International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I3P103

Volume 72 Issue 3, 13-22, March 2026

Original Article

Semi-Analytical Solutions of Fractional Differential Equations in RL and RC Circuits

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Received: 16 January 2026 Revised: 21 February 2026 Accepted: 12 March 2026 Published: 26 March 2026

Abstract *- Fractional-order differential equations provide an effective mathematical model for electrical circuits that exhibit memory and nonlocal effects. This study investigates semi-analytical methods to solve fractional-order differential equations of RL and RC electrical circuits. The semi-analytical methods, namely the Generalized Differential Transform Method (GDTM), Variational Iteration Method (VIM), Homotopy Perturbation Method (HPM), and Adomian Decomposition Method (ADM), are used to solve equations. The obtained series solutions are analyzed and compared graphically for different values of the fractional-order to illustrate their impact on circuit dynamics. The comparative results provide useful guidance for selecting an appropriate mathematical method for solving fractional-order electrical circuit models.*

Keywords *- Fractional-Order Circuits, Caputo Fractional Derivative, Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM), Generalized Differential Transform Method (GDTM).*

1. Introduction

Fractional calculus has gained significant attention from researchers in recent decades as an important mathematical tool for modeling complex dynamical systems with memory and hereditary properties. Fractional calculus generalizes the integer-order differentiation and Integration to arbitrary real or complex orders. This extension allows accurate mathematical modelling of many physical and engineering phenomena in which past states influence the present state. The fractional calculus and fractional differential equations have been widely studied in the literature. Podlubny [1] presented a comprehensive study of fractional differential equations and their applications in engineering and science. Oldham and Spanier [2] developed one of the systematic frameworks for fractional calculus and its applications to arbitrary-order differentiation and Integration. Petráš [3] presented techniques for the analysis of fractional-order nonlinear systems and discussed their advantages in describing complex dynamical systems. Kaczorek et al. [4] applied fractional-order theory to electrical circuits and demonstrated the importance of fractional calculus in electrical engineering.

Electrical circuits constitute one of the important applications in the field of fractional calculus. Traditional circuit analysis is based on integer-order differential equations derived from Kirchhoff's voltage and current laws. The electrical components, capacitors and inductors, often show non-ideal characteristics such as memory effects and frequency-dependent responses. These effects cannot be described in detail by integer-order models. Fractional-order models provide an effective model for representing these phenomena by including nonlocal properties and long-term memory effects in the mathematical formulation. Consequently, in recent years, fractional differential equations have been widely used to model RL, RC, and RLC circuits.

Several studies have investigated the application of fractional calculus in electrical circuits. Recently, the introduction of new fractional operators and analytical techniques has extended fractional-order circuit theory. Radwan et al. [5] showed that fractional models provide flexibility in describing the dynamic responses of fractional-order RC and RL circuits. Gómez-Aguilar et al. [6–8] developed fractional models and analytical solutions for RC, LC, RL, and RLC circuits using different fractional operators. Further, Gómez-Aguilar et al. [9] analysed the effectiveness of Atangana–Baleanu fractional derivatives in electrical circuits. Morales-Delgado et al. [10] derived analytical solutions of electrical circuits using conformable fractional derivatives. Gómez-Aguilar et al. [11,12] also studied analytical solutions for fractional electrical circuits. These studies established the theoretical foundation for fractional electrical circuits.

In addition to theoretical developments, several researchers have investigated mathematical solutions for the governing equations of fractional RL and RC circuits through analytical and numerical methods. Abro et al. [13] carried out a comparative mathematical analysis of RL and RC circuits using Atangana–Baleanu and Caputo–Fabrizio fractional derivatives and examined the influence of different fractional operators on circuit responses. Shah et al. [14] obtained analytic solutions for fractional RL circuit models and discussed the behavior of current in fractional-order systems. Magesh et al. [15] applied the generalized differential transform method to solve fractional RLC circuit equations. Recently, Abukhaled et al. [16] proposed a Green's function approach for solving fractional RLC circuit models. Muniswamy et al. [17] conducted numerical analysis of fractional RLC circuits using the Caputo fractional operator. Jadhav et al. [18] investigated mesh RL electrical circuits using fractional calculus.

