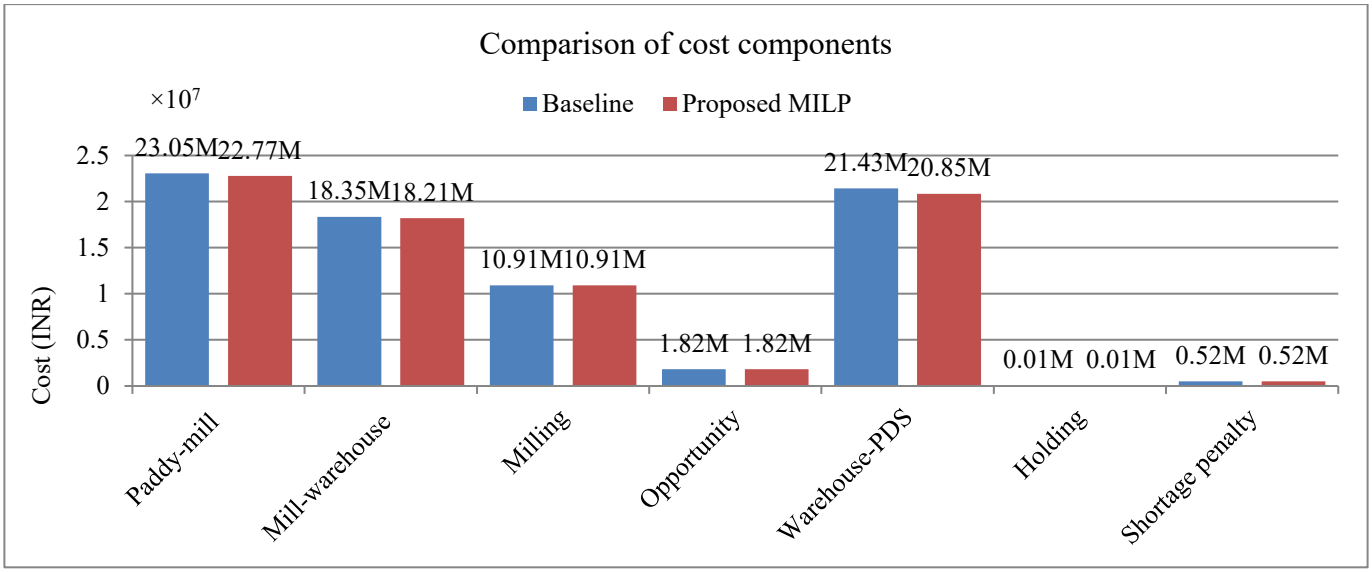


Fig. 2 Comparison of individual cost components between the baseline allocation and the proposed MILP solution

The warehouse-to-PDS stage contributes INR 577,500.38 of the total savings. The paddy-to-mill and mill-to-warehouse stages contribute INR 277,105.19 and INR 142,779.88, respectively. These three transportation reductions together explain the full system-cost difference. Since throughput and shortage do not change, the benefit in this simulation is mainly a result of better flow allocation across the network.

The MILP reached a 0% optimality gap and returned an objective value of INR 75,105,464.62. The independently reconstructed objective agreed with this value within numerical tolerance, providing an additional check on the implementation.

4.2. Sensitivity Analysis and Robustness Assessment

Sensitivity analysis was conducted to examine how changes in important supply-chain parameters affect the optimized total system cost and systemic shortage. The reference MILP solution has a total system cost of INR 75,105,464.62 and a systemic shortage of 2,098.49 Qtl. The parameters considered include warehouse holding cost, shortage penalty, opportunity cost, transportation cost, milling capacity, PDS demand, paddy availability, and milling recovery. The tested ranges and the corresponding changes in the optimized solution are summarized in Table 3.

