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Original Article

An Exploration of Absolute Mean Graceful Labeling Across Distinct Graphs

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Abstract *- This paper investigates several graphs that are absolute mean graceful graphs. We establish new results concerning absolute mean graceful graphs. The main objective of this work is to study the absolute mean graceful nature of various graph structure families. We present explicit labeling constructions to show that the eight-sprocket graph SC_n, octopus graph O_n, flower pot graph FP_n, triangular book graph $B(3,n)$, Kayak paddle graphs $KP(3,4,n)$, armed crown graph AC_n and the corona product of the path graph P_n and a complement of a complete graph $\overline{K_m}$ (i.e.,$P_n \odot \overline{K_m}$) admits an absolute mean graceful labeling.*

Keywords *- Labeling, Graceful labeling, Absolute mean graceful labeling.*

1. Introduction

Graph labeling is a significant area of graph theory, with elegant labeling being one of its most impactful variants. For a graph G with q edges, a graceful labeling is defined as an injective allocation of integers from $\{0, \pm1, \pm2, \dots, \pm q\}$ to the vertices such that the labels induced on the edges, obtained by taking the absolute difference of the vertex labels, form the set $\{1, 2, \dots, q\}$. The concept was first introduced by Rosa [9] in 1967 under the name β-valuation and later popularized by Golomb as graceful labeling. Then, after, graceful labeling has inspired wide-ranging research, moving to numerous generalizations and applications in communication networks, coding theory, and combinatorial optimization. Proceeding with this, Kaneria and Chudasama introduced the notion of Absolute Mean Graceful Labeling (AMGL), which enhances the classical framework by incorporating mean-based conditions into the assignment of labels. In AMGL, the edge labels derived from vertex differences are obtained by an absolute mean-value mechanism to achieve the complete set $\{1, 2, \dots, q\}$, hence extending the applicability of graceful labeling to new classes of graphs. Kaneria and Chudasama proved that some standard graphs are absolutely mean graceful graphs, and further investigated such graphs in the context of duplication of graph elements. Kaneria et. al. [3] proved absolute mean graceful graphs in the context of the path union of graphs. Akbari et al. [1] investigated that jewel and jellyfish-related graphs are absolutely mean graceful, while the same authors in [2] examined absolute mean graceful graphs in the context of barycentric subdivision. In this paper, we discuss the absolute mean graceful labeling of various graph structures. For all undefined terminologies, readers may refer to [4, 6, 7].

We consider a simple, connected, and undirected graph $G = (V, E)$ with p vertices and q edges. For all terminology and notation, we follow [6, 8].

Definition 1.1. A function f is called a *graceful labeling* for a graph G, if $f: V(G) \to \{0, 1, \dots, q\}$ is injective and the induced function $f^*: E(G) \to \{1, 2, \dots, q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = uv \in E(G)$. A graph G is called a graceful graph if it admits a graceful labeling.

Definition 1.2. A function f is called an absolute mean graceful labeling of a graph $G = (V(G), E(G))$, if $f: V(G) \to \{0, \pm1, \pm2, \dots, \pm q\}$ is injective and the induced function $f^*: E(G) \to \{1, 2, \dots, q\}$ defined as,
$f^*(e) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil$ is bijective, for every edge $e = uv \in E(G)$. A graph that admits an absolute mean graceful labeling is called an absolute mean graceful graph.

2. Main Results

Theorem 2.1. The eight-sprocket graph SC_n is an absolute mean graceful graph.
Proof. Let $G = SC_n$ be any eight-sprocket graph.

Let $V(G) = \{v_{i,j} \,/\, 1 \le i, j \le 2n\}$ and $E(G) = \{v_{i,1}v_{i+1,1} \,/\, 1 \le i \le 8, v_{8,1} = v_{1,1}\} \cup \{v_{1,i}v_{1,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{2,i}\, v_{2,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{3,i}v_{3,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{4,i}v_{4,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{5,i}v_{5,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{6,i}v_{6,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{7,i}v_{7,i+1} \,/\, 1 \le i \le 2n-1\} \cup \{v_{8,i}v_{8,i+1} \,/\, 1 \le i \le 2n-1\}$.

