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Original Article

[J, K] – Set Domination of Sunlet Graph and Helm Graph

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Abstract *- Domination is an important theoretical concept in graph theory. The fastest-growing area in graph theory is the study of domination and related problems. By [J, K] set domination, a subset D ⊊ V in a graph G = (V, E) is a [J, K] set if every vertex v ∈ V – D, J ≤ | N (v) ∩D | ≤ K, for non-negative integer J and K, that is every vertex v ∈ V – D is adjacent to at least J but not more than K vertices in D. The domination number of G is denoted by* $\gamma_{[J,K]}$ *(G), which is the minimum cardinality of a dominating set G. In this paper, the [J, K]-domination number of Sunlet graphs, Helm graphs, and their applications has been studied.*

Keywords *- Dominating Set, Domination Number, [J, K] – Dominating Set, [J, K] – Domination Number.*

1. Introduction

The basic concepts and introduction of graph theory have been learned from D. B. West [7]. Fundamentals of domination in graphs have been explained in T. W. Haynes et al. [2]. [1, 2] – A set of various graphs has been understood from M. Chellali et al. [4]. [1, 2] – Domination and bounds for different graphs have been explained by Xiaojing Yang and Baoyindureng Wu [8]. In 1962, Oystein Ore [5] first defined the domination number of a graph. Ernie Cockayne and Steve Hedetniemi [1] first started to study dominating sets in graphs in 1975. In 1988, E. Sampath Kumar [6] introduced the concept of [1, k] – dominating set of a graph.

2. Preliminaries

Let G = (V, E) be a simple connected graph with vertex set V, edge set E, and order n = | V |, size m = |E|. The open neighborhood, closed neighborhood, and degree of a vertex V in G are respectively denoted by N(v) = {u ∈ V | u v ∈ E}, N [v] = N (v) ∪ {v}, and d(v) = | N(v)|. The maximum and minimum degrees are denoted by Δ (G) and δ (G)

Definition 2.1: A set D ∈ V of vertices in a graph G = (V, E) is a dominating set if and only if for every vertex v ∈ V-D is adjacent to at least one vertex in D.

Definition 2.2: A set D ∈ V is called [J, K] – set domination, if for any vertex v ∈ V – D, J ≤ | N (v) ∩D | ≤ k, there are at least J vertices adjacent to v but not more than K vertices in D. The smallest cardinality of [J, K] – set is called [J, K]- dominating set. It is denoted by $\gamma_{[J,K]}$ (G).

Definition 2.3: The n-Sunlet graph is a graph on 2n vertices that is obtained by attaching n pendant edges to the cycle c_n and it is denoted by s_n.

Definition 2.4: The Helm graph H_n is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

Remarks:
Number of vertices in n-sunlet graph ---2n
Number of edges in n-sunlet graph -----2n
Maximum degree of n-sunlet graph ----3
Minimum degree of n-sunlet graph -----1

3. [J, K] – set domination of sunlet graph and helm graph

Theorem 3.1: The [1, 1] – dominating sets of the sunlet graph s_n, n = 3,4,…..is

$$D_{[1,1]}(s_n) = \begin{cases} v_{k1}, & k1 = 1, 2, 3 \ldots\ldots n \\ v_{k2}, & k2 = n+1, n+2, \ldots\ldots 2n \end{cases}$$

Proof: Let $v_1, v_2, v_3 \ldots\ldots v_n, v_{n+1}, v_{n+2}, v_{n+3} \ldots\ldots v_{2n}$ are the vertices of the sunlet graphs_n, such that v_i is adjacent to $v_{n+i}, i = 1,2,3 \ldots\ldots n$. There fore $v_1, v_2, v_3 \ldots\ldots v_n$ dominates $v_{n+1}, v_{n+2} \ldots\ldots v_{2n}$. And v_{n+1} is adjacent to $v_i, i = 1,2,3 \ldots\ldots n$. so $v_{n+1}, v_{n+2} \ldots\ldots v_{2n}$ dominates $v_1, v_2 \ldots\ldots v_n$.
There are two dominating sets with n vertices in each set.
Therefore | $D_{[1,1]}(s_n)$ | = 2 and $\gamma_{[1,1]}(s_n)$ = n
The generalization of the [1,1] – dominating sets of the sunlet graph s_n is

$$D_{[1,1]}(s_n) = \begin{cases} v_{k1}, & k1 = 1, 2, 3 \ldots\ldots n \\ v_{k2}, & k2 = n+1, n+2, \ldots\ldots 2n \end{cases}$$