Several semi-analytical methods have been developed to obtain solutions of fractional differential equations arising in electrical systems. Among these techniques, the Adomian Decomposition Method (ADM) has been widely used because it provides rapidly convergent series solutions without linearization or discretization. Adomian [19] introduced the decomposition method and observed its effectiveness in solving various types of differential equations. Hu et al. [20] and Cheng et al. [21] applied ADM to fractional differential equations and showed that it can produce accurate analytical approximations for fractional dynamical systems. Another semi-analytical method for solving fractional differential equations is the Homotopy Perturbation Method (HPM). This method combines the concepts of homotopy from topology with classical perturbation techniques to obtain approximate solutions of nonlinear problems. He [22] introduced the homotopy perturbation technique and demonstrated its wide applicability in engineering problems. Abdulaziz et al. [23] applied HPM to fractional initial value problems and demonstrated its efficiency in solving fractional differential equations. Javeed et al. [24] analyzed the performance of HPM for fractional differential equations and discussed its convergence characteristics. The Generalized Differential Transform Method (GDTM) is another semi-analytical method used to solve fractional differential equations. Odibat et al. [25] proposed the generalized differential transform method and obtained series solutions for fractional differential equations. Odibat et al. [26] studied the stability and convergence of the series solution obtained by GDTM. Similarly, the Variational Iteration Method (VIM) has been widely used for solving fractional differential equations. He et al. [27,28] developed the variational iteration method for various nonlinear problems. Yang et al. [29] studied the convergence of VIM for multi-order complex fractional differential equations.

Although fractional calculus has been widely applied to electrical circuit analysis, most of the studies focus on a single method. Comparative investigations of multiple semi-analytical techniques for solving fractional RL and RC circuit models are limited. This study presents a comparative investigation of semi-analytical methods for solving fractional-order differential equations in RL and RC electrical circuits. Four widely used semi-analytical techniques are considered. Series solutions are obtained for the fractional RL and RC circuit models using these methods. The obtained solutions are compared graphically for different values of the fractional order to examine their influence on circuit responses. The study provides a systematic comparison of these semi-analytical techniques, providing their applicability and effectiveness in solving fractional electrical circuit equations.

2. Preliminaries

The section begins by outlining key definitions and the general governing equation required for providing the theoretical foundation.

2.1. Caputo Fractional Derivative

Let $\alpha \in [n-1, n)$*, where* $n \in \mathbb{N}$*, and let* f *be an* n*-times continuously differentiable function on* $[a, t]$*. The Caputo fractional derivative of order* α *of* f *is defined by*

$$ {}_{a}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(x)}{(t-x)^{\alpha-n+1}} \, dx $$

Where $\Gamma(\cdot)$ *denotes the Gamma function.*

2.2. Caputo Fractional Integration

Let $\alpha > 0$ *and let* f *be a function defined on* $[a, t]$*. The fractional integral of order* α *of* f *is defined by*

$$ I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-x)^{\alpha-1} f(x) \, dx $$

Where $\Gamma(\cdot)$ *denotes the Gamma function.*

2.3. Governing Equation

Consider a general fractional differential equation.

$A\,D^{\alpha}y(t) + B\,y(t) = E(t),\ \ 0 < \alpha \leq 1, \quad y(0) = a_0$ (1)

D^{α} denotes the fractional Caputo derivative operator.

3. Methodology

In this section, the semi-analytical methods to solve the fractional order differential equation (1) are discussed.

3.1. Adomian Decomposition Method (ADM)

The Adomian Decomposition Method (ADM) [19- 21] is a technique used to solve linear and nonlinear differential equations without linearization or discretization.

Consider a general nonlinear differential equation

$Lu + Ru + Nu = g,$ (2)

where L is the highest-order linear operator, R is the remaining linear operator, Nu contains the nonlinear terms.

Operating the inverse operator L^{-1} on both sides of equation (2) gives

$u = \varphi + L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu)$ (3)

where φ is an integration constant satisfying the condition $L\varphi = 0$.

