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Original Article

Factorization of Integers

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Abstract - *Considering the breakthrough that happened recently in math, which established the condition for primality versus composite numbers, the decomposition of large numbers has been put to rest. To fully comprehend factorization of integers, this paper presents this issue with regard to the area of cryptography.*

A cryptosystem has two primary functions: to encipher, which means to prepare the message in a coding scheme, and to decipher the information, which means to decode the message. This decryption cannot be accomplished without knowledge of the secret deciphering key. For this reason, it is natural to look at an ancient problem of number theory, the problem of finding the complete factorization of a large composite integer whose prime factors are not known in advance.

To reach this goal, an algebraic system is presented herein that is robust enough to handle large numbers. This system will encompass all that we know about algebraic properties, including the integration of some known theories of integers (e.g., lattice points representation). Parametric representation of curves as single points is necessary because these curves, after analysis, will be a piece of information toward the limit of the function f. A pattern will be established using the gap between a point on the rational plane and a point on the irrational plane. In doing so, it will be proven that the curves under study converge to the curve that holds the key to factorization. A demo is available that will perform the principleof factorization in nlogn complexity (i.e., the length of the input of integers versus the running time of the output of the function). This is called the time complexity of thealgorithm, and it will be shown, herein, that indeed it is nlogn complexity. This work is designed to answer related problems in the field of mathematics, such as trisecting angles in geometric figures, banking issues such as transfer of funds, and communication integrity. However, the strongest modern algorithms (quadratic sieve, elliptic-curve method, and number field sieve) have been unable to resolve any of these ten numbers.

It can be stated that the current effective limit of systematic factoring is ~100 digits, but it is still instructive and rewarding to find factors of much larger numbers. This is where a sandwich could make a difference; where a triangulation can take place to control false readings in conjunction with backtracking, leveraging new computers and graphic abilities. This new math breakthrough that has occurred enables the capability of distinguishing prime numbers from composite numbers. Resolving the latter into its prime factors is the last hurdle that needs to be crossed.

Keywords *– Integers, RHO, Gauge Point Calibration.*

1. Introduction

The breakthrough in math, which established the condition for primality versus composite numbers, is put to rest. The decomposition of large numbers becomes the next challenge in sight. However, to fully comprehend the factorization of integers, this paper presents the issue of the decomposition of large numbers within the area of cryptography. The RHO method was the process utilized to attempt factorization, but this process is inefficient when attempting factorization of large numbers. The culprit is the difficulty of creating a partial order in the field of integers capable of converging with the integer solution of any lengthy numeral. This paper walks through all the critical segments necessary and sufficient to achieve partial order in the field of integers. Some of this segment includes a purge equation that manipulates the parameters of a triangle in a concentric circle. The Pell equation combines the linear indeterminant problems and the continuous fraction algorithm as one of the indices flagged in the direction of the integer solution. Finally, the gap between the rational point and the irrational point inside is associated with the Niven, Ivan limit point that controls the length of the decimal field born out of the integer field under examination. Hence, the target equation will converge where the solution set resides. The tableau proof presented here is a set of rules that govern the reduction formulae into an algorithm that is structured inside a limit point that closes all the paths toward a converging point inside the integer field that reveals the solution set being searched. These segments become mandatory after a literature review of what is available has been conducted.

2. Overview: Executive Summaries and Supportive Documents

2.1. Preservation Principle of Integers

One of the properties of algebra is the preservation principle, which is a mapping of one system to another, preserving all inherent properties of the mother system. A polynomial is always expressed in the following form: $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots a_0 x^0$. Since the 2nd degree equation contains the quadratic formula that allows it to solve for its roots, one should map the polynomial into the quadratic form.

This mapping requires two shifts:
Shift 1: To collapse the coefficient.
Shift 2: To represent the polynomial in a monic form where a = 1.

2.2. The Quadratic Formula

The quadratic formula is used to investigate the existence of roots of a quadratic polynomial, $f(x)=ax^2+bx+c$, over a field F, and describe the roots of "x" (provided they exist) in terms of the coefficients a, b, and c. It will be necessary to exclude from consideration all fields in which 1+1=0 (i.e., field Z_2). If p is an element of a field F and there exists in F an element q such that $q^2=p$, then p is called a perfect square in field F.

The following theorems are considered:

Theorem 1: Gauss
Theorem 2: Structure of a quadratic in Field F
Theorem 3: Positive definite form.
Theorem 4: There is an α in L in which ($(\alpha^2 = 2^{1/2})^2$)
Theorem 5: The linear indeterminate problem

This presentation begins with the Gauss theorem and proceeds to the Theorem of Support before the formulae for factorization are presented algorithmically. This is done in the following format in view of the executive summary presented for this discussion:

Theorem 2 (Gauss): The number of lattice points inside an ellipse is given by:

$$N=2\pi T/\sqrt{\Delta}\ 2 \quad +\text{Error} \qquad 1.0$$

Where the error is bounded by k' √Tas T. Therefore, the error has an "order of magnitude" √T

Now:

$N= 2\pi T/\sqrt{\Delta}+\text{Error}.$	Since $3m^2 = \Delta$ thus:
$N= 2\pi T/\sqrt{3m^2}$	Square both sides
$N^2= (2\pi T/\sqrt{3m^2})^2$	
$N^2= 4\pi^2 T^2/3m^2$	Letting m= T= l and N=√N 2.0
$((N)^{1/2})^2=4\pi^2/3<4/3$	Majoring the inequality

(Calculus Terminology of ε, δ (Epsilon-Delta Limit of Functions)

2.3. The Gauge Point Calibration

The above formula is the rate of a fixed single point on the plane. Thus, the rate that is needed is 4/3 if $0<b<c$, if N=4/3 for a fixed point inside the plane. This point inside the plane can be converted into an equation inside the plane. However, is this the least upper-bound equation in the F field? This question is paramount to avoid overshooting the limit point and overlooking the roots of the polynomial. An analysis of the limit point will be done. Namely, working with 4/3 so that the significance of the latter can be outlined in the work.

Furthermore, the number of lattice points inside an elliptic curve is associated with error estimations. To eliminate the possibility of errors with the choice of the starting equation, the least upper-bound equation produced by the limit rate of 4/3 needs to be investigated.

