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Original Article

Density Parameters of K_4-Snake Graph Families and Corona Product Structures

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Abstract *- In this research article, we derive density expressions explicitly for families of K_4-snake graphs and other graph structures together with their corona products. We investigate Closed-form formulas for the density of m-complete snake graphs, double and alternate snake graphs, subdivision of families of snake graphs, barbell graph, friendship graph, and double triangular snake graph. Furthermore, we obtain the density of corona products for barbell graph, friendship graph, and double triangular snake graph with complete graph K_1 and identical graph copies. These findings provide consolidated expressions that narrate how density behaves under structural changes such as double, alternation, subdivision, and corona operations. The established formulas contribute to the theoretical study of graph density and extend existing work on density behavior in composite graph operations, as they gives a deeper insight into graph density behavior under various graph structures.*

Keywords *- Alternate snake graph, Corona product, Double snake graph, Graph density, Snake graph.*

1. Introduction

Graph theory constitutes a fundamental component of discrete mathematics, providing a flexible framework for describing and evaluating interactions across several domains, including computer networks, biology, social sciences, and communication systems. Density is a highly relevant metric in graph structure analysis, as it indicates the degree of interconnection among the vertices of a graph. Investigating the variations of this metric across several graph families and through different graph operations helps enhance comprehension of connection patterns and structural complexity.

As global systems grow more interconnected, graph theory has become an essential instrument for modelling and addressing real-world issues. The swift advancement of network science and data-driven research has established graphs as a potent framework for analysing trends, behaviours, and intricate phenomena [1]. Density is a crucial statistic in graph analysis, as it indicates the extent of interconnection among nodes in a network. The corona product is an operation that profoundly affects the structural features of a graph. The corona product generates enlarged graph structures by affixing a copy of one graph to each vertex of another, resulting in densities that are frequently more complex to understand. Examining density in this context yields insights into the evolution of connectivity as graphs are expanded or amalgamated.

Graph density provides significant information about the structural properties of a graph. In social networks, density indicates the level of contact within a group or community, signifying whether the connections are sparse or highly cohesive. Density serves as a fundamental statistic essential for understanding the structure and operation of networks across several fields, including computer science, biology, communication systems, and transportation. Through the assessment of density, researchers attain a more lucid understanding of the fundamental patterns and connectivity inside a network, facilitating enhanced analysis and interpretation.

This study examines the density of various specialized graphs and their corona products. Following the exposition of essential definitions and concepts necessary for the analysis, we ascertain the density of the m-complete snake graph, the double m-complete snake graph, the alternate m-complete snake graph, the double alternate m-complete snake graph, the barbell graph, the friendship graph, and the double triangular snake graph. We then extend our analysis to the corona product of these graphs with K1, as well as to corona products formed from several identical copies of these graphs.

Previous studies have examined the density behaviour of certain basic graphs and corona products; however, closed-form density formulas for K4-snake graph families and their structural variants remain largely unexplored. We investigate density formulas for K4-snake graph families, including alternate, double alternate, and subdivision of snake graphs, corona products involving barbell graphs, friendship graphs, and double triangular snake graphs.

2. Basic Terminology

Definition 2.1: In the graph, if we denote the number of edges by $|e|$ and the number of vertices by $|v|$ then density of graph[2] is given by $\frac{2\cdot|e|}{|v|\cdot(|v|-1)}$. We will denote it as D.

Definition 2.2: An m-complete snake graph[3] is a graph formed from the path P_n by substituting each of its edges with a copy of the complete graph K_m, where $n \geq 2$, $m \geq 3$. We will denote it as K_mS_n. (see Figure 1)

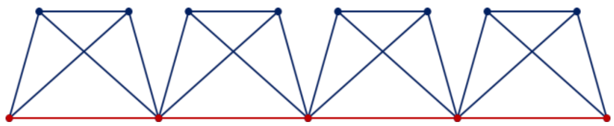

Fig. 1 4-complete snake graph K_4S_5

Definition 2.3: A double m-complete snake graph[3,4] is a graph constructed from the path P_n by substituting each of its edges with a copy of a double m-complete graph, where $n \geq 2$, $m \geq 3$. We will denote it as DK_mS_n. (see Figure 2)

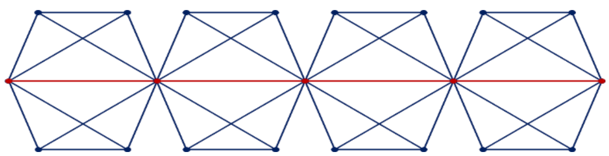

Fig. 2 Double 4-complete snake graph DK_4S_5

Definition 2.4: An alternate m-complete snake graph[3,4] is a graph constructed from the path P_n by replacing every second(alternate) edge of P_n with a copy of the complete graph K_m, where $n \geq 2$, $m \geq 3$. We will denote it as AK_mS_n.