The solution can be represented as an infinite series of the form.

$u = \sum_{n=0}^{\infty} u_n$ (4)

Also, assume that the nonlinear term Nu can be written as an infinite series using Adomian polynomials. A_n of the form

$Nu = \sum_{n=0}^{\infty} A_n$, (5)

where the Adomian polynomials A_n are evaluated using the formula

$A_n = \frac{1}{n!}\frac{d^n}{dx^n} N(\sum_{i=0}^{\infty}(\lambda^i u_i))\Big|_{\lambda=0}$ $, n = 0,1,2, \dots$ (6)

Then, using equations (4), (5), and (6) in equation (3), one gets

$\sum_{n=0}^{\infty} u_n = \varphi + L^{-1}(g) - L^{-1}(R\sum_{n=0}^{\infty} u_n) - L^{-1}(\sum_{n=0}^{\infty} A_n)$

In which each term is given by the recurrence relation

$u_0 = \varphi + L^{-1}(g),$

$u_{n+1} = \varphi + L^{-1}(Ru_n) - L^{-1}(A_n). \qquad n = 0,1,2, \dots$ (7)

Applying the fractional integral operator $D^{-\alpha}$ From equation (1), one gets

$$y(t) = a_0 + \frac{1}{A}D^{-\alpha}E(t) - \frac{B}{A}D^{-\alpha}y(t)$$

$$y(t) = \sum_{n=0}^{\infty} y_n(t)$$

$$\sum_{n=0}^{\infty} y_n(t) = a_0 + \frac{1}{A}D^{-\alpha}E(t) - \frac{B}{A}D^{-\alpha}\left[\sum_{n=0}^{\infty} I_n(t)\right]$$

$$y_0(t) = a_0 + \frac{1}{A}D^{-\alpha}E(t)$$

$y_{n+1}(t) = -\frac{B}{A}D^{-\alpha}y_n(t)$ (8)

3.2. Homotopy Perturbation Method (HPM)

For a nonlinear problem, consider

$A(y) - f = 0$ (9)

a homotopy $H(y,p)$is constructed as

$H(y,p) = (1-p)[L(y) - L(y_0)] + p[A(y) - f] = 0,$ (10)

where $p \in [0,1]$is an embedding parameter and $y_0$is an initial approximation. The solution is assumed in the form

$$y = \sum_{n=0}^{\infty} p^n\, y_n.$$

Setting $p \to 1$ yields the approximate analytical solution. HPM is computationally simple, avoids small-parameter assumptions, and provides accurate solutions with few iterations as discussed in [22-24].

Applying the fractional integral operator $D^{-\alpha}$ From equation (1), one gets

$$y(t) = a_0 + \frac{1}{A} D^{-\alpha} E(t) - \frac{B}{A} D^{-\alpha} y(t)$$

Introduce an embedding parameter $p \in [0,1]$and define the homotopy

$$y(t) = a_0 + p\left[\frac{1}{A} D^{-\alpha} E(t) - \frac{B}{A} D^{-\alpha} y(t)\right]$$

Assume the HPM series solution:

$$y(t) = \sum_{n=0}^{\infty} p^n y_n(t)$$

Substitute the series into the homotopy equation and equate like powers of p,

$$y_0(t) = a_0$$
$$y_1(t) = \frac{1}{A} D^{-\alpha} E(t) - \frac{B}{A} D^{-\alpha} y_0(t)$$

Setting $p = 1$

$$y(t) = a_0 + \tfrac{1}{A} D^{-\alpha} E(t) - \tfrac{B}{A} D^{-\alpha} y_0(t) + \cdots \tag{11}$$

3.3. Generalized Differential Transform Method (GDTM)

The generalized differential Transform as given in [25- 26] of the analytic function $y(t)$ is

$$Y_\alpha(k) = \frac{1}{\Gamma\alpha k+1}\left[\frac{d^{\alpha k}}{dt^{\alpha k}}\, y(t)\right]_{t=0} \tag{12}$$