Table 3. Summary of sensitivity analysis of the proposed MILP model

Parameter	Tested range	Total system cost change (%)	Systemic shortage range (Qtl)
Warehouse holding cost	−30% to +30%	−0.0039 to +0.0039	2,098.49 (unchanged)
Shortage penalty	−30% to +30%	−14.3794 to +0.2096	2,098.49–352,186.93
Opportunity cost	−30% to +30%	−0.7168 to +0.7266	2,098.49–2,876.48
Transportation cost	−30% to +30%	−24.7002 to +19.1282	2,098.49–222,181.79
Milling capacity	−30% to +30%	+4.2125 to −1.1336	2,098.49–85,776.49
PDS demand	−30% to +30%	−32.2430 to +36.1966	0–113,733.84
Paddy availability	−10% to +10%	+1.3221 to −0.6780	0–38,480.23
Milling recovery	0.63–0.72	+2.7573 to −2.9819	0–23,818.93

Note: Reference MILP total system cost = INR 75,105,464.62 and reference systemic shortage = 2,098.49 Qtl.

4.2.1. Sensitivity of Total System Cost

The results in Table 3 show that PDS demand and transportation cost produce relatively large changes in the optimized total system cost. Milling recovery and milling capacity also affect the objective value, particularly when their levels are reduced. In contrast, opportunity cost and warehouse holding cost produce only small changes over the tested ranges. The shortage penalty shows an asymmetric response: reducing the penalty decreases the objective value, but this reduction is accompanied by a substantial increase in systemic shortage.

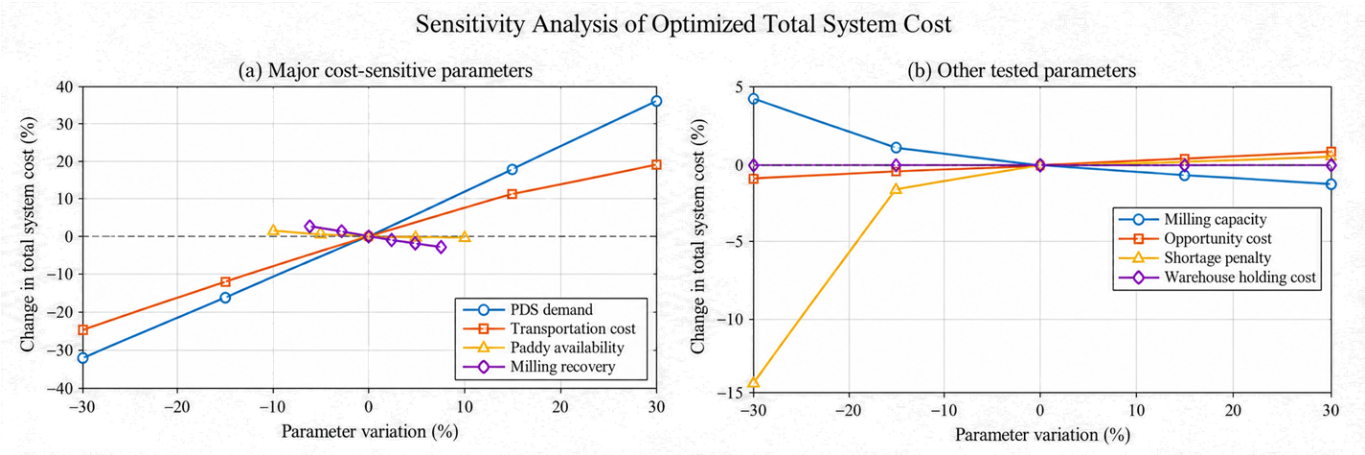


Fig. 3 Sensitivity of the optimized total system cost to variations in key supply-chain parameters

4.2.2. Sensitivity of Systemic Shortage

The shortage results indicate that system feasibility is more sensitive to increases in transportation cost, reductions in milling capacity, paddy availability, and milling recovery, as well as increases in PDS demand. A 30% increase in transportation cost raises the systemic shortage to 222,181.79 Qtl, while a 30% reduction in milling capacity increases it to 85,776.49 Qtl. Similarly,

a 30% increase in PDS demand results in a shortage of 113,733.84 Qtl. Reductions in paddy availability and milling recovery also increase the shortage. In contrast, increasing the shortage penalty does not reduce the baseline shortage within the tested range, whereas reducing the penalty results in a much larger shortage.