The vertex labeling function $f: V(G) \to \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.
$f(v_{i,j}) = (-1)^{j-1}(18n - 2ni - j + 1), \forall i = 1, 3 \;\&\; \forall j = 1, 2, \dots, n+1$
$f(v_{i,n+2}) = (-1)^{n+1}(17n - 2ni - 2), \forall i = 1, 3$
$f(v_{i,j}) = (-1)^{j-1}(18n - 2ni - j), \forall i = 1, 3 \;\&\; \forall j = n+3, \dots, 2n$
$f(v_{i,j}) = (-1)^{j-1}(18n - 2ni - j - 1), \forall i = 5, 7 \;\&\; \forall j = 1, 2, \dots, n$
$f(v_{5,n+1}) = (-1)^{n}(7n - 2),$
$f(v_{7,n+1}) = (-1)^{n}(3n - 3),$
$f(v_{i,j}) = (-1)^{j-1}(18n - 2ni - j - 2), \forall i = 5, 7 \;\&\; \forall j = n+2, \dots, 2\,n - 3$
$f(v_{5,j}) = (-1)^{j-1}(8n - j - 2), \forall j = 2n-2, 2n-1, 2n$
$f(v_{7,2n-2}) = -(2n+1), f(v_{7,2n-1}) = 2n - 2, f(v_{7,2n}) = f(v_{8,1}) = -2n,$
$f(v_{i,j}) = (-1)^{j}(18n - 2ni - j + 1), \forall i = 2, 4 \;\&\; \forall j = 1, 2, \dots, n+1$
$f(v_{2,n+2}) = (-1)^{n+2}(13n - 2), f(v_{2,j}) = (-1)^{j}(14n - j), \forall j = n+3, \dots, 2n$
$f(v_{4,n+2}) = (9n - 4), f(v_{4,n+3}) = (-9n + 3), f(v_{4,n+4}) = (9n - 5),$
$f(v_{4,j}) = (-1)^{j}(10n - j - 1), \forall j = n+5, , \dots, 2n-1$
$f(v_{6,j}) = (-1)^{j}(6n - j - 1), \forall j = 1, 2, \dots, n+1$
$f(v_{6,n+2}) = (-1)^{n+2}(5n - 4),$
$f(v_{6,j}) = (-1)^{j}(6n - j - 2), \forall j = n+3, \dots, 2n$
$f(v_{8,2}) = (2n - 4), f(v_{8,3}) = (3 - 2n), f(v_{8,4}) = (2n - 5),$
$f(v_{8,j}) = (-1)^{j}(2n - j - 1), \forall j = 5, \dots, 2n-1$

The labeling function f defined as above is one-one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.
Therefore, the eight-sprocket graph SC_n is an absolute mean graceful graph.

Illustration 2.2. Absolute Mean graceful labeling SC_6 is shown in the following Figure 1.