Inverse of $Y_\alpha(k)$ is

$$y(t) = \textstyle\sum_{k=0}^{\infty} Y_\alpha(k)\, t^k \tag{13}$$

Apply the generalized differential Transform to equation (1),

$$A\frac{\Gamma(\alpha(k+1)+1)}{\Gamma\alpha k+1} Y(k+1) + BY(k) = E(k)$$
$$Y(k+1) = \frac{\Gamma\alpha k+1}{\mathrm{A}\Gamma(\alpha(k+1)+1)}\big(E(k) - BY(k)\big)$$
$$Y(0) = a_0$$

For $k = 0, 1, 2, 3, \cdots$

$$Y(1) = \frac{\Gamma 1}{\mathrm{A}\Gamma(\alpha+1)}\,(E(0) - Ba_0)$$

$$I(2) = \frac{\Gamma(\alpha+1)}{\mathrm{A}\Gamma(2\alpha+1)}\big(E(1) - BY(1)\big)$$

$$Y(3) = \frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)}\big(E(2) - BY(2)\big)$$

By continuing in this way, one gets,

$$y(t) = \sum_{k=0}^{\infty} Y(k)\, t^{\alpha k}$$

$$y(t) = a_0 + \left[\frac{\Gamma 1}{A\Gamma(\alpha+1)}\,(E(0) - Ba_0)\right] t^{\alpha} + \left[\frac{\Gamma(\alpha+1)}{A\Gamma(2\alpha+1)}\,\big(E(1) - BY(1)\big)\right] t^{2\alpha} + \left[\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)}\,\big(E(2) - BY(2)\big)\right] t^{3\alpha} + \cdots \quad (14)$$

3.4. Variational Iteration Method (VIM)

Consider a general fractional differential equation

$$D_t^{\alpha} u(t) = L(u) + N(u) + g(t), \;\; 0 < \alpha \le 1 \quad (15)$$

Where L is the linear operator, N is a non-linear operator, and $g(t)$is a known source term.

The variational iteration correction functional for a fractional equation is given in [27-29] as

$$u_{n+1} = u_n + \int_0^t \lambda(t,\tau)[D^{\alpha} u_n(\tau) - L(u_n) - N(u_n) - g(\tau)]\, d\tau, \;\; for\ n = 0,1,2,\cdots$$

Where $\lambda(t)$ is the Lagrange Multiplier.

VIM gives a series solution as

$$u(t) = \lim_{n\to\infty} u_n(t) = u_0(t) + \sum_{k=1}^{\infty} u_k(t)$$

Each term $u_k(t)$ is obtained iteratively and usually involves fractional power terms $t^{k\alpha}$.

Now write equation (1), which can be written as

$$D^{\alpha} y(t) = \frac{1}{A} E(t) - \frac{B}{A} y(t)$$

The variational iteration correction functional for a fractional equation is

$$y_{n+1} = y_n + \int_0^t \lambda(t,\tau)\left[D^{\alpha} y_n(\tau) + \frac{B}{A} y_n(\tau) - \frac{1}{A} E(\tau)\right] d\tau$$

$\lambda(t)$- Lagrange Multiplier

$$\lambda(t,\tau) = -\frac{(t-\tau)^{\alpha-1}}{\Gamma\alpha}$$

$$y_{n+1} = y_n - \frac{1}{\Gamma\alpha}\int_0^t (t-\tau)^{\alpha-1}\left[D^{\alpha} y_n(\tau) + \frac{B}{A} y_n(\tau) - \frac{1}{A} E(\tau)\right] d\tau \quad (16)$$

Choose the zeroth approximation to satisfy the initial condition

$$y_0(t) = a_0$$

$$y_1 = a_0 - \frac{1}{\Gamma\alpha}\int_0^t (t-\tau)^{\alpha-1}\left[\frac{B}{A} a_0 - \frac{1}{A} E(\tau)\right] d\tau$$

$$y_1 = a_0 - \frac{Ba_0}{A\Gamma\alpha + 1} t^{\alpha} + \frac{1}{A\Gamma\alpha}\int_0^t (t-\tau)^{\alpha-1} E(\tau)\, d\tau$$