Note: 4/3 is a fixed point in the area of the elliptic curve. However, it is not necessarily the least upper-bound points:

$x^2 + (240\text{-}A)x + l0A + l = \sqrt{N}*(4/3)$ 3.0
Since the goal is to solve for $b^2\text{-}4ac=(\sqrt{N}*(4/3))^2$, where b=240-A and c= l 0A+1:
$(240\text{-}A)^2\text{-}4(10A+1)=(\sqrt{N}*(4/3))(\sqrt{N}*(4/3))$

2.4. The Third Analysis (The Gap Point Between the Rational Plane and the Irrational Plane)

The third analysis is to discuss the gap point between the rational plane and the irrational plane. The other point is the closest irrational point that can be found with the Pythagorean triple. To avoid the error associated with solving for x using the Dirichlet Series, the Continuous Fraction algorithm is favored. Therefore, (x, y) in the Pythagorean point becomes the seed x_1, x_2 used to estimate the irrational gap point. Thus, if p/q=209/56, then the Pythagorean point is (23408/46817,40545/46817) or (0.499989, 0.866032).

The closest irrational point is (√3/2,1/2). Where: 23408/46817=0.499989 can be approximated to ½. Using this initial value, the Pell equation can now be used ($x^2+y^2=1$) to complete the point and find the x value, √3/2. Here, m/n is defined as the slope of the points derived from the factors of the equation, thus (-n,0);(0,-m), thus 209/56= 3.73214. Furthermore, when the slope is between the unit point and the irrational point, namely (1.0);(√3./2,1/2), the result is 2 + √3 is 3.73205. There is a difference of only 0.00009, meaning the gap between the rational point and the irrational point is 0.00009.

2.5. Significance and Guide to the Theorem

To conclude the preliminary studies and enhance the application of proof system design where the goal is to close all leaves of a derivation formula, the area of Automated Deduction in non-classical logic will be used.

The reduction rules are the tableau rules. Now the significance of finding the gap points by the process of calculus, namely integration by parts, can be explained. However, the latter proved to be too difficult at best, so an alternative way to get the irrational gap slope is necessary. Therefore, the Riemann Hypothesis concept, specifically the line integral $L_i(x)$, is best for this equation. New parameters and rules needed for the final reduction formula will be established in this paper. Furthermore, this completes the work, and an inductive process can achieve factorization.

3. Linear Measurement Analysis

According to the Preliminary Studies outlined in the Extended Abstract, instead of applying Dirichlet integration by parts, a linear measurement is proposed here to identify the estimated gap slope. The closest irrational boundaries can be found by applying a linear measurement of:

$$L_1(x) + \log(n)*K= m/n \qquad 7.0$$

- In the log integral function $\int_0^x (1/logt)\, dt$, the usual symbol is $L_i(x)$. $L_i(x)$ is defined to be the area under the graph of 1/logt, from 0 to x. The x value is located where the area under the curve of the function 1/log(t) cancels the negative region of the graph of 1/log(t). This is an important value when researching the Riemann Hypothesis. Since the Pythagorean triple seed is being used, one of its special characters, Equal Side Legs, must be analyzed.

- Theorem 4: There is an α in Lin which ($\alpha^2 = 2^{1/2})^2$)

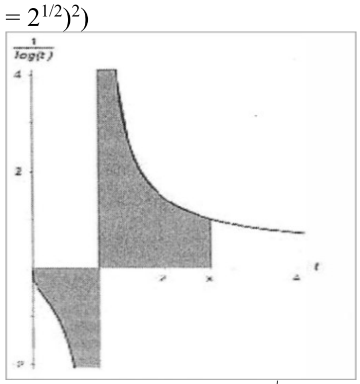

Fig. 1 Case of the Equal Side Triangle (1, 1, √2) and its Consequences

Given the 90° angle triangle with side (1, 1, √2), the ratio of the leg is 1, and the hypotenuse is √2 due to $a=m^2-n^2$; $b=2mn$; $c=m^2+n^2$ which was defined earlier, a Pythagorean triple cannot be achieved since it is given that both sides are 1. If a = 1, then b must be a number other than 1.

Since the hypotenuse of that triangle is irrational, that is, it is equal to 1.4142..., then the constant irrational needed for this must be different from 1.4142...

3.1. Why 1.4142 is Not Considered

This is analogous to finding the horizontal asymptote of a rational function.

$$\text{Suppose: } f(x) = 3/(x + 1) + 2;$$

When graphing this function by transformation, we usually say that if x could be equal to -1, then f(x) will be equal to 2. However, x can never reach -1 f(x), and HA will never reach 2. Thus, in accordance with the previously defined rules, the Pythagorean triple legs of the triangle cannot be 1. The hypotenuse of the square root of 2 will never be achieved. Therefore, no triangle from this family can be put as a Pythagorean triple.

3.2. What Can Be Said About Complex Sides (Theorem 4)?

Given a 90° angle triangle with side (1, 1, √2), the ratio of the leg is 1, and the hypotenuse is √2. Keep in mind the same requirement $a= m^2-n^2$; $b=2mn$; $c=m^2+n^2$ is needed. If all the sides are doubled (2, 2, 2 √2), side a=2 is the same as $a=1-i^2$; b=2, c=2 √2, these changes still allow for the same result as with (1, 1, √2). With this knowledge in place, the following case can be considered:

If the conjugate $a = 1 + i^2$ is used, which is essentially a=0.

If this is possible, then c=2. However, this is a contradiction because c can never reach 2 by the same argument raised earlier. However, there is an interval to consider: 1.4142 ... <"c"<2.0. The question now is, can this be improved upon?