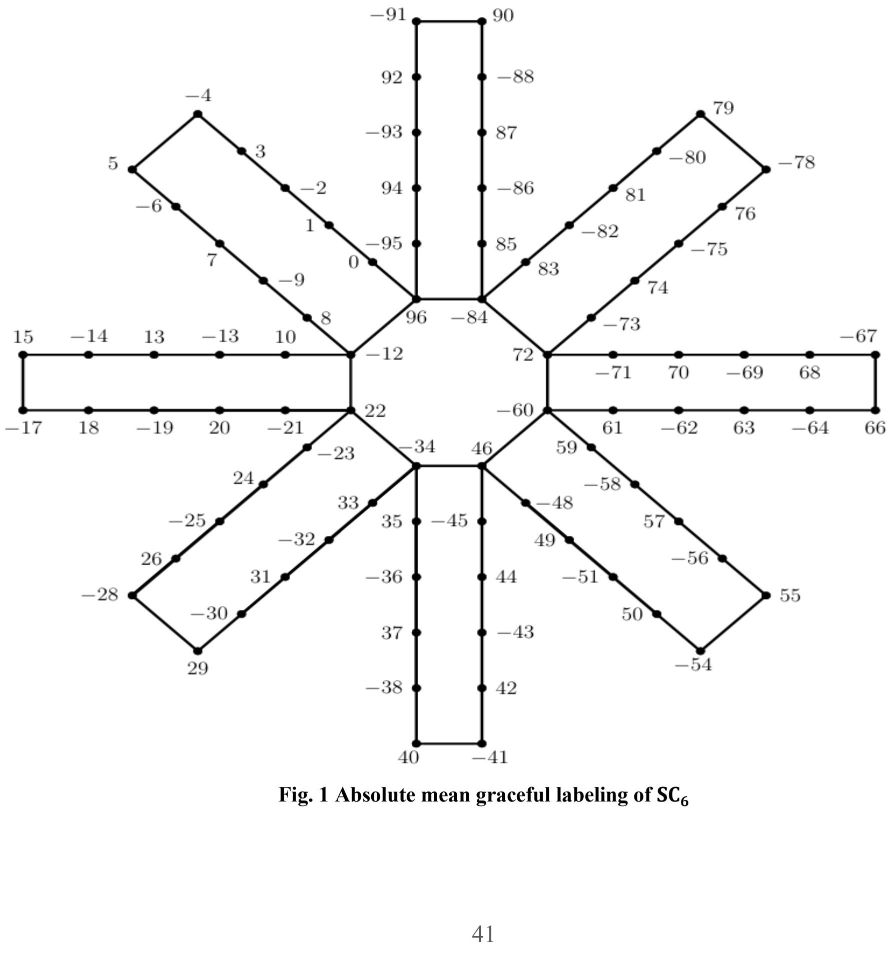

Fig. 1 Absolute mean graceful labeling of SC_6

Theorem 2.3. The octopus graph O_n is an absolute mean graceful graph.
Proof. Let $G = O_n$ be any octopus graph.
Let $V(G) = \{u_i, v_i / 1 \le i \le n\} \cup \{v\}$ and $E(G) = \{u_i v, u_{i-1}u_i, v_i v / 1 \le i \le n\}$.
The vertex labeling function $f: V(G) \to \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.
$f(v_0) = q$

$\boldsymbol{Case-1: n \equiv 0 (mod 2)}$
$f(u_i) = i - q, \forall i = 1,3, \dots, n-1$
$f(u_i) = 2n - i + 2, \forall i = 2, 4, \dots, n$
$f(v_i) = 2i - 3, \forall i = 1,2, \dots, \frac{n+2}{2}$
$f(v_i) = 2i + n - 3, \forall i = \frac{n+4}{2}, \dots, n$

$\boldsymbol{Case-2: n \equiv 1 (mod 2)}$
$f(u_i) = i - q, \forall i = 1,3, \dots, n$
$f(u_i) = 2n - i + 1, \forall i = 2, 4, \dots, n-1$
$f(v_i) = 2i - 2, \forall i = 1,2, \dots, \frac{n+1}{2}$
$f(v_i) = 2i + n - 3, \forall i = \frac{n+3}{2}, \dots, n$

The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.
Therefore, the octopus graph O_n is an absolute mean graceful graph.

Illustration 2.4. Absolute mean graceful labeling for O_8 is shown in the following Figure 2.