By continuing in this way,

$$y(t) \approx a_0 - \frac{Ba_0}{A\Gamma\alpha+1} t^{\alpha} + \frac{1}{A\Gamma\alpha}\int_0^t (t-\tau)^{\alpha-1} E(\tau)\, d\tau \quad (17)$$

4. Application to Fractional-Order Electrical Circuit
4.1. Formulation of a Problem

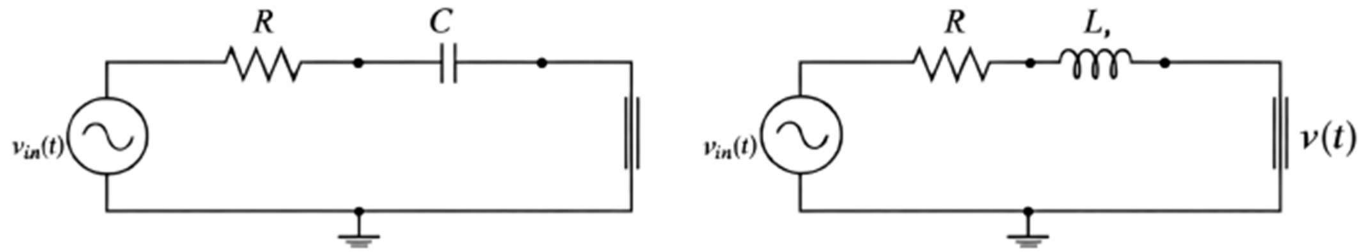

Fig. 1 RC and RL Electrical Circuits

The governing ordinary differential equations of RL and RC electrical circuits can be obtained by applying Kirchhoff's laws.

$$\frac{dI(t)}{dt} + \frac{R}{L}I(t) = \frac{E(t)}{L} \tag{18}$$

$$\frac{dV(t)}{dt} + \frac{V(t)}{RC} = \frac{E(t)}{RC} \tag{19}$$

With initial Conditions $I(0) = I_0$, $V(0) = V_0$

Here R, L, C & $E(t)$ are resistance, inductance, capacitance, and source of voltage, respectively.

Traditionally, when a mathematical model is framed using a fractional derivative, one replaces $\frac{d}{dt}$ by $\frac{d^\alpha}{dt^\alpha}$, $0 < \alpha \le 1$, where α represents the order of the derivative. It is remarkable to know that the writers of [7, 8] noted that the above replacement is not entirely accurate from a physical point of view because of the time derivative $\frac{d}{dt}$ has dimensions of inverse seconds (s^{-1}) while the time fractional derivative $\frac{d^\alpha}{dt^\alpha}$ has $(s^{-\alpha})$. According to the statement of Joseph Fourier [30], every physically valid equation will have the same dimensions on the left and right sides. Hence, to meet the prerequisites of physical reality, one needs some modifications. In order to keep up the dimension of time, the authors of [6–9] have introduced the new auxiliary parameter σ as

$$\left[\frac{1}{\sigma^{1-\alpha}}\,\frac{d^\alpha}{dt^\alpha}\right] = \frac{1}{s}, \quad 0 < \alpha \le 1 \tag{20}$$

Hence, generalize equations (18) and (19) as

$$\frac{1}{\sigma^{(1-\alpha)}} D^\alpha I(t) + \frac{R}{L}I(t) = \frac{E(t)}{L}, 0 < \alpha \le 1 \tag{21}$$

$$\frac{1}{\sigma^{(1-\alpha)}} D^\alpha V(t) + \frac{V(t)}{RC} = \frac{E(t)}{RC}, 0 < \alpha \le 1 \tag{22}$$

With initial Conditions $I(0) = I_0$, $V(0) = V_0$

Equation (21) is dimensionally reliable iff the parameter σ has the dimension of time. $[\sigma] = 1$. When $\alpha = 1$, the expression (21) becomes an ordinary derivative operator $\frac{d}{dt}$. So, they have suggested that. $\frac{d}{dt}$ would be replaced by $\left[\frac{1}{\sigma^{1-\alpha}}\,\frac{d^\alpha}{dt^\alpha}\right]$ in an ordinary differential equation to form an FODE.