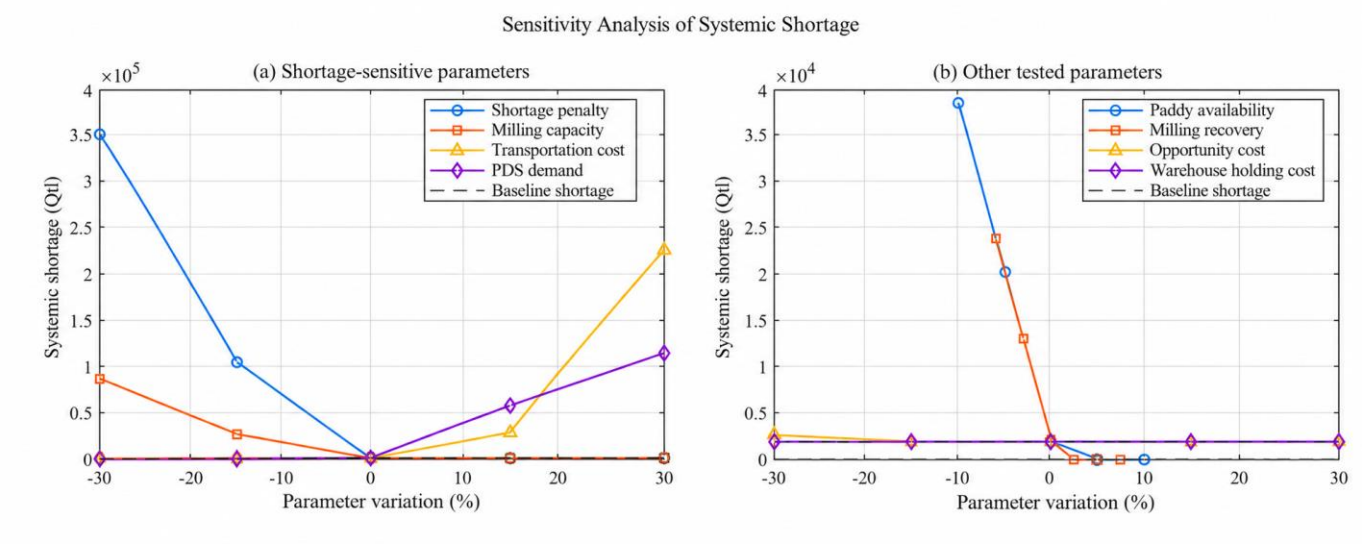


Fig. 4 Sensitivity of systemic shortage to variations in key supply-chain parameters

4.2.3. Overall Robustness Assessment

The sensitivity results show that the model does not respond equally to all parameters. PDS demand, transportation cost, paddy availability, shortage penalty, and milling recovery have stronger effects on either total system cost or systemic shortage, whereas opportunity cost and warehouse holding cost have relatively smaller effects on the objective value. The results also show that cost and shortage do not always change in the same proportion; therefore, both measures should be considered when assessing the behaviour of the optimized network.

As an additional robustness check, a zero holding-cost scenario was tested because the warehouses considered in this study are existing government storage facilities. Removing the marginal holding cost changed the total system cost by only 0.013%, while total inventory increased from 4,103.48 Qtl to 8,300.45 Qtl. This indicates that holding cost has a limited effect on the objective value under the present network conditions, although its removal can encourage higher inventory levels.

4.3. Real-Data-Driven MILP Validation of the Integrated Seraikela-Kharsawan District Network

The integrated model was next tested for Seraikela-Kharsawan using the observed procurement-stage information and the available downstream network details. The network has 15 procurement centres, 7 rice mills, 8 SFC warehouse locations, and 9 demand blocks, including Kukru. Observed Stage-1 paddy availability is 134,849 Qtl. The same Stage-1 data are used in the integrated case so that the transportation check and the downstream optimization remain consistent.

Complete historical records were not available for all downstream inputs, particularly block-level demand, individual warehouse capacities, and a complete historical road-distance dataset. For this reason, the integrated case is treated as a real-data-driven calibrated simulation. Observed procurement quantities, the procurement-centre-to-mill distance matrix, number of mills, warehouse locations, and administrative demand structure are retained. Missing downstream values are calibrated from the available district-level allocation and storage information. The same calibrated values are used in the baseline and MILP cases. Therefore, the case is not presented as a completely independent historical dataset; it is an extension of the real Stage-1 network to the downstream stages.