To answer this, the log integral function is utilized. Since the log and a constant in a linear measurement of slope were used, the constant must be located where the area under the curve of the function 1/log(t) cancels the negative region of the graph of 1/log(t); where the x value of 1.4513692348828 was found. So, L_1=1.4513692348828…

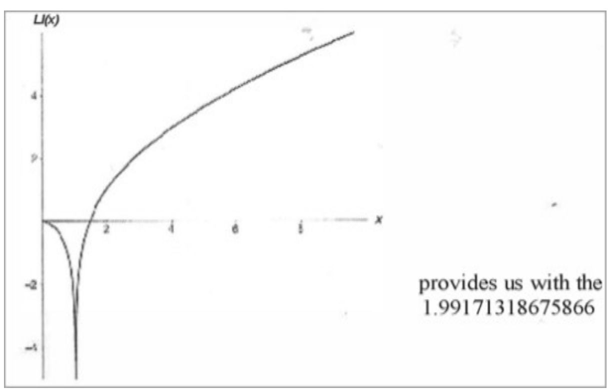

Fig. 2 The Inverse of the Log Resulting Value

The inverse of the log resulting value results in 1.9917138675866, which is extremely close to the asymptote upper boundary of 2 for the "c" evaluated discussed previously. Since the Riemann Hypothesis uses the zeta function primarily to discuss prime numbers versus composite numbers, the critical strip of the Riemann Hypothesis has a value close to the point of origin of the log integral function. Thus, this value must also be investigated.

$$\zeta(1/(2)= 1.4603545088095$$

Notice the inverse of the log provides a resulting value of 1.9833061721671. This value is not close enough to 2 and, therefore, demands no further analysis. Thus, 1.4513692348828 can be considered the point of origin, and the constant will be centered here.

3.3. What is the Gradient and How Can it Be Measured?

First, define the gradient for a sloping straight line. It is the vertical rise divided by the horizontal span. To get the gradient of a curve at any point, construct one straight line that touches the curve at that point. Plainly, there is only one such line; if the line "rolls" a little (e.g., suppose the line is a steel rod and the curve is a steel band), it touches the curve at a slightly different point. The gradient of the curve at the point is the gradient of that unique touching straight line. The gradient of log x at the argument x=10, if measured, turns out to be 1/10. Due to the log function, the gradient at any argument x is 1/x. Thus, the equation for the irrational is fully defined to be:

$$L_1 + \log(n)*K = m/n \qquad 7.0$$

3.4. How Does the $L_1 + \log(n)*K =$ min Equation Relate To The Tables?

Given the irrational slope that is found, it measures the closest distance to the slope of the curve m/n, and that another irrational slope can be found to be the closest distance to the Pythagorean triple curve using the minimum distance of the seed: $x_1=(m^2-n^2)/(m^2+n^2)$ or $x_2=2mn/(m^2+n^2)$ with the Pell equation $x^2+ y^2 = 1$.

The goal is a strip found in the Sandwich. The two curves (the original curve under study and the Pythagorean curve) will provide the closest estimated slope of the irrational. Thus, calculating their deviation should be small in comparison to the slope of the curve m/n and the minimum point estimate between these two seeds ($x_1 = (m^2 -n^2)/(m^2+n^2)$ or $x_2 =2mn/(m^2+n^2)$); in the table named "P_2 and alternate P_2 (Alt P_2)."

Strip of Convergence Equation Between:

$\text{Min } (L_1+ \log(n)*K= (x_1=(m^2-n^2)/(m^2+n^2), x_2=2mn/(m^2+n^2))$ 8.0

And, $L_1+ \log(n)*K= m/n$;

It is likely to find deviations that are not small. Curves having this characteristic should be eliminated from the "Sandwich Consideration." In fact, if the Pythagorean triple seed is estimated to two decimal places, then the deviation should have an accuracy level of at least 4 decimal places. In general, the deviation is estimated to be 2p, where p stands for the number of decimal places (see tables for illustration).

3.5. Proof

In using the Pell Equation, namely $x^2 + y^2 = 1$, and the Pythagorean triple value as a seed mapping two decimal places into the equation, it will give a y value of at least 5 decimal places. Given that when a square root is taken, the value reaches its peak growth for small decimals:

- Decrease in value until it doubles in value when it reaches 0.5.
- Decrease in significance until it reaches a growth almost equivalent to the seed value. Thus, for an irrational deviation slope to be significant, it must match the deviation of the Pythagorean counterpart up to 2p. Therefore, for a two-decimal expansion accuracy, the map to the Pythagorean equation will reveal a value that is significant to at least a length of 5. Therefore, the 5^{th} place will hold a significant digit. Any slope outside that range will be dropped from consideration. The false values that were apparent in parameter P1 in the linear indeterminate value problem illustration will now be under control (ref. Analyses of Linear Indeterminate Problems).

3.6. Theory of Linear Indeterminate Problems (Theorem 5)

When "a" and "b" are relatively prime, it is possible to find other integers such as "x" and "y" that represent:

$$ax + by = l \qquad 9.0$$

This may be stated differently by saying that unity is a linear combination of "a" and "b." The proof is an immediate application of Euclid's algorithm. To make the notation more systematic, it is supposed that a > b.

Write $a = r_1$, and $b = r_2$, and perform the algorithm for each element in field F:

$$r_1=q_1r_2 +r_3$$
$$r_2=q_2r_3+ r_4$$

****
$r_{n-3}=q_{n-3}\bullet r_{n-2} + r_{n-1}$
$r_{n-2}=q_{n-2}r_{n-1} + 1$

The last remainder is 1. Since “a” and “b” are relatively prime, it can be reconstructed by a stepwise process, “a” and “b.” Begin at the bottom and write 1 as a linear combination of r_{n-2} and r_{n-1}, as follows:

$$1= r_{n-2}-q_{n-2}r_{n-1}$$

Here, substitute from the next to the last division:

$$r_{n-1}=r_{n-3} - q_{n-3}r_{n-2}$$

After the arrangement, the following is obtained:

$$1 = q_{n-2}r_{n-3} + (1+q_{n-2}q_{n-3})r_{n-2}$$

So that 1 has been represented as a linear combination of r_{n-3} and r_{n-2}. From the third to last relation, introduce the following:

$$r_{n-2}=r_{n-4}-q_{n-4}r_{n-3}$$

Moreover, in a similar manner, express 1 linearly by means of rn4 and rn3. This process continues, as the Theorem requires, until arriving at a linear combination of r1=a and r2=b equal to 1. This process is needed to avoid using integration and to find the irrational point. The Euclidean Algorithm provides the necessary tool to find the closest rational to the gap point. However, how to use this information and find a pattern between all the equations in field F is the next challenge. For example, to factor the numeral 901, N = 901 is implied.

Example 1: Using (4/3):

Suppose N=901, x=10, then:

Step 1: $9x^2 + 0x + 1$ According to Shift 1
Step 2: $x^2 + 80x + 1$. According to Shift 2
Step 3: $x^2 + (80-A) x + 10A + 1 = \sqrt{901}*(4/3)$ Boundary of equation

Now, to solve for $b^2-4ac = (\sqrt{N}*(4/3))^2$ where b = 80-A, and c = 10A + 1.

