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Original Article

Introducing a New Specific Instance of the Trapezium

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Abstract *- In this research article, we introduce and define a new specific instance of a trapezium, called Kashif's Trapezium. The paper details the derivation of its slant height and the determination of two distinct values for its altitude (height). Also, present a new theorem proving that Kashif's Trapezium is a cyclic quadrilateral. Moreover, the semi-perimeter of the trapezium is examined, where a novel characteristic is found.*

Keywords *- Algebraic Geometry, Kashif's Trapezium, Equidecomposability, Equiareal.*

1. Introduction

The study of Polygons is a lynchpin of Euclidean geometry and provides the foundational framework for complex spatial analysis and architectural design. Euclidean geometry relies on specific axioms. Polygons are the first closed shapes that allow us to test and prove these axioms in practice. In the foundation of measurement, concepts like perimeter and area are first defined through polygons before being applied to curves or 3D solids. This paper introduces a new specific instance: Kashif's Trapezium, characterized by its unique properties among area, altitude, semi-perimeter, and many more. The idea comes from the concept of equiareal figures. Consider in two-dimensional space a right-angled triangle is equiareal with a circle, then we can easily determine its radius by keeping $\pi r^2 = \frac{ab}{2}$ For a square, the side length is determinable, but what about a trapezium? What should be the lengths of the parallel sides and altitude? The objectives of this research are to define Kashif's Trapezium formally. This trapezium gives positive integer lengths of parallel sides and altitude if a, b, and c are Pythagorean triplets.

The history of equiareal figures goes back thousands of years, starting from practical land measurement in ancient civilizations to sophisticated mathematical fields in modern study of cartography, as well as in physics. Egyptians in 3000-1500 BC [1] developed empirical methods and rules for calculating areas of different two-dimensional shapes to adjust agricultural rent after Nile flooding. They used the cut-and-paste technique to rearrange shapes like squares and rectangles for the solution of area-related problems. Greek mathematicians shifted from empirical rules to logical and analytical proofs. Euclid's Elements formalized the study of figures with equal area in 300 BC. In our modern era, equiareality moved into the study of linear transformations and global equity. In 1805, Karl B. Mollweide introduced the elliptical Mollweide projection [2] to improve the comparison with cylindrical models. In transformation theory, mathematicians like Felix Klein categorized equiareal maps with squeeze mappings and shear mappings as those with a concept of matrix determinant. Applications of equiareal figures cover a wide range of studies like cartography and map projections in which thematic mappings and equal earth projections come, geographic information systems and remote sensing, computer graphics with panoramic display, power system engineering, geological structural analysis, etc[4].

2. Preliminaries

Before starting the actual definition, let us see the following diagrams.

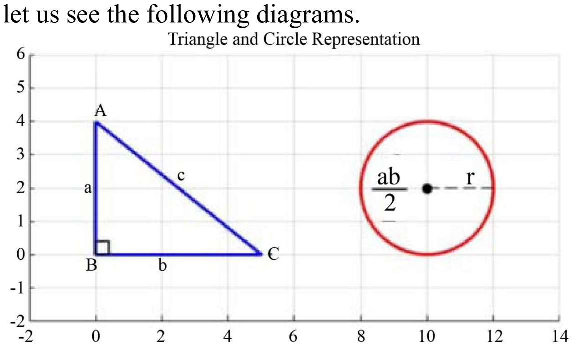

Diagram 1.1

Showing a Right-Angle Triangle and Circle with Equal Area. r is Determinable

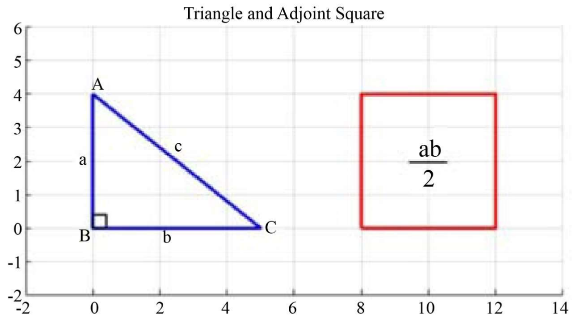

Diagram: 1.2

Showing a Right-Angle Triangle and a Square with Equal Area. Side is Determinable

In the above right-angle triangle, c is $c = \sqrt{a^2 + b^2}$, also used in Kashif's Trapezium.

