International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I4P101

Volume 72 Issue 4, 1-8, April 2026

Original Article

Absolute Mean Cordial Labeling of Join of Several Structure

N. A. Parmar[1], M. P. Rupani[2], P. Z. Akbari[3]

[1,2]Shree H. N. Shukla College of I. T. & Mgmt., Saurashtra University, Rajkot, Gujarat, India.
[3]Department of Mathematics, Saurashtra University, Rajkot, Gujarat, India.

[1]Corresponding Author : nishabaparmar24@gmail.com

Received: 18 February 2026 Revised: 22 March 2026 Accepted: 11 April 2026 Published: 25 April 2026

Abstract - *A graph $G = (V, E)$ is called an absolute mean cordial graph if there exists a one-to-one function f from $V(G)$ to the set $\left\{0, \pm 2, \pm 4, \dots, \pm \left\lceil \frac{q}{2} \right\rceil\right\}$ such that each edge $uv \in E(G)$ is assigned the label 1 when $\left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil \leq \left\lceil \frac{q}{2} \right\rceil$, and 0 otherwise, with the labeling satisfying the condition $\left|e_f(0) - e_f(1)\right| \leq 1$. In this paper, we investigate the join of several graphs that are absolute mean cordial graphs. We establish new results concerning absolute mean cordial graphs.*

Keywords - *Labeling, Cordial Labeling, Absolute Mean Cordial Labeling, Sunflower Graph, Helm graph.*

1. Introduction

Graph labeling is a fundamental area of graph theory with broad applications in computer science, communication networks, and combinatorial optimization. Over the years, many types of labeling schemes have been introduced, each with unique constraints and significance. Among these, graceful labeling has been studied widely due to its simple idea and theoretical importance. It has also inspired many other labeling methods and variations. A. Rosa[11] introduced the concept of labeling with the name of β –valuation. S. Golomb [7] named such labeling as graceful labeling.

Kanani and Chudasama[9] introduced a new labeling, namely absolute mean cordial labeling, derived from two labelings – absolute mean graceful labeling and cordial labeling. They proved the triangular snake $T_\beta, \forall \beta \geq 3$, cycle $C_\xi, \xi \neq 5$, friendship graph $F_\chi, \forall \chi \geq 2$, helm graph $H_\lambda, \forall \lambda \geq 3$, alternate triangular snake $AT_\beta, \forall \beta \geq 3$, double triangular snake $DT_\beta, \forall \beta \geq 3$, double alternate triangular snake DAT_β, $\forall \beta \geq 3$ are absolute mean cordial graphs. Further, Akbari et. Al.[1] proved a complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n, double step grid graph DSt_n, wheel graph W_n, sunlet graph $C_n \odot K_1$, gear graph G_n, swastik graph $SW_n, n \neq 1$, sunflower graph SF_n, flower graph Fl_n, extended friendship graph $Fr_{4,n}$ admits absolute mean cordial labeling. In this paper, we proved that the join of several graphs admits absolute mean cordial labeling. First of all, we need some definitions that are useful for this paper.

Definition 1.1. A function f is called a cordial labeling of the graph $G = \big(V(G), E(G)\big)$, if $f: V(G) \to \{0,1\}$ is injective and the induced function $f^*: E(G) \to \{0,1\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1, respectively, under f, and let $e_f(0)$, $e_f(1)$ the number of edges of G having edge labels 0 and 1, respectively, under f^*. A binary vertex labeling f of a graph is called a cordial labeling if $\left|v_f(0) - v_f(1)\right| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called a cordial graph if it admits a cordial labeling.

Definition 1.2. A function f is called an absolute mean cordial labeling of a graph $G = (V(G), E(G))$, if $f: V(G) \to \left\{0, \pm 2, \pm 4, \dots, \pm \left\lceil \frac{q}{2} \right\rceil\right\}$ is one-one and the induced function $f^*: E(G) \to \{0,1\}$ defined as,

$$f^*(e) = \begin{cases} 1\,;\ if\ \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil \leq \left\lceil \frac{q}{2} \right\rceil \\ 0\,;\ otherwise \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ is onto for every edge $e = (u, v) \in E(G)$. A graph is called an absolute mean cordial if it admits an absolute mean cordial labeling.

Definition 1.3. The join of two graphs G and H is the graph obtained by taking one copy of G & H, and joining each vertex of G to each vertex in H.

We consider a simple, connected, and undirected graph $G = (V, E)$ with p vertices and q edges. For all terminology and notation, we follow [6, 8].