Solving:

$(80-A)^2 - 4 (10A + 1) = (\sqrt{901}*(4/3)) (\sqrt{901}*(4/3))$
$A^2-160A + 6400-40A- 4= (40)(40)$

Simplifying:

$A^2 - 200A + 4796 = 0$

Solving for A:

$A= (-b-\sqrt{b^2-4ac})/2a$

(This value is not considered because it is beyond the upper-bound)

$A = (-b+ \sqrt{b^2-4ac})/2a$
$A = (-200+ \sqrt{40000-19184})/2$
$| A | = 28$Using Step 3, replace A by its value:$x^2 + (80-28) x + 10*28+1=0$

Thus, the new maximum value is:

$x^2 + 52x + 291 = 0$

However, since it is not known in advance where the equation with an integer solution lies, we can see that the equation associated with the rate 4/3 is too close and there is a risk of overshooting. The Sandwich goes as follows (going back to 901 and the equations in field F):

```
Procedure de(var a,b,dl,c,bl,cl:real; var
fa,nombre,si,n23:longint);
var
  i :integer;
  begin
    repeat
      begin
        b: = b-1;
        c: = c+lO;
        detl: = sqrt (((b*b)-4*c)); factl: = abs ((-b+detl)/ 2);
        fact2: = abs ((-b+detl)/2);
        bl: = b;
        cl: = c;
        dl := c;
        end;
        until (factl >= l) and (fact2>3) ;
  end.
```

Using the procedure "de," it was found that the least upper bound that satisfied the field is $x^2+62x+181=0$. This exceeds the one identified by the limit point, namely $x^2+52x+291=0$, due to the error associated with the estimation of the area. However, now add the new Max into the Sandwich.

When one factors $x^2 + 62x + 181 = 0$, the result is considered to be the lattice point representation of the equation in parametric form, i.e., 58.928480088 and -3.0715199125. A second transformation needs to take place to view these points as the indeterminate problem stipulated earlier.

First, truncate the decimal portion of the numeral -58, -3. Second, the points must be considered as fractions, and so the continuous fraction algorithm must be performed. Therefore, 58 and 3 are the only positive integers and will be the only integers of concern going forward. However, according to the continuous fraction algorithm, only numbers that are prime to each other can be worked with. Therefore, the following must be done:

a) Find the greatest common divisor between (58, 3) = 1
b) Perform the continuous fraction algorithm:

58/3
58: = 3*19+ 1
3= 1*3 +0

c) Rebuild "a" and "b" systematically according to the theory of indeterminate problems: Transposing 58: = 3*19 to the other side of the equation, we get: 58-57 = 1
Transposing 57= 3*19 again we find that 58-57= 1
Decomposing "b" into b_1*b_2 allows us to have the largest factor with respect to the ratio b_2 / b = 19/57.
Note: $b_1 = 3$, $b_2 = 19$

Now:

$$x^2+41x + 391 = 0$$

Factors into:

-25.9058326913, -15.091673087

Using truncation -25 – 15, then using the absolute value obtained [25, 15]:

CGD (25, 15) =5

Now the continuous fraction algorithm is working with 3/5:

3-2*1 = 1
3*2-5 = 1
Ratio b_2/b vs l/n where b=b1 *b2
2/6= 1/3 vs 1/15.

The sides of the first triangle generated by the roots of the equation, following the given rules: Let m, n imply the roots of the first equation, then the sides of the triangle will be:

$$a= m^2-n^2;\ b= 2mn;\ c= m^2+n^2$$

This is how to map a triangle of an inscribed circle.

Having these sets now allows the sides to be mapped as a point inside a circle. Considering the gap point as an initial value that obeys the scheme used for the Pythagorean values, $x_1 = (m^2 - n^2)/(m^2+n^2)$;$x_2 = 2mn/(m^2+n^2)$, the initial value for the Pell equation "$x^2+y^2= \pm 1$" can be estimated. Meaning $x = (m^2- n^2)/(m^2+n^2)$. Using this x, a y value can be found in accordance with the Pell equation ($x^2 + y^2= \pm 1$). This is how the irrational point mentioned earlier is found, and it needs to be done for each equation. In this document, the 4th table of convergence of 901.

4. Convergence Tables

4.1. First Iteration Table

Table 1: 1st Iteration 901 $X^2 + 62x +181=0$ and $x^2+4\,lx+391=0$ 0.85696017509 dcvl 0.81665382639 dcv2 (*dcvl,dcv2: mean decimal deviation of eq1 or eq_2)
0.3255717319 1/n 0.33333333b_2/b diff 0.00761620145 **P_1 (**Parameter one for Sandwich) eq_1 ↓ according to the results of Indicators 2 and 3, namely P_2 and P_3, tentative orientation of P_1 0.066261705660 1/n 0.333333333 b_2/b diff 0.26707162767 **P_1 (**Parameter one for Sandwich) The first two roots of the equation: -58.928480088; -3.0715199125 The second two factors: -25.9058326913; -15.091673087 The sides of the first triangle generated by the roots of the equation are: -3463.1315309 362 3482 The gap point is as followed $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$ estimated to two decimal points. Now using the Pell equation ($x^2+y^2 = 1$) will give you the gap point for y: 3463.1315309/3482=0.99458l=0.99=.9801=1-.9801=.0199^.5=0.141067 Using the (0,1) point and the gap approximation point in a slope formula, we have: (0,1);(.99,.141067)=14.1067 Alt P_2 This is considered because there are 2 choices of x, y values for the seed: 19.1854-14.1067=5.0787 L_1+ log(x)*K= $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$

Table 1 *(continued)*: 1st Iteration 901 $X^2 + 62x +181=0$ and $x^2+4\,lx+391=0$ 0.85696017509 dcvl 0.81665382639 dcv2 (*dcvl, dcv2: mean decimal deviation of eq1 or eq_2)
L_1+ log(x)*K= m/n Finding k for 58.928480088:

1.4513692348828+(1/58.928480088) *k = 14.1067; k = 745.76
1.4513692348828+(1/58.928480088) *k =19.1854; k = l045.04
Gap for 58.928480088:
1.4513692348828+(1/58.928480088) *745. 76= 14.106710063058
Boundaries of original curve m/n:
1.4513692348828+(1/58,928480088) *1045.04= 19.185408844243
Deviation for 58.928480088 of irrational with Pythagorean:
14.1067-14.106710063058=0.000010063058
Deviation for 58.928480088 with curve m/n:
l9,185408844243-19.1854=0.000009
Estimate slope for 58.928480088 of irrational boundaries:
19.185408844243-14.10671006358=5.078698780663
Net deviation in the strip for 58,928480088:
5.0787-5.078698780663=0.000001219337* (ok curve)
The sides of the second triangle generated by the roots of the equation are:
443.48280688 781.925 898.871
The gap point is as follows: $x_1 =(m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$ estimated to two decimals points.
Now using the Pell equation ($x^2+y^2 = 1$) will give you the gap point for y:
443.48280688/898.871=.493378=.49=.49^2=.2401=1-.2401= 7599^.5=.871722
(.0,1);(.49,.871722)= .3,81722 vs 1,71656 slope diff=2.10327* P_2
781.925/898.871=0.869897=0.87^2=0.7569= 1-0. 7569=$.2431^{0.5}$=0.493052
Using the (0,1) and the gap approximation point in a slope formula:
(0, 1) ;(0.87,0.493052) =1.71615 Alt P_2
1. 716556-1.71615=0.000405
Purge Equation:
L_1+ log(x)*K= $x_1 =(m^2-n^2)/(m^2+n^2)$ or $x_2 =2mn/(m^2+n^2)$
L_1+ log(x)*K= min
Finding k for 25.9058326913:
l .4513692348828+(1/25.9058326913) *k=l.71615; k=6.86
1.4513692348828+(1/25.9058326913)*k=l.716556; k=6.87
Gap for 25.9058326913:
1.4513692348828 + (1/25.9058326913) *6.86=1.7161744654942
Deviation for 25.9058326913:
1.7161744654942-1.71615=0.000024465494
Deviation for 25.9058326913 of irrational with curve m/n:
1. 7165604789499-1. 716556=0.0000044789499
Estimates the slope of irrational for 25.90:
1.7165604789499-1.7161744654942=0.0003860134557
Net deviation in the strip for 25.90:
0.000405-0.0003860134557=0.0000189865443 (Drop curve from sandwich consideration P_1 is not going in the same direction.)
The Slopes are m/n:
19.1854 1.716556

4.2. Fourth Iteration

Finally, the analysis of x2+50x+301 with x2+49x+311

Factor: -43,-7

GCD(43,7) = 1

p/q = 7/43

43 - 7*6 = 1

The ratio of b_2/b_1 = 6/42 versus 1/n= 1/7 they are equal so their diff=0

Table 2. (4th Iteration 901)
0.000000 devf 0.014702703425dcv2
(*dcv'1,dcv2: mean decimal deviation of eq_1 or eq_2)

0.14285714286 1/n 0.14285714286 a_2/a diff 0.000000000 (converges to the curve that has the integer roots as we have predicted earlier.)
P_1 (parameter one for Sandwich)
0.13346415276 1/n 0.14285714286 a_2/adiff 0.00093929906009
The first factors associated with the first equation are:
-41.507351352 7.4926486483
The second factor associated with the first equation is:
-43 -7
The sides of the first equation transform into a triangle according to the Pythagorean triple, which are:
-1666.7204325 622 1779
The gap point is as follows: $x_1 =(m^2-n^2)/$ m2+n2) or $x_2 = 2mn/(m^2+n^2)$ estimated
to two decimal points. Now, using the Pell equation ($x^2 +y^2= 1$) will give you the gap point for y:
1666.7204325/1779, 622/1779) = 0.94^2= 0.8836= 1-0.8836= 0.1 $1640^{0.5}$= 0.341174
Using the (0,l) point and the gap approximation point in a slope formula, we have: (0,1);(0.94,0.341174) = 1.42678 P_2
Deviation of P_2 versus m/n:
5.53974-1.42678=4.11296 (drop P_2)
Using the (0,1) point and the gap approximation point in a slope formula, we have:
622/1779= 0.349635 = 0.35^2 = 0.1225 = 1-0.1225 = $0.8775^{0.5}$=0.93675 (0,1);(0.35,0.93675)=5.53357 Alt P_2 indicator
Deviation of Alt P_2 versus m/n:
5.53974-5.53357=0.00617 (Keep the Alternate indicator deviation)
The sides of the second equation transform into a triangle according to the Pythagorean triple are: -1800, 602,1898
The gap point is as follows: $x_1 =(m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$ estimated to two decimal points. Now, using the Pell equation $x^2+y^2= 1$ will give you the gap point for y:
(1800/1898, 602/1898) = $(0.95)^2$ and (1-0.9025) =.(0.0975)^0.5=0.31225
The gap point is approximated to be (0.95,0.31224989991992)
Using the (0,1) point and the gap approximation point in a slope formula, we have:
(1,0), (0.95,0.31224989991992) = 6.14286 vs 6.245=.10214 $P_{2,}$ which is too far from zero.
Now, using y as a seed:
602/1898 = 0.317176 = 0.32^2 = 0.1024 = 1-0.1024 = $0.8976^{0.5}$ = 0.947418
Using the (0,1) point and the gap approximation point, alternate P_2 in a slope formula:
(0,1);(0.32,0.947418) = 6.08568 Alt P_2
Deviation of Alt P2 with m/n:
6.14286-6.08568=0.05718 P_2 Alt keep this indicator deviation
Purge Equations:
L_1+ log(x)*K = $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$
L_1+ log(x)*K= m/n
1.4513692348828 ... +(l/7)*k = 6.08568; thus k = 32.44;
1.4513692348828 ... +(1/7)*k = 6.14286; thus k = 32.84(mln)
Gap for 1/7:
6.08568-6.0856549491684 ...... = 0.000025…
6.14286-6.1427978063113 ...... = 0.000062
Irrational slope deviation estimates:
6.1427980631 l 3 - 6.085654991684 = 0.0571428146273
0.0571428146273 - 0.05718 = 0.00003 7 P_1
The slope m/n are:
5.53974 6.14286