In this way, if a right-angle triangle is equiareal with a trapezium, then what would be the lengths of its parallel sides and the length of the altitude. This is discussed in detail in the following.

Definition 2.1Trapezium: A trapezium is a planar polygon consisting of four edges and four vertices (a quadrilateral) in which at least one pair of opposite sides is parallel.

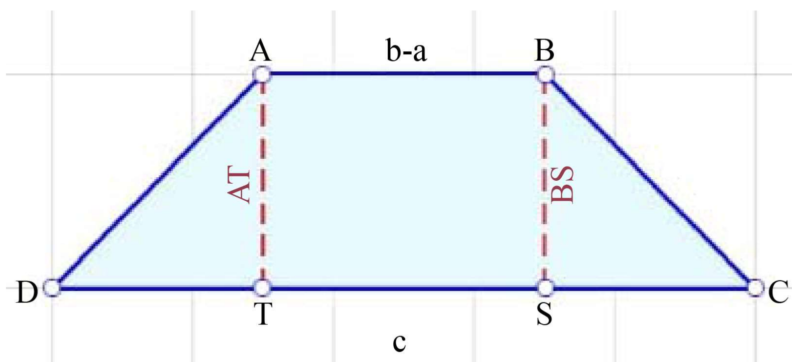

Fig. 1 Kashif's Trapezium

Definition 2.2 Kashif's Trapezium: Kashif's Trapezium is a trapezium with an arrangement of parallel side lengths and its altitude, which gives the area exactly equal to the right-angle triangle, i.e., let □ABCD is a trapezium with AB|| DC with AB<DC and altitude AT with the values AB = $b - a$, DC = c, and AT = $\frac{c-(b-a)}{2}$ as shown in Figure 1.3**.** $c \text{ is nothing but } c = \sqrt{a^2 + b^2}$.

This is a new equiareal figure. That gives all parameters which are essential to calculate the area as natural numbers if a, b, and c are Pythagorean triplets with interesting mathematical properties.

3. Derivation of Slant Height

As we know, a trapezium has two slant heights (sometimes called legs) and their lengths can be changed according to the nature of the trapezium, but it does not affect the area of the trapezium; hence, most of the time we consider a trapezium as an isosceles trapezium

Therefore, the two angles that touch the same leg (one at the top and one at the bottom) always add up to 180, which is called the supplementary angles property of an isosceles trapezium

In accordance with Figure 1.

$$Note\ that\ altitudes\ of\ a\ trapezium\ is\ defined\ as\ the\ perpendicular\ distance\ between\ its\ two\ parallel\ sides$$

$$\therefore this\ clearifies\ \angle ATD + \angle BSC = 90.$$

$$since\ the\ slant\ heights\ and\ altitudes\ of\ \Delta DAT\ and\ \Delta CBS\ are\ equal$$

$$So\ in\ accordance\ with\ [Rightangle, Hypoteneous, Side]\ congruence\ postulate/test$$

$$\Rightarrow DT = CS$$

$$now\ observe\ that\ DC = DT + TS + CS$$

$$c = 2DT + (b - a)$$

$$\Rightarrow DT = \frac{c - (b - a)}{2} = h$$

$$i.e. DT = AT$$

$\Rightarrow \measuredangle DAT\ is$ 45-45-90 …(a)