4.2. Application 1: Fractional R-L Circuit

$$\frac{1}{\sigma^{(1-\alpha)}} D^\alpha I(t) + \frac{R}{L}I(t) = \frac{E(t)}{L}, 0 < \alpha \le 1 \tag{23}$$

With initial Conditions $I(0) = I_0$
Rewrite the above equation as

$$D^{\alpha} I(t) = \frac{\sigma^{(1-\alpha)}}{L} E(t) - \frac{\sigma^{(1-\alpha)} R}{L} I(t)$$

Define $a = \frac{\sigma^{(1-\alpha)}}{L}, \quad b = \frac{\sigma^{(1-\alpha)} R}{L}$

Solution of the proposed problem (23) by using the above-discussed methods is given as follows:

The solution obtained by the Adomin Decomposition Method is

$$I(t) = I_0 + aD^{-\alpha} E(t) - bD^{-\alpha}[I_0(t)] + (-b)^2 D^{-2\alpha}[I_0(t)] + \cdots$$

The solution obtained by the Homotopy Perturbation Method is

$$I(t) = I_0 + aD^{-\alpha} E(t) - \frac{b I_0 t^{\alpha}}{\Gamma\alpha + 1} - abD^{-2\alpha} E(t) + \frac{b^2 I_0 t^{2\alpha}}{\Gamma 2\alpha + 1} + \cdots$$

The solution obtained by the Generalized Differential Transform Method is

$$I(t) = I_0 + \left[\frac{\Gamma 1}{\Gamma(\alpha+1)} \left(aE(0) - bI(0)\right)\right] t^{\alpha} + \left[\frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} \left(aE(1) - bI(1)\right)\right] t^{2\alpha} + \left[\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)} \left(aE(2) - bI(2)\right)\right] t^{3\alpha} + \cdots$$

The solution obtained by the Variational Iteration Method is

$$I(t) = I_0 - \frac{b I_0}{\Gamma\alpha + 1} t^{\alpha} + \frac{a}{\Gamma\alpha} \int_0^t (t-\tau)^{\alpha-1} E(\tau)\, d\tau + \cdots$$

For transient response plots, set the circuit parameters to $R = 100\ \Omega$, $L = 50$ mH, and step input voltage $E(t) = 10$ V, with initial current $I(0) = 0$. Fractional orders $\alpha = 0.8, 0.9, 1.0$ were used to illustrate the transition from fractional to classical dynamics, in accordance with existing studies on fractional-order R–L circuits.