Theorem 3.2: The [1, 2] – dominating sets of the sunlet graph s_n, n = 3,4,5…..is

$$\boldsymbol{D_{[1,2]}(s_n)} = \begin{cases} \{v_{r1}, v_{r2}|_{r_2=k2+s}^{r_1=k1+s-2} & \text{If } k1+s-2 \le n,\ k2+s \le 2n \\ \text{K1}= n,\ (n-1) \ldots\ldots (r+2),\ k2 = 2n,\ (2n-1) \ldots\ldots (2n+1-r),\ r = 1,2,3 \ldots \\ \text{S is a fixed integer.} \\ \{\{v_{q1}, v_{q2}|_{k2+s=\ \ p2n+q_2}^{k_1+s-2\ =p2n+q_1} & \text{If } k1+s-2 > n,\ k2+s > 2n \\ \text{For fixed p} \\ \text{K1}= n,\ (n-1) \ldots\ldots (r+2),\ k2 = 2n,\ (2n-1) \ldots\ldots (2n+1-r),\ r = 1,2,3 \ldots \\ \text{Where s = 0, 1, 2, ……., (n-1)} \end{cases}$$

Proof: let $\boldsymbol{v_1, v_2, v_3} \ldots\ldots v_n$ *and* $v_{n+1}, v_{n+2 \ldots\ldots\ldots v_{2n}}$ are the vertices of the sunlet graph s_n. The dominating sets are { $v_{k1-2}, v_{2k2-1}, v_{2k2}$}, { $v_{k1-3}, v_{2k2-2}, v_{2k2-1}, V_{2k2}$}……..{$v_1, v_{k2+2}, v_{k2+3} \ldots\ldots v_{2k2}$}

Where K1$= n,\ (n-1) \ldots\ldots (r+2),\ k2 = 2n,\ (2n-1) \ldots\ldots (2n+1-r),\ r = 1,2,3 \ldots$

Suppose if n=8 in s_8 We can get six dominating sets; each dominating set contains eight dominating vertices.
We have
$\{v_1, v_2, v_3, v_4, v_5, v_6, v_{15}, v_{16}\}, \{v_1, v_2, v_3, v_4, v_5, v_{14}, v_{15}, v_{16}\}$
$\{v_1, v_2, v_3, v_4, v_{13}, v_{14}, v_{15}, v_{16}\}, \{v_1, v_2, v_3, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$
$\{v_1, v_2, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}, \{v_1, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$

There are (n-2) number of [1, 2] dominating sets in the sunlet graph s_n. In each dominating set, we can find n different dominating vertices.

Thus |$D_{[1,2]}(s_n)$ | = n-2 and $\gamma_{[1,2]}(s_n)$ = n
So, the generalization of [1, 2] – dominating sets of the sunlet graph s_n is

$$D_{[1,2]}(s_n) = \begin{cases} \{v_{r1}, v_{r2}|_{r_2=k2+s}^{r_1=k1+s-2} & \text{If k1}+s-2 \le n,\ k2+s \le 2n \\ \text{K1}= n,\ (n-1) \ldots\ldots (r+2),\ k2 = 2n,\ (2n-1) \ldots\ldots (2n+1-r),\ r = 1,2,3 \ldots \\ \text{S is a fixed integer\}} \\ \{\{v_{q1}, v_{q2}|_{k2+s=\ \ p2n+q_2}^{k1+s-2\ =p2n+q_1} & \text{I } k1+s-2 > n,\ k2+s > 2n \text{ f} \\ \text{For fixed p} \\ \text{K1}= n,\ (n-1) \ldots\ldots (r+2),\ k2 = 2n,\ (2n-1) \ldots\ldots (2n+1-r),\ r = 1,2,3 \ldots \\ \text{Where s = 0, 1, 2, ……., (n-1)} \end{cases}$$

Similarly, we can find the generalization of [1, 2] dominating sets for the remaining (n-1) dominating sets.