For any fixed $m \geq 3$, an alternate m-complete snake graph can occur in three forms depending on the parity of n and placement of the replaced edges in the path P_n. We will denote an alternate type-i m-complete snake graph as $A_iK_mS_n$. (see Figures 3, 4, 5)

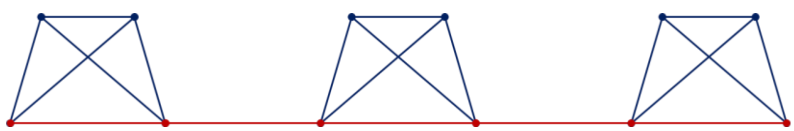

Fig. 3 Alternate type-1 4-complete snake graph $A_1K_4S_6$

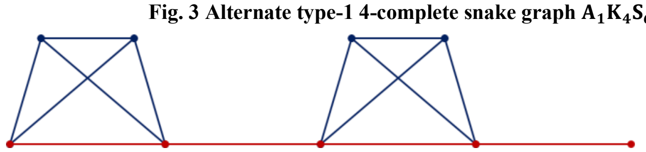

Fig. 4 Alternate type-2 4-complete snake graph $A_2K_4S_5$

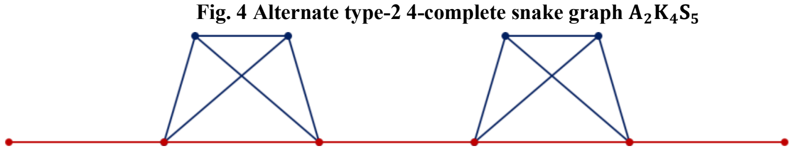

Fig. 5 Alternate type-3 4-complete snake graph $A_3K_4S_6$

Definition 2.5: A subdivision graph[4] is a graph derived from a base graph G by systematically performing an edge subdivision operation on every single edge belonging to G. We will denote it as $S(G)$. (see Figure 6)

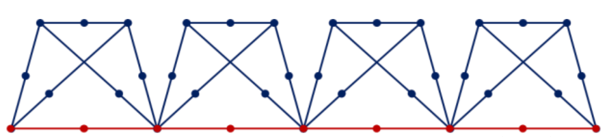

Fig. 6 Subdivision of 4-complete snake graph $S(K_4S_5)$

Definition 2.6: A barbell graph[5,6] is formed by taking two disjoint copies of the complete graph $\boldsymbol{K_n}$ (where $\boldsymbol{n \geq 3}$) and linking them with a single edge. We will denote it as B_n. (see Figure 7)

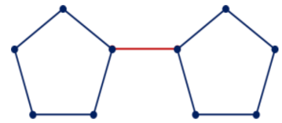

Fig. 7 Barbell graph B_5

Definition 2.7: A friendship graph[7] is constructed by joining $\boldsymbol{n}$ copies of the cycle $\boldsymbol{C_3}$ at a single common vertex, where $\boldsymbol{n \geq 2}$. We will denote it as $\boldsymbol{F_{rn}}$. (see Figure 8)

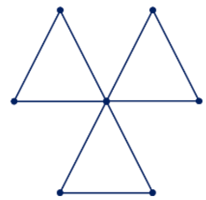

Fig. 8 Friendship graph Fr_3

Definition 2.8: A double triangular snake graph[4] is constructed by combining two triangular snake graphs that share the same underlying path. We will denote it as DT_n. (see Figure 9)

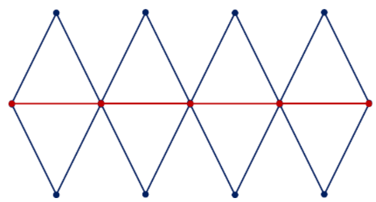

Fig. 9 Double triangular snake graph DT_5

Definition 2.9: Let G and H be two graphs. The corona product[8,9] of G with H is obtained by taking one copy of G and $|V(G)|$ copies of H, and then joining each vertex of G to every vertex in the corresponding copy of H. We will denote it as $G \circ H$. (see Figure 10)