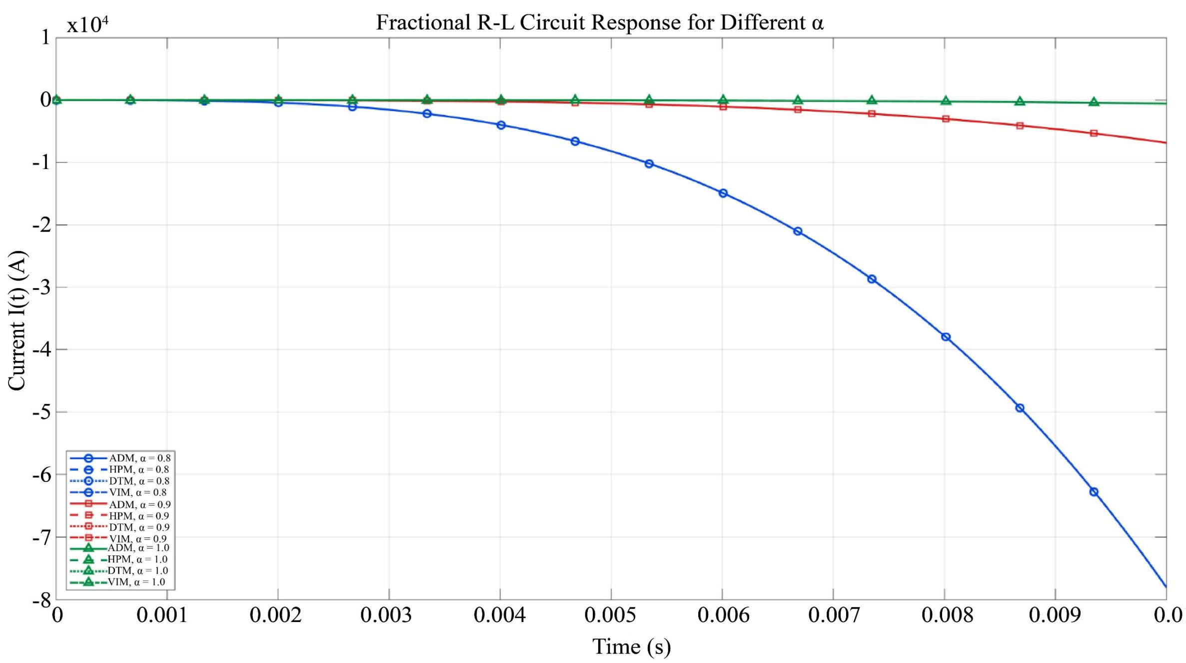

Fig. 2 Comparison of Semi-Analytical Solutions Obtained by ADM, HPM, GDTM, and VIM for Fractional Orders α = 0.8, 0.9, and Integer Order α = 1

The fractional-order responses ($\alpha = 0.8, 0.9$) exhibit delayed current evolution compared to the integer-order case, reflecting the presence of hereditary effects in the system. The overlap of curves obtained by using ADM, HPM, GDTM, and VIM demonstrates the consistency and convergence of the employed semi-analytical methods.

4.3. Application 2: Fractional R-C Circuit

$$\frac{1}{\sigma^{(1-\alpha)}}D^{\alpha}V(t)+\frac{V(t)}{RC}=\frac{E(t)}{RC}, 0<\alpha\leq 1 \quad (24)$$

With initial Conditions $V(0)=V_0$

Rewrite the above equation as

$$D^{\alpha}V(t)=\frac{\sigma^{(1-\alpha)}}{RC}E(t)-\frac{\sigma^{(1-\alpha)}R}{RC}V(t)$$

Define $a=\frac{\sigma^{(1-\alpha)}}{RC}$, $b=\frac{\sigma^{(1-\alpha)}R}{RC}$

Solution of the proposed problem (24) by using the above-discussed methods is given as follows:

The solution obtained by the Adomian Decomposition Method is

$$V(t)=I_0+aD^{-\alpha}E(t)-bD^{-\alpha}[I_0(t)]+(-b)^2D^{-2\alpha}[I_0(t)]+\cdots$$

The solution obtained by the Homotopy Perturbation Method is

$$V(t)=I_0+aD^{-\alpha}E(t)-\frac{bI_0t^{\alpha}}{\Gamma\alpha+1}-abD^{-2\alpha}E(t)+\frac{b^2I_0t^{2\alpha}}{\Gamma 2\alpha+1}+\cdots$$

The solution obtained by the Generalized Differential Transform Method is

$$V(t)=I_0+\left[\frac{\Gamma 1}{\Gamma(\alpha+1)}\left(aE(0)-bI(0)\right)\right]t^{\alpha}+\left[\frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}\left(aE(1)-bI(1)\right)\right]t^{2\alpha}+\left[\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)}\left(aE(2)-bI(2)\right)\right]t^{3\alpha}+\cdots$$

The solution obtained by the Variational Iteration Method is

$$V(t)=I_0-\frac{bI_0}{\Gamma\alpha+1}t^{\alpha}+\frac{a}{\Gamma\alpha}\int_0^t(t-\tau)^{\alpha-1}E(\tau)\,d\tau+\cdots$$

For the transient voltage response of the fractional-order R–C circuit, the circuit parameters are chosen as $R=100$ and $C=100$, with a step input voltage $E(t)=10$ V and zero initial capacitor voltage $V(0)=0$. Fractional orders $\alpha=0.8, 0.9,$ and 1.0 are considered to demonstrate the transition from fractional to classical charging response. These parameter values are commonly employed in experimental and theoretical studies of fractional-order R–C circuits and allow a clear comparison between memory-dependent fractional dynamics and the conventional integer-order response.