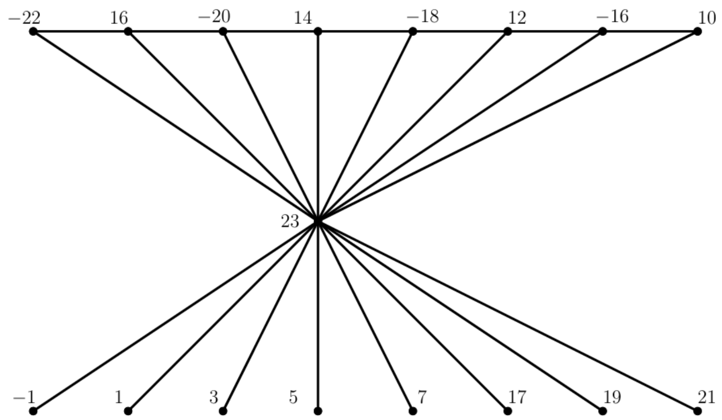

Fig. 2 Absolute mean garceful labeling of O_8

Theorem 2.5. The Flowerpot graph FP_n is an absolute mean graceful graph.
Proof. Let $G = FP_n$ be any flowerpot graph.
Let V(G) = $\{u_i, v_i / 1 \le i \le n, v_{n+1} = v_1\}$ and E(G) = $\{v_i u_i, v_{i-1} v_i / 1 \le i \le n\}$.
To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., $\pm\left\lceil\frac{q}{2}\right\rceil$}, we take the following cases.
Case-1: $n \equiv 0 (mod 2)$
$f(v_n) = -4,$
$f(v_i) = (-1)^{i+1}(q - i + 1), 1 \le i \le n-1$
$f(u_i) = (2i - 3), 1 \le i \le \frac{n-4}{2}$
$f(u_i) = (2i - 1), \frac{n-2}{2} \le i \le n$
Case-2: $n \equiv 1 (mod 2)$
$f(v_1) = q,$
$f(v_i) = (-1)^{i+1}(q - i + 1), 1 \le i \le n$

$f(u_i) = (2i - 3), 1 \leq i \leq \frac{n+3}{2}$

$f(u_i) = (2i - 1), \frac{n+5}{2} \leq i \leq n - 1$

The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.
Therefore, the flowerpot graph FP_n is an absolute mean graceful graph.

Illustration 2.6. Absolute mean graceful labeling for FP_7 is shown in the following Figure 3.

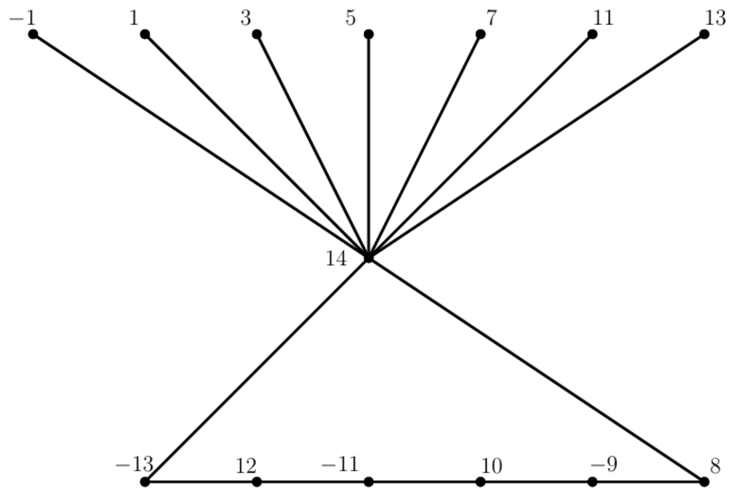

Fig. 3 Absolute mean graceful labeling of FP_7

Theorem 2.7. The triangular book graph $B(3, n)$ is an absolute mean graceful graph.
Proof. Let $G = B(3, n)$ be any triangular book graph.
Let $V(G) = \{v_i / 1 \leq i \leq n\} \cup \{x, y\}$ and $E(G) = \{xv_i, yv_i / 1 \leq i \leq n\} \cup \{xy\}$.
The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\}$ defined as follows.
$f(x) = q$,
$f(y) = q - 2$,
$f(v_i) = 4i - q - 3$
The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.

Therefore, the triangular book graph $B(3, n)$ is an absolute mean graceful graph.

Illustration 2.8. Absolute mean graceful labeling for $B(3,5)$ is shown in the following Figure 4.