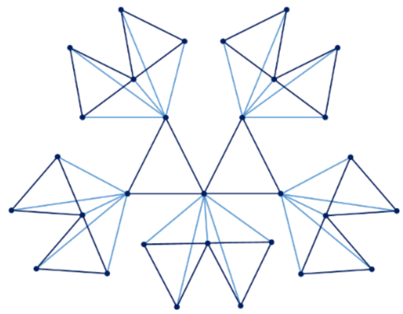

Fig. 10 Corona product of Fr_2 and Fr_2, $Fr_2 \circ Fr_2$

Definition 2.10: Main Results and Examples

Theorem 3.1: The expression for calculating the density of the 4-complete snake graph K_4S_n is given by $\frac{4}{3n-2}$;$n \geq 2$.

Proof: Based on the property of the 4-complete snake graph K_4S_n,

$|v| = 3n - 2$ and $|e| = 6(n-1)$

$$\therefore D(K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 6(n-1)}{(3n-2)(3n-2-1)} = \frac{4}{3n-2}.$$

Theorem 3.2: The expression for calculating the density of the double 4-complete snake graph DK_4S_n is given by $\frac{22}{5(5n-4)}$;n ≥ 2.

Proof: Based on the property of the double 4-complete snake graph DK_4S_n,

$|v| = 5n - 4$ and $|e| = 11(n - 1)$

$$\therefore D(DK_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 11(n-1)}{(5n-4)(5n-4-1)} = \frac{22}{5(5n-4)}.$$

Theorem 3.3: The expression for calculating the density of the alternate type-1 4-complete snake graph $A_1K_4S_n$ is given by $\frac{7n-2}{2n(2n-1)}$;n ≥ 2 is even.

Proof: Based on the property of the alternate type-1 4-complete snake graph $A_1K_4S_n$,

$|v| = 2n$ and $|e| = \frac{7n-2}{2}$

$$\therefore D(A_1K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot \frac{7n-2}{2}}{2n(2n-1)} = \frac{7n-2}{2n(2n-1)}.$$

Theorem 3.4: The expression for calculating the density of the double alternate type-1 4-complete snake graph $DA_1K_4S_n$ is given by $\frac{2(6n-1)}{3n(3n-1)}$;n ≥ 2 is even.

Proof: Based on the property of the double alternate type-1 4-complete snake graph $DA_1K_4S_n$,

$|v| = 3n$ and $|e| = 6n - 1$

$$\therefore D(DA_1K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (6n-1)}{3n(3n-1)}.$$

Theorem 3.5: The expression for calculating the density of the alternate type-2 4-complete snake graph $A_2K_4S_n$ is given by $\frac{7}{2(2n-1)}$;n ≥ 2 is odd.

Proof: Based on the property of the alternate type-2 4-complete snake graph $A_2K_4S_n$,

$|v| = 2n - 1$ and $|e| = \frac{7(n-1)}{2}$

$$\therefore D(A_2K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot \frac{7(n-1)}{2}}{(2n-1)(2n-1-1)} = \frac{7}{2(2n-1)}.$$

Theorem 3.6: The expression for calculating the density of the double alternate type-2 4-complete snake graph $DA_2K_4S_n$ is given by $\frac{4}{3n-2}$;n ≥ 2 is odd.

Proof: Based on the property of the double alternate type-2 4-complete snake graph $DA_2K_4S_n$,

$|v| = 3n - 2$ and $|e| = 6(n - 1)$

$$\therefore D(DA_2K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 6(n-1)}{(3n-2)(3n-2-1)} = \frac{4}{3n-2}.$$