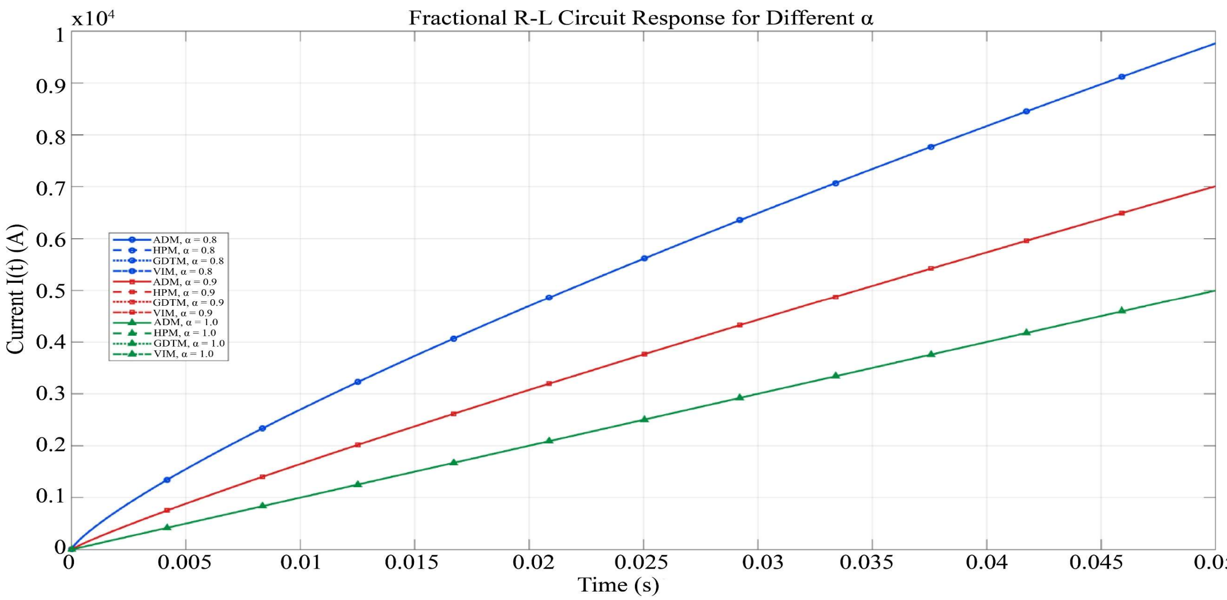

Fig. 3 Comparison of Semi-Analytical Solutions Obtained by ADM, HPM, GDTM, and VIM for Fractional Orders α = 0.8, 0.9, and Integer Order α = 1.

In the above figure, the transient voltage response shows that lower fractional orders (α = 0.8, 0.9) exhibit a slower rise compared to the classical integer-order case (α = 1.0). It reflects the memory-dependent nature of fractional capacitors. The delayed voltage responses observed for $\alpha < 1$. It is directly correlated with dielectric relaxation, anomalous diffusion, electrode polarization, and visco-electric behaviour.

5. Conclusion

This study presented a comparative analysis of four semi-analytical techniques for solving fractional-order RL and RC circuits. Transient responses were evaluated for $\alpha = 0.8, 0.9, and\ 1.0,$ demonstrating the smooth transition from fractional to classical electrical response. The results show that all methods produce identical numerical solutions when the same number of series terms are utilized, all methods yield the same numerical solutions. For linear fractional circuits, ADM and HPM provide computational efficiency and rapid convergence. For complex nonlinear systems, GDTM and VIM are useful alternatives. The study illustrates that semi-analytical methods provide reliable and efficient tools for analyzing fractional-order electrical circuit models in applied mathematics and electrical engineering.

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