For the calibrated case, the district rice-allocation anchor is 39,722.48 Qtl, with a milling recovery factor of 0.67. The aggregate warehouse-capacity benchmark is 57,000 Qtl across eight warehouse locations. The required paddy throughput is 58,299.16 Qtl. Opening inventory is 662.04 Qtl, and milled rice generated during the planning horizon is 39,060.44 Qtl, giving total PDS dispatch of 39,722.48 Qtl. Under these common conditions, the baseline cost is INR 7,813,080.15, and the MILP cost is INR 7,114,689.99, a reduction of 8.94%. Both cases process 58,299.16 Qtl of paddy, generate 39,060.44 Qtl of milled rice, and meet the calibrated rice requirement without shortage.

Table 4. Baseline versus Proposed MILP: Seraikela-Kharsawan Integrated Network

Performance Indicator	Baseline	Proposed MILP
Total system cost (INR)	7,813,080.15	7,114,689.99
Paddy processed (Qtl)	58,299.16	58,299.16
Milled rice (Qtl)	39,060.44	39,060.44
PDS dispatch (Qtl)	39,722.48	39,722.48
Shortage (Qtl)	0	0
Cost reduction (%)	—	**8.94**

The MILP, therefore, gives a lower cost for the same prescribed throughput and calibrated downstream demand. The 8.94% reduction is a result of the calibrated Seraikela-Kharsawan configuration; it should not be described as a directly observed historical cost saving for 2021–22. The distinction between observed district information and calibrated inputs is kept explicit so that the scope of the validation is clear.

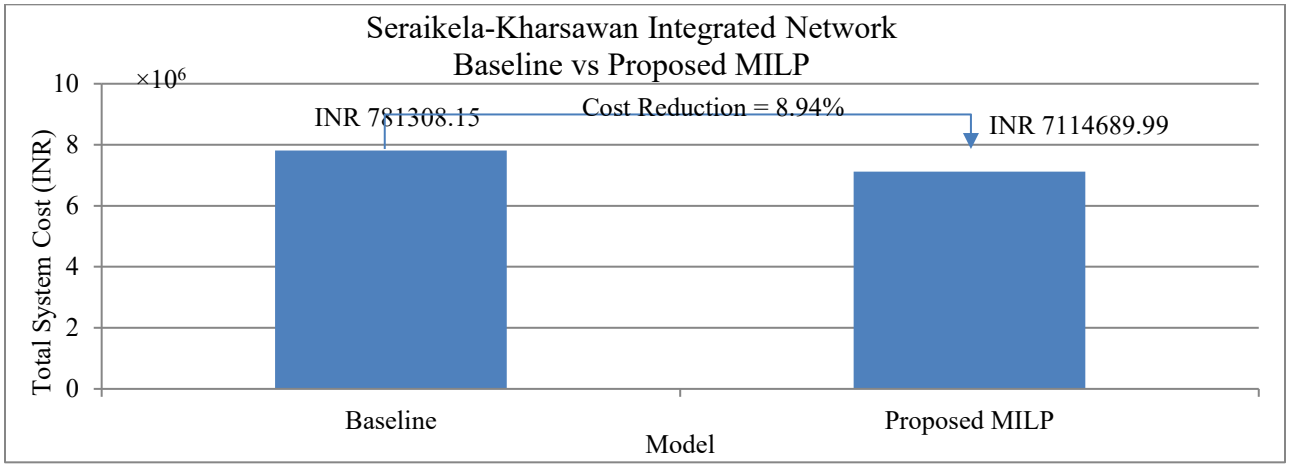


Fig. 5 Comparison of total system cost between the baseline allocation and the proposed MILP for the calibrated Seraikela-Kharsawan network

4.4. Stage-1 Real-Data Validation: Manual and MATLAB-Based Modified VAM

As an independent computational verification of the procurement-to-mill transportation component, the Modified Vogel's Approximation Method (VAM) was evaluated using the same real-data configuration. This validation is presented separately from the integrated MILP results because the Modified VAM addresses only the Stage-1 transportation allocation, whereas the proposed MILP simultaneously represents procurement, milling, warehousing, inventory, and PDS distribution decisions. The purpose of this comparison is therefore to verify the computational implementation of the Stage-1 transportation procedure and not to claim a direct optimization comparison between the two approaches. The Stage-1 problem comprises 15 procurement centres and 7 rice mills, with a total paddy supply of 134,849 Qtl and an aggregate mill requirement of 176,500 Qtl. The same input data, distance matrix, and transportation-cost structure were used in both the manual and MATLAB-based implementations. The computational implementation and numerical experiments were carried out using MATLAB (Higham & Higham, 2016) [8].