Corollary 3.1: From Theorem 3.1 and Theorem 3.6, the density of the 4-complete snake graph K_4S_n and double alternate type-2 4-complete snake graph $DA_2K_4S_n$ are same. (see Figure 11)

$$\therefore D(K_4S_n) = D(DA_2K_4S_n) = \frac{4}{3n-2}$$

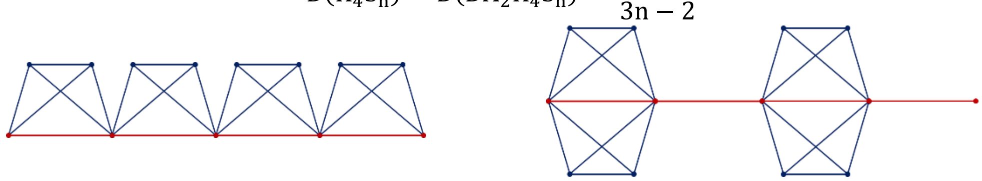

Fig. 11 K_4S_5 and $DA_2K_4S_5$

Theorem 3.7: The expression for calculating the density of the alternate type-3 4-complete snake graph $A_3K_4S_n$ is given by $\frac{7n-12}{2(n-1)(2n-3)}$;n ≥ 2 is even.

Proof: Based on the property of the alternate type-3 4-complete snake graph $A_3K_4S_n$,

$|v| = 2n - 2$ and $|e| = \frac{7n-12}{2}$

$$\therefore D(A_3K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot \frac{7n-12}{2}}{(2n-2)(2n-2-1)} = \frac{7n-12}{2(n-1)(2n-3)}.$$

Theorem 3.8: The expression for calculating the density of the double alternate type-3 4-complete snake graph $DA_3K_4S_n$ is given by $\frac{2(6n-11)}{(3n-4)(3n-5)}$;n ≥ 2 is even.

Proof: Based on the property of the double alternate type-3 4-complete snake graph $DA_3K_4S_n$,

$|v| = 3n - 4$ and $|e| = 6n - 11$

$$\therefore D(DA_3K_4S_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (6n-11)}{(3n-4)(3n-4-1)} = \frac{2(6n-11)}{(3n-4)(3n-5)}.$$

Theorem 3.9: The expression for calculating the density of the subdivision of the 4-complete snake graph $S(K_4S_n)$ is given by $\frac{8}{3(9n-8)}$;n ≥ 2.

Proof: Based on the property of the subdivision of a 4-complete snake graph $S(K_4S_n)$,

$|v| = 9n - 8$ and $|e| = 12(n - 1)$

$$\therefore D\big(S(K_4S_n)\big) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 12(n-1)}{(9n-8)(9n-8-1)} = \frac{8}{3(9n-8)}.$$

Theorem 3.10: The expression for calculating the density of the subdivision of the double 4-complete snake graph $S(DK_4S_n)$ is given by $\frac{11}{4(16n-15)}$;n ≥ 2.

Proof: Based on the property of the subdivision of a double 4-complete snake graph $S(DK_4S_n)$,

$|v| = 16n - 15$ and $|e| = 22(n - 1)$

$$\therefore D\big(S(DK_4S_n)\big) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 22(n-1)}{(16n-15)(16n-15-1)} = \frac{11}{4(16n-15)}.$$

Theorem 3.11: The expression for calculating the density of the subdivision of an alternate type-1 4-complete snake graph $S(A_1K_4S_n)$ is given by $\frac{8(7n-2)}{(11n-2)(11n-4)}$;n ≥ 2 is even.

Proof: Based on the property of the subdivision of an alternate type-1 4-complete snake graph $S(A_1K_4S_n)$,

$|v| = \frac{11n-2}{2}$ and $|e| = 7n - 2$

$$\therefore D\big(S(A_1K_4S_n)\big) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (7n-2)}{\left(\frac{11n-2}{2}\right)\left(\frac{11n-2}{2} - 1\right)} = \frac{8(7n-2)}{(11n-2)(11n-4)}.$$

Theorem 3.12: The expression for calculating the density of the subdivision of a double alternate type-1 4-complete snake graph $S(DA_1K_4S_n)$ is given by $\frac{4(6n-1)}{(9n-1)(9n-2)}$;n ≥ 2 is even.

Proof: Based on the property of the subdivision of a double alternate type-1 4-complete snake graph $S(DA_1K_4S_n)$,

$|v| = 9n - 1$ & $|e| = 12n - 2$

$$\therefore D\big(S(DA_1K_4S_n)\big) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 2(6n-1)}{(9n-1)(9n-1-1)} = \frac{4(6n-1)}{(9n-1)(9n-2)}.$$

Theorem 3.13: The expression for calculating the density of the subdivision of an alternate type-2 4-complete snake graph $S(A_2K_4S_n)$ is given by $\frac{56}{11(11n-9)}$;n ≥ 2 is odd.