SUNLET GRAPH

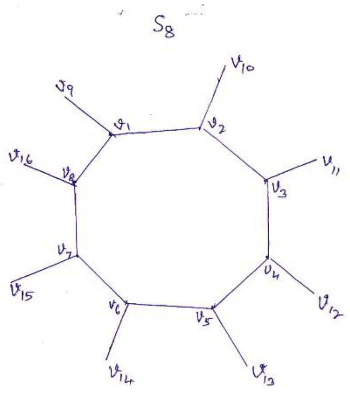


FIG (i)

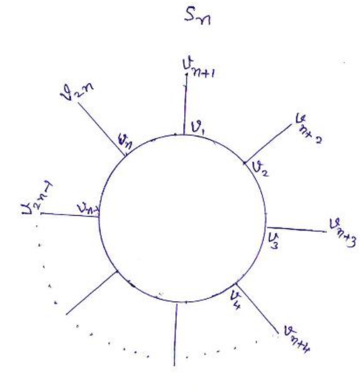


FIG (ii)

Remarks:

Number of vertices in Helm graph - 2n+5
Number of edges in Helm graph - 3n+6
Maximum degree of Helm graph - 4
Minimum degree of Helm graph - 1

Theorem 3.3: The [1, 1] – dominating sets of the helm graph H_n, n = 3, 4, 5…is
$D_{[1,1]}$ (H_n) = { $v_{k1},\ k1 = 1,2,3 \dots \dots . n$ }

Proof: Let $v_1, v_2, v_3 \dots \dots \dots . v_n, v_{n+1}, v_{n+2}, \dots \dots \dots \dots v_{2n}$ and v_{2n+1} are the vertices of the helm graph H_n.
v_i Is adjacent to v_{n+i}, I = 1, 2,…n, there fore $v_1, v_2, v_3 \dots \dots v_n$ dominates the vertices $v_{n+1}, v_{n+2} \dots \dots . v_{2n}$
And $v_1, v_2 \dots \dots . v_n$ adjacent to v_{2n+1} So these vertices dominate the other vertices. v_{2n+1}. There is one dominating set with n vertices.
Thus $D_{[1,1]}$ (H_n) = 1 and ɣ$_{[1,1]}H_n$ = n
The generalization of the [1, 1] – dominating set of the sunlet graph H_n is
$D_{[1,1]}$ (H_n) = {$v_{k1},\ k1$ = 1, 2, 3……n}

Theorem 3.4: The [1, 2] – dominating sets of the helm graphH_n, n $= 3, 4, 5 \dots .$ Is

$D_{[1,1]}$ (H_n) =

$\{v_{r1}, v_{r2}|_{r_2=k2+s}^{r_1=k1+s-2}$ If $k1 + s - 2 \le n,\ s_2 + s \le 2n$
K1= $n,\ (n-1) \dots \dots . (r+2),\ k2 = 2n,\ (2n-1) \dots \dots (2n+1-r),\ r = 1,2,3 \dots$
S is a fixed integer.
$\{v_{q1}, v_{q2}|_{k2+s\ =\ p2n+q_2}^{k1+s-2\ =pn+q_1}$
For fixed p}
K1= $n,\ (n-1) \dots \dots . (r+2),\ k2 = 2n,\ (2n-1) \dots \dots (2n+1-r),\ r = 1,2,3 \dots$
Where s= 0, 1, 2…… (n-1)

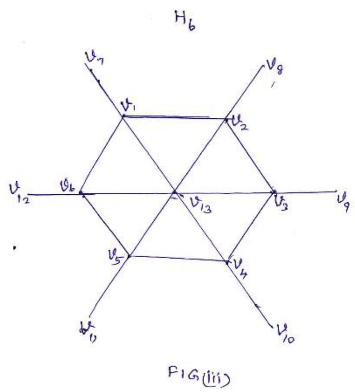
Proof: Let $v_1, v_2, v_3 \dots \dots \dots . v_n, v_{n+1}, v_{n+2} \dots . v_{2n}$ and v_{2n+1} are the vertices of the helm graphH_n.
The dominating sets are
$\{v_{k1-2}, v_{2k2-1}, v_{2k2}\}$, $\{v_{k1-3}, v_{2k-2}, v_{2k2-1}, v_{2k2}\}$,……… $\{v_1, v_{k2+2}, v_{k2+3} \dots \dots . v_{2k2}\}$
K1= $n,\ (n-1) \dots \dots . (r+2),\ k2 = 2n,\ (2n-1) \dots \dots (2n+1-r),\ r = 1,2,3 \dots$
Suppose if n=6 in H_6 We can get four dominating sets; each dominating set contains six dominating vertices.
We have.
$\{v_1, v_2, v_3, v_4, v_{11}, v_{12,}\}$, $\{v_1, v_2, v_3, v_{10}, v_{11}, v_{12,}\}$, $\{v_1, v_2, v_9, v_{10}, v_{11}, v_{12,}\}$, $\{v_1, v_8, v_9, v_{10}, v_{11}, v_{12,}\}$,
There are (n-2) number of dominating sets in the helm graph.H_n. In each dominating set, we can find n different dominating vertices.
Thus $|D_{[1,1]}$ (H_n) |= n-2 and ɣ$_{[1,2]}$ (H_n) = n