$$now\ using\ pythagorus\ theorem\ in\ \Delta \mathrm{DAT}$$

$$DA^2 = DT^2 + AT^2$$

$$DA = \sqrt{2DT^2}$$

$$DA = \sqrt{2h^2}$$

$$\therefore DA = h\sqrt{2}$$

$$\therefore we\ found\ the\ slant\ height\ DA = CB = h\sqrt{2}$$

4. Determination of another value of Altitude

We know the area of Kashif's Trapezium is $\frac{ab}{2}$

$$A(\square ABCD) = \frac{1}{2} \times AT \times (DC + AB)$$

$$\Rightarrow \frac{ab}{2} = \frac{1}{2} \times h \times (c + (b - a))$$

$$\Rightarrow ab = h(c + (b - a))$$

$$\Rightarrow h = \frac{ab}{c + (b - a)}$$

$$in\ this\ way\ we\ obtain\ another\ value\ for\ h$$

Illustration let $a = 6, b = 8\ \Rightarrow c = \sqrt{6^2 + 8^2} = 10$

$$h = \frac{c - (b - a)}{2}$$

$$h = \frac{10 - (8 - 6)}{2}$$

$$\therefore h = 4$$

By other equality, $h = \frac{ab}{c+(b-a)}$

$$h = \frac{48}{10 + (8 - 6)}$$

$$\therefore h = 4$$

5. Semi-Perimeter of Kashif's Trapezium.

Since, $AB = b - a$, $BC = h\sqrt{2}$, $DC = c$ and $AD = h\sqrt{2}$ are determined.

Therefore, its semi-perimeter can be evaluated, let its semi-perimeter= s

$$s = \frac{\left[(b-a) + h\sqrt{2} + c + h\sqrt{2}\right]}{2}$$

$$s = \frac{\left[2h\sqrt{2} + c + (b-a)\right]}{2}$$

$$s = \frac{2h\sqrt{2}}{2} + \frac{c - (b-a) + (b-a) + (b-a)}{2}$$

$$s = h\sqrt{2} + \frac{c-(b-a)}{2} + \frac{2(b-a)}{2}$$

$$s = h\sqrt{2} + h + (b-a) \dots (1)$$

$$but\ we\ know\ h = \frac{c-(b-a)}{2}$$

$$\Rightarrow\ 2h + (b-a) = c$$

$$\Rightarrow h + h + (b-a) = c$$

$$\Rightarrow h + (b-a) = c - h \dots (2)$$

$$from\ (1)\&(2)$$

$$s = c + h\sqrt{2} - h \dots (3)$$

Hence, in this way, we obtain two equalities of semi-perimeter, see equations (1) and (3). We can also derive this height length by another geometrical method.

6. Theorem: Kashif's Trapezium is a Cyclic Quadrilateral

Proof: The most common way to check whether a given quadrilateral is cyclic or not is to measure whether the interior opposite angles are supplementary or not. If one pair of opposite angles adds up to 180, then the other pair will automatically add up to 180.

And from the equation (a)

$\angle CDA + (\angle DAB = \angle DAT + \angle BAT) = 45 + 45 + 90 = 180$. Hence cyclic.

7. Conclusion

Now, the Kashif's Trapezium has become a mathematical marvel that inherits various beautiful mathematical properties. Interestingly, also the height of Kashif's Trapezium can be calculated by the formula $h = \frac{c - \sqrt{c^2 - 2ab}}{2}$ Also, we can derive the quadratic equation $2h^2 - 2hc + ab = 0$. One can determine its angles, and one can also find the lengths of the diagonals of Kashif's Trapezium and the ratio of the diagonals. Altogether, the circumradius R and many more. It will be our further research. This trapezium can also be formed by a geometrical approach.

A new theorem can be established that $2|\ (c - (b-a))$, that is, if a, b, c are Pythagorean triplets, then the difference $\left(c - (b-a)\right)$ is divisible by 2. Its application will include complex spatial analysis, architectural design, for instance, in physics, consider a chair with four triangular legs, if we replace the legs with this trapezium, it increases the stability of the chair, etc.

Civil engineers calculate the "cut and fill" volumes for the conversion of a triangular cross-section of a hill into the trapezoidal embankment of the same area, which helps in balancing material efficiency. In mechanical engineering, the center of pressure or distribution of force on a surface, to simplify a triangular load in an equiareal trapezium, and in Mesh simplification, also a concept of equiareal trapezium comes.

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