4.3. Table of Linear Deviation from Last to First (Linear Order Established)

Table 3. Table of Linear Deviation from Last to First (Linear Order Established)
0.000037 P_1 0.00000000-sol equation $x^2+50x+301$ =041736
0.0002514037727 (drop) $x^2+51x + 291=0$ P_1 0.18057081986
0.000010702214 3dec x^2 + 48x +321=0 P_1 0.0004868553810 0.000521850336
0.0000189865443 (drop) x^2+4 1x+391=0 P_1 0.26707162767
0.000001219337 $x^2 + 62x +181 = 0$ P_1 0.00761620145
0.0000142687923 3dec x^2+ 49x+311 =0 P_1 0.00093929906009 0.00081671

4.4. Graph of 901 According to All Tables

The Purge equation strip of irrational boundary estimates:

Min (L_1+ log(n)*K= ($x_1 =(m^2-n^2)/(m^2+n^2)$, $x_2 =2mn/(n^2+n^2)$)
And L_1+ log(n)*K= m/n;
DIS means the deviation of the irrational slope from the Pythagorean slope.

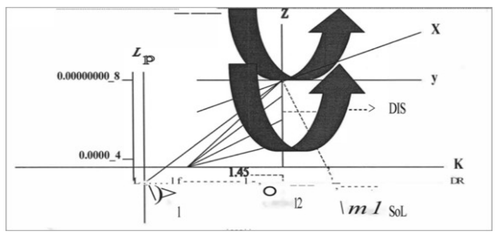

Fig. 3 Graph of 901 to All Tables

Introduction to limit point proof and implication:

The irrational slope has a limit point $-1/(\surd 5)* n^2) < \lambda - m/n < 1/((\surd)*n^2)$ 10.0

Here it is proven that any irrational number λ can be approximated by a rational m/n such that:

$$-1/((\surd 5)n^2) < \lambda - m/n < 1/((\surd 5)n^2) \quad 10.0$$

Concentrating on the last expression, there is a limit on the approximation of the irrational slope by a rational number. Furthermore, √5 is the constant that yields the best possible approximation, meaning if √5 is replaced by any larger constant, the statement becomes false.

For the integer solution n/m= 1800/ 1898 of 901

$-1.2414309874738E-7<\lambda-(1800/1898) <1.2414309874738E-7$
$- 0.948367 <\lambda,<0.948367$ The limit of λ,
$-. 0.94833 <\lambda< 0.948333$ the irrational found is within this range.
$x^2+50x+301$ - .0.948367 <λ,<0.948367 The limit of λ_1,
The limit net deviation between λ and λ is: 000034
for n/m= 602/ 1898;
$-0.31717585056712<\lambda<0.31717585056712$ the limit λ_2(drop)
3dec x^2+ 49x+ 311; 622/1779
$-1.41307E-7<\lambda-(1666.7204325/1779)<1.41307E-7$
$-0.93688599275727<\lambda<0.93688599275727$ The limit of λ_1,
Using the result deviation for the equation
$-0.93687186527178<\lambda<0.93687186527178$
The limited deviation net result between λ and λ_1 is 0.000014
for side n/m= 622/ 1779
$-0.34963448488768<\lambda<0.34963448488768$ the limit of λ_2
Using result deviation:

```
0.34963862550589<λ<0.34963862550589
x² +48x +321; use deviation of 582/2019
           m/n= 1933.2969249/2019
-1.09709E-7<λ-(1933.2969249/2019)<1.09709E-7
-0.95755161139052<)λ<0.95755161139052 limit of λ1
```

Using result:

-0.95754101888555<)λ< 0.95754101888555

The limited deviation result between λ and λ_1 is: 0.000011

For the side of: 582/2019

-0.28826140589275<λ<0.28826140589275 limit λ_2

Using result:

```
-0.28825081335778<λ<0.28825081335778
x² + 62x +181 n/m= 3463.1315309/3482
2.71108E-7<λ-(3463.1315309/3482) <2.71108E-7
-0.99458 l 17729346<λ<0.99458117729346 Limit of λ
Using result:
0.99457992107081<λ<0.99457992107081
The limit net deviation result is: 0.00000125622265
```

Three cases are identified:

- Case 1: The data converges in descending order. This case uses data from 901.
- Case 2: The data converges in ascending order. This case uses data from 2501.
- Case 3: All curves follow a linear order. This case uses the other integers.

Now, we need to show that the largest value, which identifies the equations with integer solutions, cannot be iterated any further using the data found earlier, combined with the forward iteration.

4.5. Proof 1 Outline

Given the tables, the output is already in linear order. The goal is to show that one more iteration does not imply better accuracy because of the limit point.

Having this goal in mind, if you try 0.948 as a seed, you should get a better result, specifically, y = 0.31827, and the slope is 6.12058 with a rate of change of 0.022274.

Apply the linear strip:

L_1+ log(x)*K= $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$

L_1+ log(x)*K= m/n

1.4513692348828 ... + (1/7) *k = 6.12058; thus k = 32.684;

1.4513692348828 ... + (1/7)*k = 6.14286; thus k = 32.84(m/n)

Gap for 1/7 range expanded to prove linear order:

1.4513692348828 + (1/7)*32.684 = 6.1205120920256

1.4513692348828+ (1/7) *32.84 = 6.1427978063113

Pythagorean deviation and irrational estimates slope:

6.12058-6, 1205120920256 = 0.000068

Original curve m/n and irrational estimated slope:

6.14286-6.1427978063113 ... = 0.000062

Estimate of irrational:

6.1427978063113 – 6.1205120920256 = 0.02222857142857

Net strip deviation:

0.02222857142857 - 0.022274 = 0.0000117142857

-0.948378<λ< 0 948378; - 0.948367 < λ <0.948367 The limit of λ.

Since the limit for λ = 0.948367 and the result for a new iteration is λ< 0 948378, this new iteration cannot be achieved. Seeing as this was the largest deviation, going one more iteration is intended to make the accuracy better as it gets closer to the limit value. However, this could not be achieved. Thus, it can be concluded that the other results are much closer to their limit point, and going one more iteration will go beyond their limit point. Therefore, linear order is achieved.

4.6. Case/Example: Data Can Converge in Descending Order (Set of Tables for 2501)

For the sake of clarity, Let N=2501, to demonstrate how to get the lowest possible bound: a = 25, b = 0, c = l (i.e., $25x^2 + 0x + 1$). Applying shift two, the result is $x^2 + 240x + 1$. This means a = l, b = 240, c = l. Furthermore, this equation is the highest upper bound. Similarly, the lowest bound can be found if the field is expanded up to b^2-4ac=0.