2. Main Results

Theorem 2.1. The graph $SF_n + K_1$ join of the sunflower graph SF_n with a complete graph K_1, it is an absolutely mean cordial graph.

Proof. Let $G = SF_n + K_1$ is a join of any sunflower graph SF_n with a complete graph K_1.

Let V(SF_n) = $\{u_i, v_i$, w / 1≤ $i \leq n,\ u_{n+1} = u_1, v_{n+1} = v_1\}$ and

E(SF_n) = $\{u_iu_{i+1}/1 \leq i \leq n\} \cup \{v_iu_i, v_iu_{i+1}\ /\ 1 \leq i \leq n\} \cup \{u_iw/1 \leq i \leq n\}$.

Therefore, V(G) = $\{v_i, u_i\ /\ 1 \leq i \leq n, u_{n+1} = u_1, v_{n+1} = v_1\} \cup \{w, v\}$ and

E(G) = $\{u_iu_{i+1}/1 \leq i \leq n\} \cup \{v_iu_i, v_iu_{i+1}/1 \leq i \leq n\} \cup \{u_iw/1 \leq i \leq n\} \cup \{u_iv/1 \leq i \leq n\} \cup \{v_iv/1 \leq i \leq n\}$.

To obtain vertex labeling function f:V(G)→{0,±2,±4, ..., ±$\left\lfloor\frac{q}{2}\right\rfloor$}, we take the following cases.

Case-1: $n \equiv 0(mod2)$

$f(w) = 1 - q, f(v) = -(3n + 3),$

$f(u_i) = q - 2i + 1, 1 \leq i \leq n,$

$f(v_i) = 1 + 2i - q, 1 \leq i \leq \frac{n}{2}$

$f\left(v_{\frac{n}{2}+i}\right) = q - 3n - 2i + 1, 1 \leq i \leq \frac{n}{2}$

Case-2: $n \equiv 1(mod2)$

$f(w) = 1 - q, f(v) = -(3n + 3),$

$f(u_i) = q - 2i + 1, 1 \leq i \leq n,$

$f(v_i) = 1 + 2i - q, 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor$

$f\left(v_{\left\lfloor\frac{n}{2}\right\rfloor+1}\right) = q - 2\left\lfloor\frac{n}{2}\right\rfloor - 2n + 1,$

$f\left(v_{\left\lfloor\frac{n}{2}\right\rfloor+i}\right) = q - 2\left\lfloor\frac{n}{2}\right\rfloor - 2n + 3 - 2i, 1 \leq i \leq 2\left\lfloor\frac{n}{2}\right\rfloor + 1$

The labeling function f, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check the edge labeling function. $f^* : E(G) \rightarrow \{0,1\}$ is onto. Also, f satisfies the condition. $|e_f(0) - e_f(1)| \leq 1$. Thus, f is an absolute mean cordial labeling for a given graph.

Therefore, the graph $SF_n + K_1$ is an absolute mean cordial graph.

Illustration 2.2. Absolute mean cordial labeling for $SF_5 + K_1$ is shown in the following Figure 1.

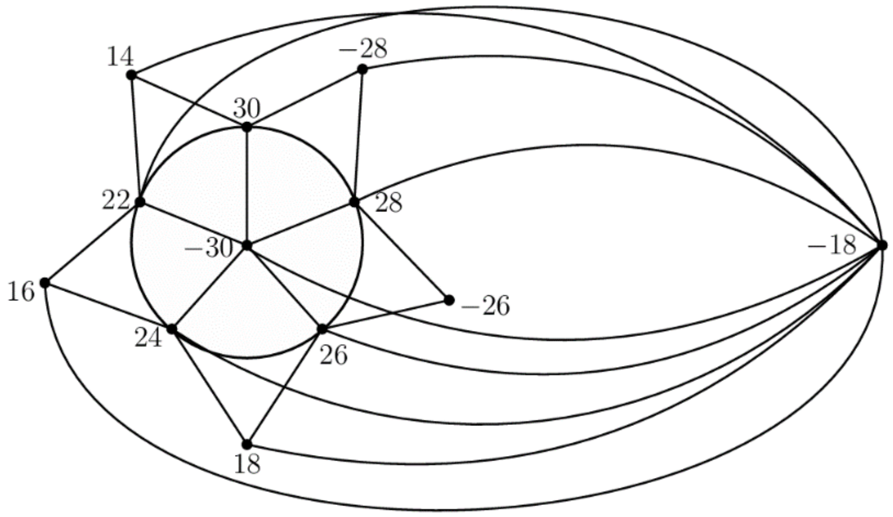

Fig. 1 Absolute mean Cordial labeling of $SF_5 + K_1$

Theorem 2.3. The graph $B_{m,n} + K_1$ join of a bistar graph $B_{m,n}$ with a complete graph K_1 is an absolute mean cordial graph.