The manual Modified VAM resulted in a total transportation cost of INR 6,941,561.86, whereas the MATLAB-based implementation yielded INR 6,883,138.17, giving a difference of INR 58,423.69 (0.84%).

Table 5. Stage-1 Transportation: Manual Modified VAM versus MATLAB-Based Modified VAM

Performance Indicator	Manual Modified VAM	MATLAB-Based Modified VAM
Procurement centres	15	15
Rice mills	7	7
Total paddy supply (Qtl)	134,849	134,849
Total mill requirement (Qtl)	176,500	176,500
Transportation cost (INR)	6,941,561.86	6,883,138.17
Difference (INR)	—	58,423.69
Difference (%)	—	0.84

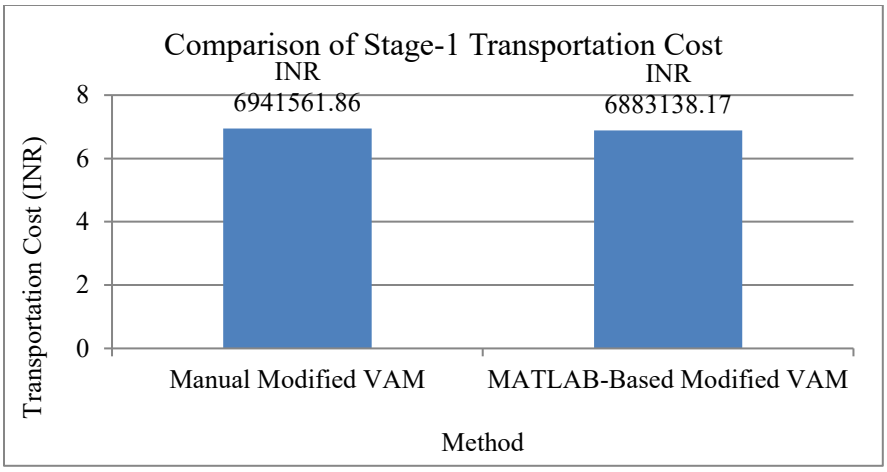


Fig. 6 Comparison of Stage-1 transportation cost obtained using manual and MATLAB-based Modified VAM implementations

The small difference is attributable to the allocation sequence generated during the solution of the unbalanced transportation problem.

5. Conclusion

This study developed a Dynamic Mixed-Integer Linear Programming (MILP) model for a four-stage food-grain supply chain supporting the Public Distribution System in Jharkhand. The model connects procurement centres, rice mills, warehouses, and PDS demand points and includes milling recovery, period-wise milling capacity, procurement availability, route activation, warehouse capacity, inventory balance, transportation cost, milling cost, opportunity cost, and shortage penalty.

Transportation costs are linked with inter-node distances through the applicable Jharkhand government rate structure. The multi-period formulation also allows grain to move and be stored across successive periods while maintaining the required material balance.

In the East Singhbhum simulation, the MILP reduced total system cost from INR 76,102,850.08 to INR 75,105,464.62, giving a 1.31% reduction under the same throughput and shortage conditions. In the Seraikela-Kharsawan calibrated case, the cost decreased from INR 7,813,080.15 to INR 7,114,689.99, or 8.94%, while the prescribed paddy throughput and zero shortage were maintained. The latter value is a result of the calibrated configuration and is not treated as an observed historical saving.

The Stage-1 transportation calculation was also checked using Modified VAM. The manual and MATLAB-based costs were INR 6,941,561.86 and INR 6,883,138.17, respectively, with a difference of 0.84%. This separate check supports the numerical implementation of the procurement-to-mill transportation component.

The model provides a common framework for studying procurement, milling, transportation, inventory, storage, and PDS distribution in one network. The results also show that paddy availability, PDS demand, transportation cost, milling capacity, shortage penalty, and milling recovery can have different effects on cost and service levels. Future work can introduce explicit post-harvest-loss and quality-deterioration variables, multiple food grains, stochastic supply and demand, vehicle-routing decisions, carbon-emission objectives, and larger state-level networks as more detailed operational data become available.

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