$(240-A)^2-4(10A+ l) = O$, Where a= l, b= 240-A, c= l0A+ l

$A^2-480A+57600-40A+4=0$

$A^2 -520A+ 57596=0$

$A= (-b \pm\sqrt{b^2-4ac})/2a$

$A=-(520\pm\sqrt{((520)2-4(1)(57\ 596))})/2(1)$

$A_1= 160$

$A_2= 360$ (out of bounds since our upper value is 240)

The new upper bound equation, instead of $x^2 + l00x + 1401$, becomes $x^2 + 185x + 551$. This is in accordance with the requirement to guard from the elliptic curve error estimates of the limit point. The polynomial with integer solution is: $x^2 + 82x + 1581$.

Here, we only need to provide two table analyses. However, we would include other tables with the solution set to show that, regardless of the random choice of the equation under study, a linear path will be established. Hence, convergence is at hand. See the outline for further details below.

5. Convergence Tables: Example – Method of Illustration

5.1. Group of Seven Tables Iteration for the Convergence of 2501

Table 4. Convergence for x^2+185x+551=0 and x^2+81x+1591

0.94412535790 dcvl 0.0356688447617 dcv2

(*dcvl,dcv2) mean decimal deviation

Table 5. Convergence for 2501 x^2 +133x +1071=0 and x^2+82x +1581)

0.22003627568 dcvl

0. 00000000 dcv2

(*dcvl ,dcv2: mean decimal deviation of eq_1 or eq_2)

-51 -31

The sides of the first equation transformed into a triangle according to the Pythagorean triple are:

15398.735175 2142 15547

The gap point is as follows: $x = (m^2-n^2)/(m^2+n^2)$ or $y = 2mn/(m^2+n^2)$ estimated to two decimal points. Now, using the Pell equation $x^2+y^2 = 1$ will give you the gap point for y: 15398.735175/15547 = 0.990463 = $(0.99)^2$ = .9801= (1-0.9801) = $(0.0\ 199)^2$ = .14067

Using the (0, 1) point and the gap approximation point in a slope formula, we have: (0,1);(0.99,0.14067) = 14.1067

Deviation with the slope of P_2 and the slope of m/n:

14.4471-14.1067= 0.3404 P_2

-51 -31

The sides of the first equation transformed into a triangle according to the Pythagorean triple are:

15398.735175 2142 15547

The gap point is as follows: $x = (m^2-n^2)/(m^2+n^2)$ or $y = 2mn/(m^2+n^2)$ estimated to two decimal points. Now, using the Pell equation $x^2+y^2 = 1$ will give you the gap point for y: 15398.735175/15547 = 0.990463 = $(0.99)^2$ = .9801= (1-0.9801) = $(0.0\ 199)^2$ = .14067

Using the (0, 1) point and the gap approximation point in a slope formula, we have: (0,1);(0.99,0.14067) = 14.1067

Deviation with the slope of P_2 and the slope of m/n:

14.4471-14.1067= 0.3404 P_2

L_1+ log(x)*K= m/n

1.4513692348828 ... +(1/8.6100181378) *k=14.4471; thus k= 111.89

L_1+ log(x)*K= $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$

1.4513692348828 ... +(1/8.6100181378) *k= 14.107; thus k=108.97

Gap:

14.4471-14.46696098223= 0.0304003901777...

Table 5. Convergence for 2501 x^2 +133x +1071=0 and x^2+82x +1581) ***(continued)***
0.22003627568 dcvl 0. 00000000 dcv2 (*dcvl ,dcv2: mean decimal deviation of eq_1 or eq_2)
14.107556278392-14.107=0.000556278392 Irrational Slope deviation estimates: 14.46696098223-14.107556278392 = 0.359404703838 Net deviation in the strip: 0.359404703838-0.3404 = 0.019004703838 **(Bad drop curve from consideration. The sides of the second equation transform into triangles according to Pythagorean triple.) 1640 3162 3562 **Note:** Must use the alternate between x_1 or x_2 seed: 3162/3562 = 0.887704= 0.89^2=0.7921=1-0.1921 = $0.2079^{0.5}$=0.455961 Using the (0,1) point and the gap approximation point in a slope formula, we have: (0, 1);(0.89,0.455961) = 1.63591 1.64516.:1.63591 = 0.009249 keep P_2 deviation keep orientation of P_1 L_1+ log(x)*k= x_1 = $(m^2-n^2)/(m^2+n^2)$ or x_2 = $2mn/(m^2+n^2)$ L_1+ log(x)*k = m/n 1.4513692348828 ... +(1/3 l)*k= l .63591; thus k= 5.72; 1.4513692348828 ... +(1/31)*k= l .64516; thus k= 6.01(min) Gap for 1/31: 1.63591-1.6358853639151 .... = 0.000025 .... 1.64516-1,6451434284312 ...... = 0.0000165715688... 1.64516-1.63591 =0.009249 keep P_2 deviation keep orientation of P_1

Purge equation strip of irrational boundaries estimates:

Min (L_1+log(n)*K=(x_1=(m^2-n^2) , x_2=$2mn/(m^2+n^2)$)
And L_1+log(n)*K=m/n;
DIS means the deviation of the irrational slope from the Pythagorean slope.