Proof. Let $G = B_{m,n} + K_1$ is a join of any bistar graph $B_{m,n}$ with a complete graph K_1.

Let V($B_{m,n}$) = $\{u_i, v_j$ / 1≤ $i \leq m, 1 \leq j \leq n\} \cup \{u, v\}$ and E($B_{m,n}$) = $\{uu_i, vv_j/\ 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{uv\}$

Therefore, V(G) = $\{u_i, v_j\ /\ 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{u, v, w\}$ and

E(G) = $\{uu_i, wu_i/1 \leq i \leq m\} \cup \{vv_j, wv_j\ /\ 1 \leq j \leq n\} \cup \{uv, uw, wv\}$.

To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., ±$\left\lfloor\frac{q}{2}\right\rfloor$}, we take the following cases.

Case-1: $B_{m,1} + K_1$
$f(u) = q - 1, f(v) = q - 3,$
$f(w) = 5 - q, f(v_1) = 2,$
$f(u_1) = 1 - q, f(u_2) = 3 - q,$
$f(u_i) = 1 - q + 2i, 3 \leq i \leq m$

Case-2: $B_{m,n} + K_1 (n \neq 1)$
$f(u) = q - 1, f(v) = 1 - q,$
$f(w) = 3 - q,$
$f(u_i) = 3 - q + 2i, 1 \leq i \leq m$
$f(v_j) = q - 2m - 4n + 4i - 3, 1 \leq j \leq n$
The labeling function *f,* defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, *f* satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. Thus, *f* is an absolute mean cordial labeling for a given graph.
Therefore, the graph $B_{m,n} + K_1$ is an absolute mean cordial graph.

Illustration 2.4. Absolute mean cordial labeling for $B_{4,5} + K_1$ is shown in the following Figure 2.

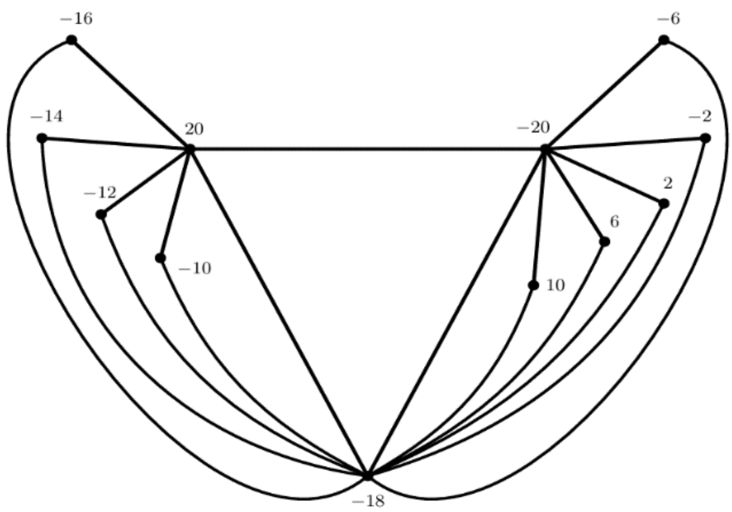

Fig. 2 Absolute mean Cordial labeling of $B_{4,5} + K_1$

Theorem 2.5. The graph $W_n + K_1$ join of wheel graph W_n with a complete graph K_1 is an absolute mean cordial graph.

Proof. Let $G = W_n + K_1$ is a join of any wheel graph W_n with a complete graph K_1.
Let V(W_n) = $\{u_i \,/\, 0 \leq i \leq n, u_{n+1} = u_1\}$ and E(W_n) = $\{u_0u_i, u_iu_{i+1} / \, 1 \leq i \leq n\}$.
Therefore, V(G) = $\{u_i \;/\; 0 \leq i \leq n, u_{n+1} = u_1\} \cup \{v\}$ and E(G) = $\{u_0u_i, u_iu_{i+1}, u_iv \,/ 1 \leq i \leq n\} \cup \{u_0v\}$.
To obtain vertex labeling function *f* : V(G)→{0,±2,±4, ..., ±$\left\lfloor\frac{q}{2}\right\rfloor$}, we take the following cases.