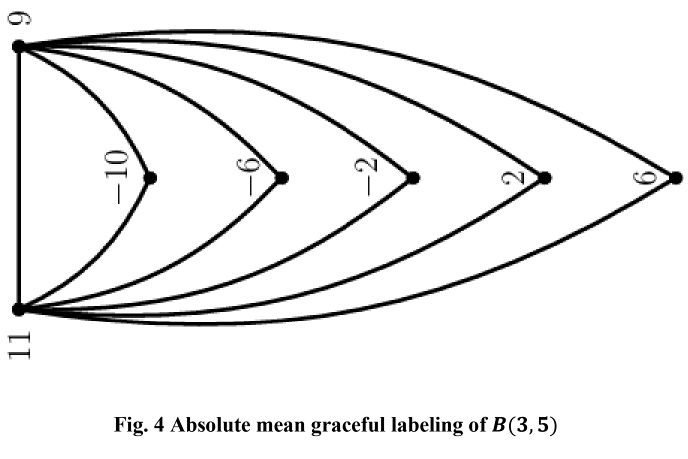

Fig. 4 Absolute mean graceful labeling of $B(3, 5)$

Theorem 2.9. The kayak paddle graph $KP(3,4,n)$ is an absolute mean graceful graph.
Proof. Let $= KP(3,4,n)$ be any kayak paddle graph.
Let V(G) = $\{u_i \ / \ 1 \le i \le n+1\} \cup \{x, y, w, u, v\}$ and
E(G) = $\{xy, yw, uv, xu_1, wu_1, u_{n+1}u, u_{n+1}v\} \cup \{u_i u_{i+1} \ / \ 1 \le i \le n\}$.
To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., ±$\left\lceil\frac{q}{2}\right\rceil$}, we take the following cases.

Case-1: $n \equiv 0(mod2)$
$f(u) = 1, f(v) = 3,$
$f(x) = q - 13, f(y) = q - 3,$
$f(w) = q - 11,$
$f(u_i) = q - i + 1, if\ i = 1,3,5,\dots,n+1$
$f(u_i) = -q + i - 2, if\ i = 2,4,\dots,n$

Case-2: $n \equiv 1(mod2)$
$f(u) = -13, f(v) = -11,$
$f(x) = q - 13, f(y) = q - 3,$
$f(w) = q - 11,$
$f(u_i) = q - i + 1, if\ i = 1,3,5,\dots,n$
$f(u_i) = -q + i - 2, if\ i = 2,4,\dots,n+1$

The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.
Therefore, the kayak paddle graph $KP(3,4,n)$ is an absolute mean graceful graph.

Illustration 2.10. Absolute mean graceful labeling for $KP(3,4,5)$ is shown in the following Figure 5.

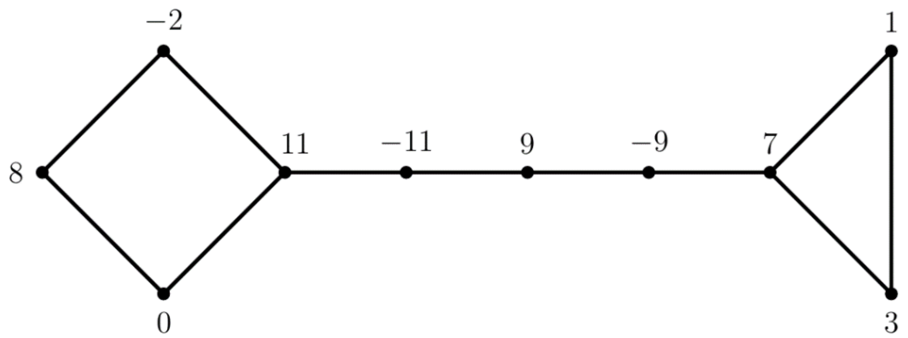

Fig. 5 Absolute mean graceful labeling of $KP(3,4,5)$

Theorem 2.11. The armed crown graph AC_n is an absolute, mean graceful graph.
Proof. Let $= AC_n$ be any armed crown graph.
Let V(G) = $\{u_i, v_i, w_i \ / \ 1 \le i \le n\}$ and E(G) = $\{u_i u_{i+1}, v_i u_i, w_i v_i \ / \ 1 \le i \le n, u_{n+1} = u_1\}$.
To obtain vertex labeling function f: V(G)→{0,±2,±4, ..., ±$\left\lceil\frac{q}{2}\right\rceil$}, we take the following cases.