Proof: Based on the property of the subdivision of an alternate type-2 4-complete snake graph $S(A_2K_4S_n)$,

$|v| = \frac{11n-9}{2}$ and $|e| = 7(n - 1)$

$$\therefore D\big(S(A_2K_4S_n)\big) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 7(n-1)}{\left(\frac{11n-9}{2}\right)\left(\frac{11n-9}{2} - 1\right)} = \frac{56}{11(11n-9)}.$$

Theorem 3.14: The expression for calculating the density of the subdivision of a double alternate type-2 4-complete snake graph $S(DA_2K_4S_n)$ is given by $\frac{8}{3(9n-8)}$;$n \geq 2$ is odd.

Proof: Based on the property of the subdivision of a double alternate type-2 4-complete snake graph $S(DA_2K_4S_n)$,

$|v| = 9n - 8$ & $|e| = 12(n-1)$

$$\therefore D(S(DA_2K_4S_n)) = \frac{2 \cdot |e|}{|v| \cdot (|v|-1)} = \frac{2 \cdot 12(n-1)}{(9n-8)(9n-8-1)} = \frac{8}{3(9n-8)}.$$

Corollary 3.2: From Theorem 3.9 and Theorem 3.14, density of subdivision of 4-complete snake graph $S(K_4S_n)$ and subdivision of double alternate type-2 4-complete snake graph $DA_2K_4S_n$ are same. (see Figure 12)

$$\therefore D(S(K_4S_n)) = D(S(DA_2K_4S_n)) = \frac{8}{3(9n-8)}$$

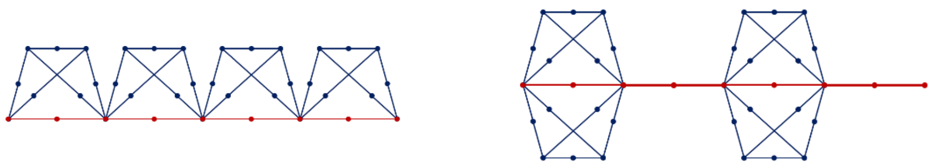

Fig. 12 $S(K_4S_5)$ and $S(DA_2K_4S_5)$

Theorem 3.15: The expression for calculating the density of the subdivision of an alternate type-3 4-complete snake graph $S(A_3K_4S_n)$ is given by $\frac{8(7n-12)}{(11n-16)(11n-18)}$;$n \geq 2$ is even.

Proof: Based on the property of the subdivision of an alternate type-3 4-complete snake graph $S(A_3K_4S_n)$,

$|v| = \frac{11n-16}{2}$ and $|e| = 7n - 12$

$$\therefore D(S(A_3K_4S_n)) = \frac{2 \cdot |e|}{|v| \cdot (|v|-1)} = \frac{2 \cdot (7n-12)}{\left(\frac{11n-16}{2}\right)\left(\frac{11n-16}{2}-1\right)} = \frac{8(7n-12)}{(11n-16)(11n-18)}.$$

Theorem 3.16: The expression for calculating the density of the subdivision of a double alternate type-3 4-complete snake graph $S(DA_3K_4S_n)$ is given by $\frac{4(6n-11)}{(9n-15)(9n-16)}$;$n \geq 2$ is even.

Proof: Based on the property of the subdivision of a double alternate type-3 4-complete snake graph $S(DA_3K_4S_n)$,

$|v| = 9n - 15$ & $|e| = 2(6n-11)$

$$\therefore D(S(DA_3K_4S_n)) = \frac{2 \cdot |e|}{|v| \cdot (|v|-1)} = \frac{2 \cdot 2(6n-11)}{(9n-15)(9n-15-1)} = \frac{4(6n-11)}{(9n-15)(9n-16)}.$$

Theorem 3.17: The expression for calculating the density of the corona product of B_n and K_1 is given by $\frac{4n+1}{2n(4n-1)}$;$n \geq 3$.

Proof: Based on the property of the corona product of B_n and K_1,

$|v| = 4n$ and $|e| = 4n + 1$

$$\therefore D(B_n \circ K_1) = \frac{2 \cdot |e|}{|v| \cdot (|v|-1)} = \frac{2 \cdot (4n+1)}{4n(4n-1)} = \frac{4n+1}{2n(4n-1)}.$$

Theorem 3.18: The expression for calculating the density of the corona product of B_n and B_n is given by $\frac{8n^2+4n+1}{n(2n+1)(4n^2+2n-1)}$;$n \geq 3$.