Case-1: $n \equiv 0 (mod 2)$
$f(u) = 1 - q, f(v) = n + 2,$
$f(u_i) = q - 2i + 1, 1 \leq i \leq n$

Case-2: $n \equiv 1 (mod 2)$
$f(u) = -q, f(v) = -2\left\lceil\frac{q}{2}\right\rceil,$
$f(u_i) = q - 2i + 2, 1 \leq i \leq n$
The labeling function *f,* defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, *f* satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. Thus, *f* is an absolute mean cordial labeling for a given graph.
Therefore, the graph $W_n + K_1$ is an absolute mean cordial graph.

Illustration 2.6. Absolute mean cordial labeling for $W_8 + K_1$ is shown in the following Figure 3.

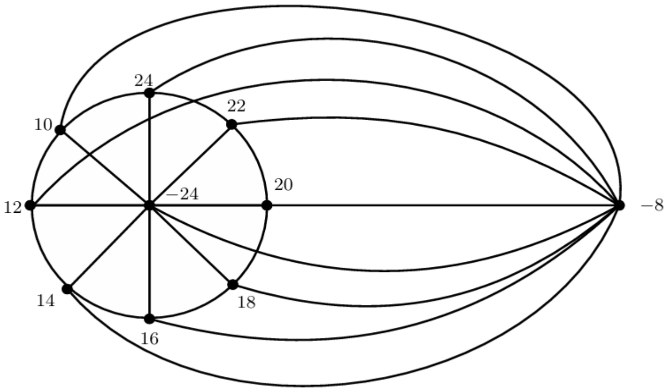

Fig. 3 Absolute mean Cordial labeling of $W_8 + K_1$

Theorem 2.7. The graph $J_{m,n} + K_1$ join of jellyfish graph $J_{m,n}$ with a complete graph K_1 is an absolute mean cordial graph.
Proof. Let $G = J_{m,n} + K_1$ is a join of any jellyfish graph $J_{m,n}$ with a complete graph K_1.
Let V($J_{m,n}$) = $\{u_i, v_j \ /1 \le i \le m, 1 \le j \le n\} \cup \{u, v, x, y\}$ and
E($J_{m,n}$) = $\{uu_i, vv_j \ /1 \le i \le m, 1 \le j \le n\} \cup \{ux, xv, vy, yu, xy\}$.
Therefore, V(G) = $\{u_i, v_j \ /1 \le i \le m, 1 \le j \le n\} \cup \{u, v, x, y, w\}$and
E(G) = $\{uu_i, wu_i \ /1 \le i \le m\} \cup \{vv_j, wv_j \ / \ 1 \le j \le n\} \cup \{ux, xv, vy, yu, xw, yw, uw, vw, xy\}$.
The vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lfloor\frac{q}{2}\right\rfloor$} is defined as follows.
$f(u) = q, f(y) = -q,$
$f(x) = q - 2, f(v) = 2 - q,$
$f(w) = -2(m + n),$
$f(u_1) = 4 - q, f(u_2) = 6 - q,$
$f(u_i) = 4 - q + 2i, 3 \le i \le m$
$f(v_j) = q - 2m + 4j - 4n - 4, 1 \le j \le n$
The labeling function f, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, f satisfies the condition $|e_f(0) - e_f(1)| \le 1$. Thus, f is an absolute mean cordial labeling for a given graph.
Therefore, the graph $J_{m,n} + K_1$ is an absolute mean cordial graph.

Illustration 2.8. Absolute mean cordial labeling for $J_{3,5} + K_1$ is shown in the following Figure 4.

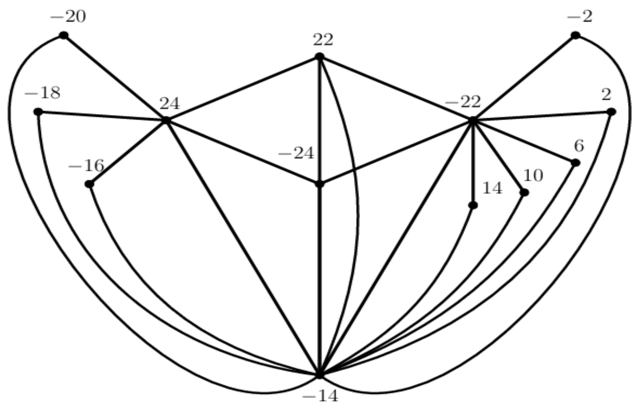

Fig. 4 Absolute mean Cordial labeling of $J_{3,5} + K_1$

Corollary 2.9. The graph $J^*_{m,n} + K_1$ join of jellyfish graph without a prime edge $J^*_{m,n}$ with a complete graph K_1 is an absolute mean cordial graph.
Proof. Let $G = J^*_{m,n} + K_1$ is a join of any jellyfish graph without a prime edge $J^*_{m,n}$ with a complete graph K_1.
Let V($J^*_{m,n}$) = $\{u_i, v_j \ /1 \le i \le m, 1 \le j \le n\} \cup \{u, v, x, y\}$ and
E($J_{m,n}$) = $\{uu_i, vv_j \ /1 \le i \le m, 1 \le j \le n\} \cup \{ux, xv, vy, yu\}$.