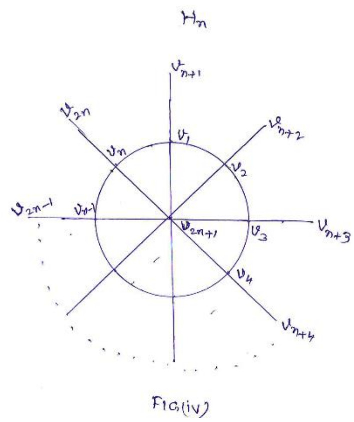
So the generalization of [1, 2] – dominating sets of the helm graph H_n is

$$D_{[1,2]}(H_n) = \begin{cases} \{v_{r1}, v_{r2}\Big|_{r_2=k2+s}^{r_1=k1+s-2} \quad \text{If } k1+s-2 \le n,\ k2+s \le 2n \\ \text{K1} = n,\ (n-1) \dots \dots (r+2),\ k2 = 2n,\ (2n-1) \dots \dots (2n+1-r),\ r = 1,2,3 \dots \\ \quad \text{S is a fixed integer.} \\ \{v_{q1}, v_{q2}\Big|_{k2+s\,=\,p2n+q_2}^{k1+s-2\,=pn+q_1} \\ \text{For fixed p}\} \\ \text{K1} = n,\ (n-1) \dots \dots (r+2),\ k2 = 2n,\ (2n-1) \dots \dots (2n+1-r),\ r = 1,2,3 \dots \\ \text{Where s} = 0, 1, 2 \dots\dots (n-1) \end{cases}$$

Similarly, we can find the generalization of [1, 2] – dominating sets for the remaining (n-2) dominating sets.

HELM GRAPH





Applications

There are many applications of domination in graphs used in land surveying, Electrical networks, networking, routing problems, coding theory, and computer communication.
We must construct a route for a school bus in such a way that the child has to get the school bus at the nearest stop from the residence. More children residing at the place have to be picked up by the two buses (dominated).

To enhance the performance of an organization each untrained staff should be seated nearer to the trained staff in [1, 1] - Domination. In some specific cases, one untrained staff member can be seated nearer to the two trained staff in [1, 2] - Domination.

4. Acknowledgement

The authors are highly thankful to everyone who contributed to the success of this work.

5. Conclusion

In this paper, we have generalized the [J, K]- dominating number of the sunlet graph, Helm graph, and its applications. Similarly, we can study [J, K]- dominating number of the Sun graph and the Grid graph.

References

[1] E.J. Cockayne, and S.T. Hedetniemi, "Towards a Theory of Domination in Graphs," *Networks*, vol. 7, pp. 247-261, 1977. [CrossRef] [Google Scholar] [Publisher Link]

[2] Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater, "Fundamentals of Domination in Graphs," Marcel Dekkar, New York, 1998. [Google Scholar]

[3] Mustapha Chellali et al., "[1, 2] – Sets in Graphs," *Discrete Applied Mathematics*, vol. 162, no. 18, pp. 2885-2893, 2013. [CrossRef] [Google Scholar] [Publisher Link]

[4] Øystein Ore, *Theory of Graphs*, American Mathematical Society, pp. 206-212, 1962. [Google Scholar]

[5] E. Sampathkumar, [1,k] – Domination in a Graph, *Journal of Mathematical and Physical Sciences*, vol. 22, pp. 613-619, 1988. [Online]. Avaliable: https://www.researchgate.net/publication/341591826_1_k-domination_in_a_graph#:~:text=(PDF)%20(1%2C%20k,Book

[6] Douglas Brent West, “Introduction to Graph Theory,” Prentice Hall, 1996. [Google Scholar] [Publisher Link]

[7] Xiaojing Yang, and Baoyindureng Wu, “[1, 2] – Domination in Graphs,” *Discrete Applied Mathematics*, vol. 175, pp. 79-86, 2014. [CrossRef] [Google Scholar] [Publisher Link]