Proof: Based on the property of the corona product of B_n and B_n,

$|v| = 4n^2 + 2n$ and $|e| = 8n^2 + 4n + 1$

$$\therefore D(B_n \circ B_n) = \frac{2 \cdot |e|}{|v| \cdot (|v|-1)} = \frac{2 \cdot (8n^2+4n+1)}{(4n^2+2n)(4n^2+2n-1)} = \frac{8n^2+4n+1}{n(2n+1)(4n^2+2n-1)}.$$

Theorem 3.19: The expression for calculating the density of the corona product of Fr_n and K_1 is given by $\frac{5n+1}{(2n+1)(4n+1)}$;$n \geq 2$.

Proof: Based on the property of the corona product of Fr_n and K_1,

$|v| = 2(2n+1)$ and $|e| = 5n + 1$

$$\therefore D(Fr_n \circ K_1) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (5n + 1)}{\big(2(2n + 1)\big)(2(2n + 1) - 1)} = \frac{5n + 1}{(2n + 1)(4n + 1)}.$$

Theorem 3.20: The expression for calculating the density of the corona product of Fr_n and Fr_n is given by $\frac{10n^2+10n+1}{(n+1)(2n+1)(4n^2+6n+1)}$;$n \geq 2$.

Proof: Based on the property of the corona product of Fr_n and Fr_n,

$|v| = 4n^2 + 6n + 2$ and $|e| = 10n^2 + 10n + 1$

$$\therefore D(Fr_n \circ Fr_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (10n^2 + 10n + 1)}{(4n^2 + 6n + 2)(4n^2 + 6n + 2 - 1)} = \frac{10n^2 + 10n + 1}{(n + 1)(2n + 1)(4n^2 + 6n + 1)}.$$

Theorem 3.21: The expression for calculating the density of the double triangular snake graph DT_n is given by $\frac{10}{3(3n-2)}$;$n \geq 2$.

Proof: Based on the property of the double triangular snake graph DT_n,

$|v| = 3n - 2$ and $|e| = 5(n - 1)$

$$\therefore D(DT_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot 5(n - 1)}{(3n - 2)(3n - 2 - 1)} = \frac{10}{3(3n - 2)}.$$

Theorem 3.22: The expression for calculating the density of the corona product of DT_n and K_1 is given by $\frac{8n-7}{(3n-2)(6n-5)}$;$n \geq 2$.

Proof: Based on the property of the corona product of DT_n and K_1,

$|v| = 6n - 4$ and $|e| = 8n - 7$

$$\therefore D(DT_n \circ K_1) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot (8n - 7)}{(6n - 4)(6n - 4 - 1)} = \frac{8n - 7}{(3n - 2)(6n - 5)}.$$

Theorem 3.23: The expression for calculating the density of the corona product of DT_n and DT_n is given by $\frac{48n^2-64n+18}{(9n^2-9n+2)(9n^2-9n+1)}$;$n \geq 2$.

Proof: Based on the property of the corona product of DT_n and DT_n,

$|v| = 9n^2 - 9n + 2$ and $|e| = (8n - 7)(3n - 2) + 5(n - 1)$

$$\therefore D(DT_n \circ DT_n) = \frac{2 \cdot |e|}{|v| \cdot (|v| - 1)} = \frac{2 \cdot \big((8n - 7)(3n - 2) + 5(n - 1)\big)}{(9n^2 - 9n + 2)(9n^2 - 9n + 2 - 1)} = \frac{48n^2 - 64n + 18}{(9n^2 - 9n + 2)(9n^2 - 9n + 1)}.$$

4. Conclusion

In this paper, we investigated explicit closed-form formulas for the graph density of several graph families and their corona products. Density formulas were obtained for several families of K_4-snake graphs, including alternate, double alternate, and subdivision of the snake graph. Furthermore, density expressions were derived for the corona products involving barbell graphs, friendship graphs, and double triangular snake graphs with K_1 and identical copies of these graphs. These results help in understanding the graph density theoretically and provide a basis for further investigations for broader classes of graphs and the generalization of operations. This may include extending the present approach to generalized graphs, higher corona operations, and other graph products.

Conflicts of Interest

The author(s) declare that there is no conflict of interest regarding the publication of this paper.

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