Therefore, V(G) = $\{u_i, v_j /1 \leq i \leq m, 1 \leq j \leq n\} \cup \{u, v, x, y, w\}$ and
E(G) = $\{uu_i, wu_i /1 \leq i \leq m\} \cup \{vv_j, wv_j / 1 \leq j \leq n\} \cup \{ux, xv, vy, yu, xw, yw, uw, vw\}$.
The vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lfloor\frac{q}{2}\right\rfloor$} is defined as follows.
$f(u) = q, f(y) = -q,$
$f(x) = q - 2, f(v) = 2 - q,$
$f(w) = -2(m + n),$
$f(u_1) = 4 - q, f(u_2) = 6 - q,$
$f(u_i) = 4 - q + 2i, 3 \leq i \leq m$
$f(v_j) = q - 2m + 4j - 4n - 4, 1 \leq j \leq n$
The labeling function f, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, f satisfies the condition $\left|e_f(0) - e_f(1)\right| \leq 1$. Thus, f is an absolute mean cordial labeling for a given graph.
Therefore, the graph $J^*_{m,n} + K_1$ is an absolute mean cordial graph.

Illustration 2.10. Absolute mean cordial labeling for $J^*_{2,5} + K_1$ is shown in the following Figure 5.

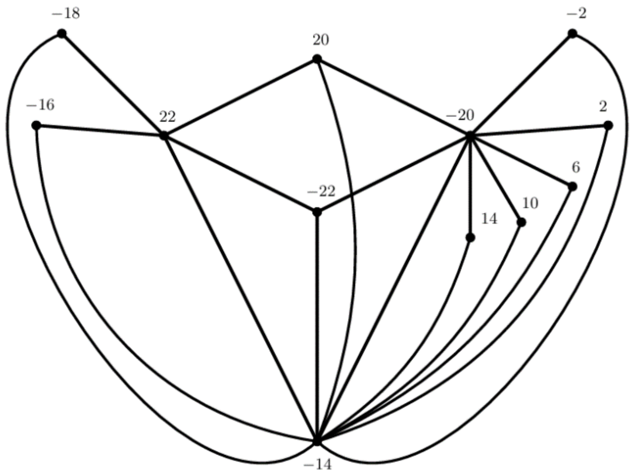

Fig. 5 Absolute mean Cordial labeling of $J^*_{2,5} + K_1$

Theorem 2.11. The graph $F_n + K_1$ join of a fan graph F_n with a complete graph K_1 is an absolute mean cordial graph.
Proof. Let $G = F_n + K_1$ is a join of any fan graph F_n with a complete graph K_1.
Let V(F_n) = $\{v_i /0 \leq i \leq 2n\}$ and E(F_n) = $\{v_0 v_i/1 \leq i \leq 2n\} \cup \{v_i v_{i+1}/ \ i = 1, 3, \dots, 2n - 1\}$.
Therefore, V(G) = $\{ v_i /0 \leq i \leq 2n\} \cup \{w\}$ and
E(G) = $\{v_0 v_i/1 \leq i \leq 2n\} \cup \{v_i v_{i+1} \ / \ i = 1,3, \dots, 2n - 1\} \cup \{wv_i \ / \ 0 \leq i \leq 2n\}$
To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lfloor\frac{q}{2}\right\rfloor$}, we take the following cases.

Case 1: $n \equiv 0 (mod 2)$
$f(w) = -(n + 4), f(v_0) = q,$
$f(v_2) = 2,$
$f(v_i) = i - q, if \ i = 1,3, \dots, 2n - 1$
$f(v_i) = q - i + 1, if \ i = 4,6, \dots, 2n$

Case 2: $n \equiv 1 (mod 2)$
$f(w) = -2\left\lceil\frac{n}{2}\right\rceil, f(v_0) = q,$
$f(v_2) = 2,$
$f(v_i) = i - q - 1, if \ i = 1,3, \dots, 2n - 1$
$f(v_i) = q - i + 2, if \ i = 4,6, \dots, 2n$
The labeling function f, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, f satisfies the condition $\left|e_f(0) - e_f(1)\right| \leq 1$. Thus, f is an absolute mean cordial labeling for a given graph.
Therefore, the graph $F_n + K_1$ is an absolute mean cordial graph.