The irrational slope has a limit point that is $1/((\sqrt{5})*n^2) < \lambda - m/n < 1 - m/n < 1((/5)*n^2$

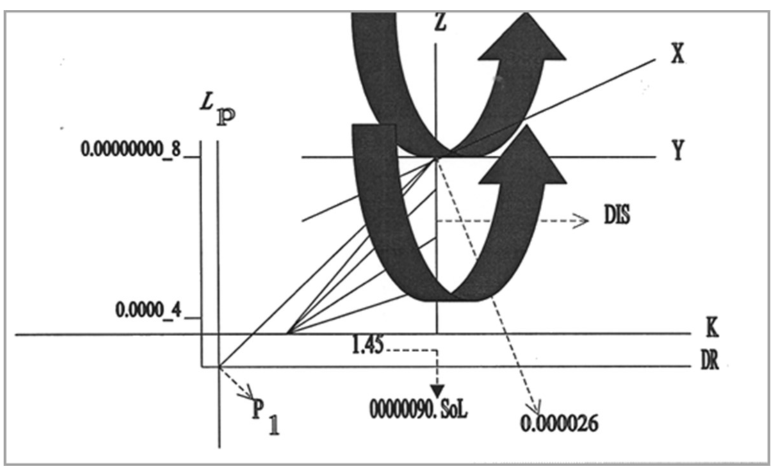

Fig. 4 Irrational Slope Limit Point

The irrational slope has a limit point $-1/(\sqrt{5})* n^2) < \lambda - m/n < 1/((\sqrt{5})*n^2)$ 10.0

Here it is proven that any irrational number λ can be approximated by a rational m/n such that:

$$-1/((\sqrt{})n^2) < \lambda - m/n < 1/((\sqrt{5})n^2) \quad 10.0$$

Table 6: Table of Linear Order in Ascending Orientation
DIS
0.0000090645161 2501 $x^2+82x+1581$
3162/3562
$-0.88770357258598 < \lambda < 0.88770357258598$ limit of λ
Using result $-0.88769447282247 < \lambda < 0.88769447282247$, λ is beyond the limit already. Note that the value is significant in the 5th digit. This implies that since this is the smallest value, linear convergence is achieved.
$0.000012\ x^2 + 84x + 1561$
3122/3934
$-0.79359433494639 < \lambda < 0.79359433494639$ limit of λ
Using result:
$-0.79358230604982 < \lambda < 0.79358230604982$
$0.0001652423271\ x^2 + 94x + 1461$ (drop) P_1 Not in the same arrange accuracy.
$0.000026\ x^2 + 107x + 1331$
8374.0745758/8787
$-0.95300724100318 < \lambda < 0.95300724100318$ limit of λ
Using result:
$0.95298123521111 < \lambda < 0.95298123521111$
0.001684712049: 2501 x^2 +185x +551 (drop accuracy not improving even with more decimal consequence of limit point requirement.

λ is beyond the limit and, therefore, must have reversed iteration. Note the value of the net deviation of the linear boundary is significant in the 5th digit. This implies that since this is converging in ascending order, the smallest value is already achieved. If the iteration is reversed, the net deviation of the slope of the Pythagorean triple seed versus the irrational slope estimate is too large, thus the curve cannot move from its location. All the other values will never be smaller than the first value because if they are, they will be beyond their limit value. Thus, linear convergence is achieved (see Figure 4).

5.2. Proof 2 Outline

Given the table output for 25.01 is already linear, the goal is to show one more iteration going backwards. This will decrease the accuracy, and the value of the deviation will be too large in comparison to the limit point. To begin, the Pythagorean seed under consideration is 3162/3562:

$3162/3562 = 0.887704 = 0.8^2 = 0.64 = 1-0.64 = 0.36^{0.5} = 0.6$

Slope of Pythagorean seed:

(0,1) ;(0.8,0.6) = 2

Thus, if 0.8 is tried as a seed, a better result will be achieved, namely y = 0.6, with a slope of 2 and a rate of change of 0.35284. Net deviation is 2.1475918354542. Drop the curve and stay in position.

5.3. Case 3: Proof of Linear Convergence for Any Integer

This proof is done through geometrical construction in accordance with the data illustrated above, namely cases 1 and 2, and supported theorems of the Purge Equation Strips.

Given that the irrational numbers are dense and given that the rational set is a dense set less than the irrational set, there exists a rational number close to the irrational counterpart. Using Dirichlet Boundary problems, a circle of radius can be established with the seeds of the Pythagorean triangle, with a minimum distance at the point of unity, a slope can be formed, and an irrational slope can be estimated. This will form a strip in field F.

Many such strips can be constructed within the limit point of accuracy, prescribed earlier. Given that these sets of lines are slanted, a set of intersecting points can be found crossing linearly up or down to the integer solution strip. The strip, which obeys linear convergence, must be irrational since the irrational field is an extremely dense set. A purge linear set must be formed in conjunction with the indeterminate ratio sets b_2/b_1 deviation with 1/n, which is essentially parameter P_1. Meaning, they have the same orientation. Thus, by geometrical construction, any integer can be factored into its prime elements.

6. Conclusion

1st Step: The F field either converges through the arithmetic means or converges toward the geometric mean.

2nd Step: The ratio b_2/b approaches 1/n. This could happen either in descending or ascending order.
Note: $a = a_1*a_2$ and $b = b_1*b_2$, remember this process refers to ax-by = l, the indeterminate case.

3rd Step: To guard against drop points in the decimal expansion in the rational plane, always compare them with the irrational expansion. They always go in the same direction toward the limit point zero. The seed of the Pythagorean value $x_1 = (m^2-n^2)/(m^2+n^2)$ or $x_2 = 2mn/(m^2+n^2)$ is used when y is calculated using the Pell equation. This is because the value that gets closest to zero could be located on either side of the triangle. The side this process is looking for is the smallest value that will identify the integer equation. This consideration is crucial when analyzing extremely large numbers (for example, numbers that are 2000 digits long). The Pythagorean triple value is still used as an indicator; however, backtracking is used to control the parameter.

4th Step: To guard from false readings, one should always compare the slope of the Pythagorean triple rate of change to the irrational gap slopes in accordance with the boundary strips established.

Rotation of curves: Due to decimal error, each curve that is selected must obey the following rules:

- The range of the deviation of the irrational slope must be at least four digits long. This means that the significant digit begins in the 5th position.
- If the parameter P_1 has the same extension in decimal range as the slope deviation, then a 2nd iteration of the table is needed. This is due to a premature error in rounding decimals up or down in accordance with rounding up rules.
- The seed of the Pythagorean point is expanded to 3 so that the slope can be recalculated with unity and used in the linear measurement trip equations to get the new irrational counterpart. Since the irrational slope has a limit point, namely $-1/((\sqrt{5})*n^2)<\lambda-m/n < 1/((\sqrt{5})*n^2)$, a linear order will be established. See linear tables for each numeral under study and proof of limit for further details.

This is important since a partial order environment must be established for the Sandwich to take place. With these rules, a binary search tree can be implemented, and this algorithm has a complexity. The Theorem in application solves the decomposition of any number or of any length, which is the cornerstone of cryptography.

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