Illustration 2.12. Absolute mean cordial labeling for $F_8 + K_1$ is shown in the following Figure 6.

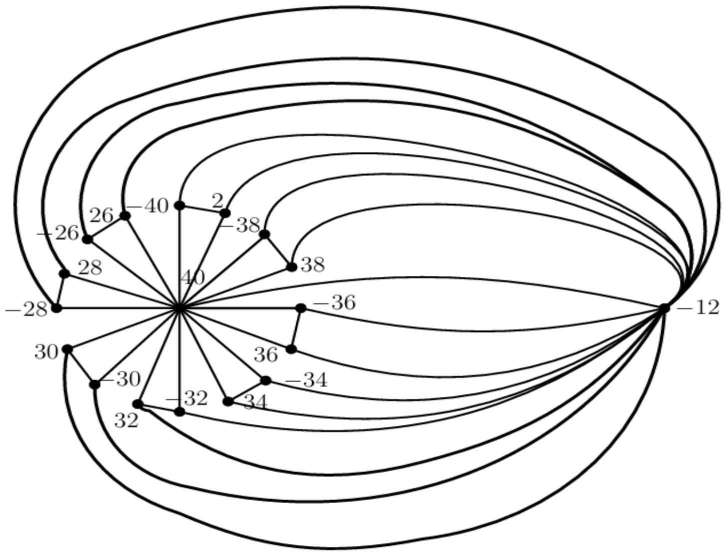

Fig. 6 Absolute mean Cordial labeling of $\mathbf{F_8 + K_1}$

Theorem 2.13. The graph $Fl_n + K_1$ join of a flower graph Fl_n with a complete graph K_1 is an absolute mean cordial graph.
Proof. Let $G = Fl_n + K_1$ is a join of any flower graph Fl_n with a complete graph K_1.
Let V(Fl_n) = $\{v_i \ / \ 0 \leq i \leq n\} \cup \{v'_i \ / 1 \leq i \leq n\}$ and
E(Fl_n) = $\{v_0 v_i, v_0 v'_i, v_i v'_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} \ / \ 1 \leq i \leq n-1\}$.

Therefore, V(G) = $\{ v_i \ /0 \leq i \leq n\} \cup \{v'_i \ / \ 1 \leq i \leq n\} \cup \{w\}$ and
E(G) = $\{v_0 v_i, v_0 v'_i, v_i v'_i \ / \ 1 \leq i \leq n\} \cup \{v_i v_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{v_i w, v'_i w \ / \ 1 \leq i \leq n \ \} \cup \{v_0 w\}$
The vertex labeling function *f* : V(G)→$\{0, \pm 2, \pm 4, ..., \pm \left\lfloor \frac{q}{2} \right\rfloor\}$ is defined as follows.

$f(w) = -2(n+1)$,
$f(v_0) = 1 - q$,
$f(v_i) = q - 2i + 1, 1 \leq i \leq n$
$f(v'_i) = 4n - 2i + 2, 1 \leq i \leq n$

The labeling function *f*, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \rightarrow \{0,1\}$ is onto. Also, *f* satisfies the condition $\left|e_f(0) - e_f(1)\right| \leq 1$.
Thus, *f* is an absolute mean cordial labeling for a given graph.
Therefore, the graph $Fl_n + K_1$ is an absolute mean cordial graph.

Illustration 2.14. Absolute mean cordial labeling for $Fl_5 + K_1$ is shown in the following Figure 7.

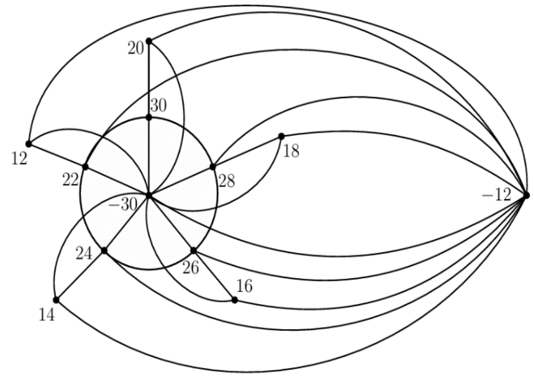

Fig. 7 Absolute mean Cordial labeling of $\mathbf{Fl_5 + K_1}$

Theorem 2.15. The graph $S_n + K_1$ join of sunlet graph S_n with a complete graph K_1 is an absolute mean cordial graph.
Proof. Let $G = S_n + K_1$ is a join of any sunlet graph S_n with a complete graph K_1.
Let V(S_n) = $\{u_i, u'_i \ / \ 0 \le i \le n, u_{n+1} = u_1, u'_{n+1} = u'_1\}$ and E(S_n) = $\{u_i u'_i, u_i u_{i+1} / 1 \le i \le n\}$.
Therefore, V(G) = $\{ u_i, u'_i / 1 \le i \le n\} \cup \{w\}$ and E(G) = $\{ u_i u'_i , u_i u_{i+1} \ / \ 1 \le i \le n\} \cup \{u_i w, u'_i w \ / \ 1 \le i \le n\}$
The vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lfloor\frac{q}{2}\right\rfloor$} is defined as follows.
$f(w) = -2n,$
$f(u_i) = q - 2i + 2, 1 \le i \le n$
$f(u'_i) = 2i - q - 2, 1 \le i \le n$

The labeling function *f*, defined as above, is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also, *f* satisfies the condition $|e_f(0) - e_f(1)| \le 1$. Thus, *f* is an absolute mean cordial labeling for a given graph.
Therefore, the graph $S_n + K_1$ is an absolute mean cordial graph.

Illustration 2.16. Absolute mean cordial labeling for $S_5 + K_1$ is shown in the following Figure 8.

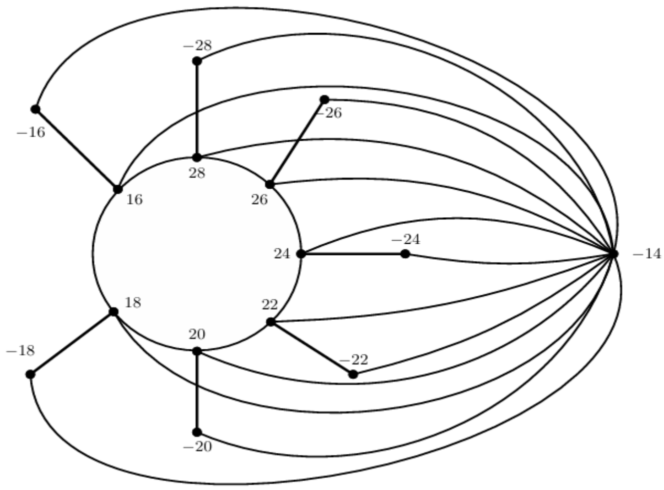

Fig. 8 Absolute mean Cordial labeling of $S_5 + K_1$

Theorem 2.17. The graph $H_n + K_1$ join of helm graph H_n with complete graph K_1 is absolute mean cordial graph.
Proof. Let $G = H_n + K_1$ is a join of any helm graph H_n with complete graph K_1.
Let V(H_n) = $\{u_i \ / \ 0 \le i \le n, u_{n+1} = u_1\} \cup \{u'_i \ / \ 1 \le i \le n, u'_{n+1} = u'_1\}$ and E(H_n) = $\{u_0 u_i, u_i u'_i, u_i u_{i+1} \ / \ 1 \le i \le n\}$.
Therefore, V(G) = $\{ u_i \ / \ 0 \le i \le n\} \cup \{u'_i \ / \ 1 \le i \le n\} \cup \{w\}$ and
E(G) = $\{ u_0 u_i , u_i u'_i \ / \ 1 \le i \le n\} \cup \{u_i u_{i+1} \ / \ 1 \le i \le n\} \cup \{u_i w, u'_i w \ / \ 1 \le i \le n\} \cup \{u_0 w\}$.
To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lfloor\frac{q}{2}\right\rfloor$} we take following cases.

Case-1: $n \equiv 0(mod 2)$
$f(u_0) = 1 - q,$
$f(w) = -4n,$
$f(u_i) = q - 2i + 1, 1 \le i \le n$
$f(u'_i) = 2(n - i + 2), 1 \le i \le n$

Case-2: $n \equiv 1(mod 2)$
$f(u_0) = -q,$
$f(w) = -4n,$
$f(u_i) = q - 2i + 2, 1 \le i \le n$
$f(u'_i) = 2(n - i + 2), 1 \le i \le n$
The labeling function *f* defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function $f^* : E(G) \to \{0,1\}$ is onto. Also *f* satisfies the condition $|e_f(0) - e_f(1)| \le 1$.
Thus, *f* is an absolute mean cordial labeling for given graph.
Therefore, the graph $H_n + K_1$ is an absolute mean cordial graph.

Illustration 2.18. Absolute mean cordial labeling for $H_5 + K_1$ is shown in following Figure 9.

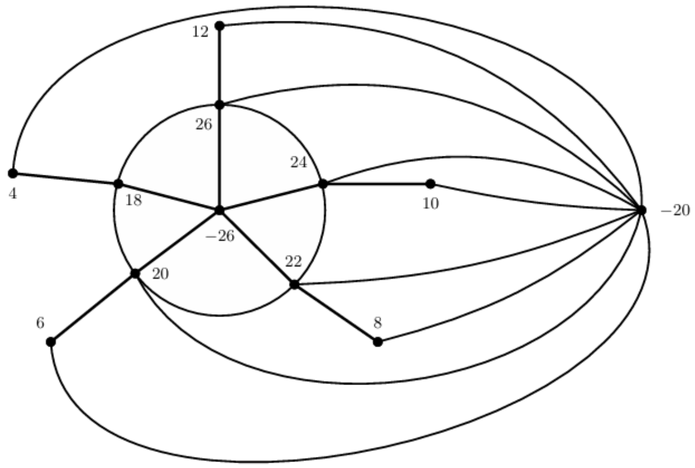

Fig. 9 Absolute mean Cordial labeling of $H_5 + K_1$

3. Conclusion

In this work, we establish that the join of various graph structures can admit absolute mean cordial labeling. Furthermore, we investigate how different graph operations — such as union, Cartesian product, and other binary operations — affect the existence of absolute mean cordial labelings. For each operation, we determine whether the resulting graph admits such a labeling, thereby extending the scope of cordial labeling theory to new structural contexts.

References

[1] P.Z. Akbari, J.C. Kanani, and V.J. Kaneria, "A Study on Absolute Mean Cordial Labeling of Various Graphs," *Educational Administration: Theory and Practice*, vol. 29, no. 4, pp. 3094–3102, Dec. 2023. [CrossRef] [Publisher Link]

[2] P.Z. Akbari, V.J. Kaneria, and N.A. Parmar, "Absolute Mean Graceful Labeling of Jewel Graph and Jelly Fish Graph," *International Journal of Mathematics Trends and Technology*, vol. 68, no. 1, pp. 86-93, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[3] P.Z. Akbari, V.J. Kaneria, and N.A. Parmar, "Some Mean Graceful Graphs in the Context of Barycentric Subdivision," *International Journal of Mathematics Trends and Technology*, vol. 68, no. 5, pp. 51-60, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[4] I. Cahit, "Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs," *Ars Combinatoria*, vol. 23, pp. 201-207, 1987. [Google Scholar]

[5] Ibrahim Cahit, "On Cordial and 3-Equitable Labelings of Graphs," *Utilitas Mathematica*, vol. 37, pp. 129-137, 1990. [Google Scholar]

[6] Joseph A. Gallian, "A Dynamic Survey of Graph Labeling," *The Electronic Journal of Combinatorics*, vol. 28, Article DS6, 2025. [CrossRef] [Google Scholar] [Publisher Link]

[7] Solomon W. Golomb, "How to Number a Graph," *Graph Theory and Computing*, Academic Press, pp. 23-37, 1972. [CrossRef] [Google Scholar] [Google Scholar]

[8] F. Harary, *Graph Theory*, New Delhi, India: Narosa Publishing House, 2001. [Google Scholar]

[9] J.C. Kanani, and H.P. Chudasama, "Absolute Mean Cordial Labeling," *Educational Administration: Theory and Practice*, vol. 29, no. 4, pp. 3119-3130, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[10] H.P. Chudasama, and V.J. Kaneria, "Absolute Mean Graceful Labeling in Path Union of Various Graphs," *Mathematical Journal of Interdisciplinary Sciences*, vol. 7, no. 1, pp. 51–56, 2018. [Google Scholar]

[11] A. Rosa, "On Certain Valuation of the Vertices of the Graph," *Theory of Graphs: International Symposium*, Gordon and Breach, pp. 349-355, 1964. [Google Scholar]

[12] S.K. Vaidya et al., "Cordial and 3-Equitable Labeling for Some Shell Related Graphs," *Journal of Scientific Research*, vol. 1, no. 3, pp. 438-449, 2009. [CrossRef] [Google Scholar] [Google Scholar]