Case-1: $n \equiv 1(mod2)$
$f(u_i) = (-1)^{i+1}(q - i + 1)\ , 1 \le i \le n - 1,$
$f(u_n) = -(n + 4),$
$f(v_i) = (-1)^{i}(n - i + 3), 1 \le i \le n - 1$
$f(v_n) = -(2n + 1),$
$f(w_i) = (-1)^{i+1}(n - i + 3), 1 \le i \le \left\lceil\frac{n}{2}\right\rceil + 2$
$f(w_i) = (-1)^{i+1}\left(2\left\lceil\frac{n}{2}\right\rceil - i - 2\right), \left\lceil\frac{n}{2}\right\rceil + 3 \le i \le n - 1$
$f(w_n) = -(2n - 1)$

Case-2: $n \equiv 0(mod2)$
$f(u_i) = (-1)^{i+1}(q - i + 1)\ , 1 \le i \le n - 1,$
$f(u_n) = -(n + 4),$

$f(v_i) = (-1)^i(n - i + 3), 1 \le i \le \left(\frac{n}{2} - 2\right)$
$f(v_i) = (-1)^i(n + 1 - i), \left(\frac{n}{2} - 1\right) \le i \le n$
$f(w_i) = (-1)^{i+1}(n - i - 2), 1 \le i \le n$

The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.
Therefore, the armed crown graph AC_n is an absolute mean graceful graph.

Illustration 2.12. Absolute mean graceful labeling for AC_{10} is shown in the following Figure 6.

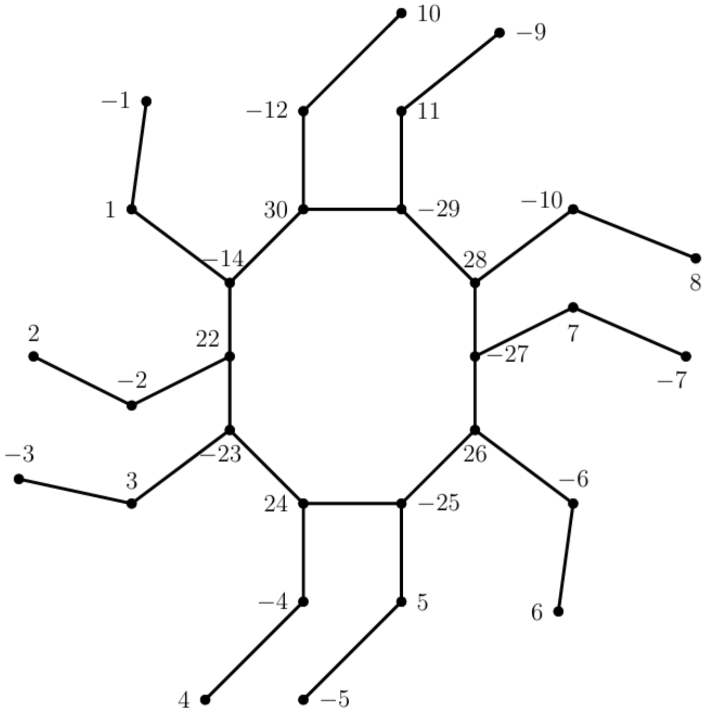

Fig. 6 Absolute mean graceful labeling of AC_{10}

Theorem 2.13. The graph $P_n \odot \overline{K_m}$ Corona product of path P_n and a complement of a complete graph $\overline{K_m}$ is an absolute mean graceful graph.
Proof. Let $G = P_n \odot \overline{K_m}$ be a corona product of any path graph P_n and a complement of a complete graph $\overline{K_m}$ graph.
Let V(P_n) = $\{u_i \ / \ 1 \le i \le n\}$ and E(P_n) = $\{u_i u_{i+1} \ / \ 1 \le i \le n - 1\}$.
Therefore, V(G) = $\{ u_i, u_{i,j} \ / \ 1 \le i \le n, 1 \le j \le m\}$ and
E(G) = $\{u_i u_{i+1} \ / \ 1 \le i \le n - 1\} \cup \{u_i u_{i,j} \ / \ 1 \le i \le n, 1 \le j \le m\}$
To obtain vertex labeling function f: V(G)→{0,±1,±2, ..., ±q}, we take following cases.
$f(u_i) = (-1)^{n+i}(q - n + i), 1 \le i \le n$

Case-1: $n \equiv 1(mod 2)$

$$f(u_{i,j}) = \begin{cases} u_1 + 2j & ; if\ j = 1,2,\dots,m\ and\ i = 1 \\ (-1)^i(|u_{i-1,m}| - 1 + 2j) & ; if\ j = 1,2,\dots,m\ and\ i = 2,3,\dots,\left\lceil\frac{n+1}{2}\right\rceil \\ (-1)^{i+1}(|u_{i-1,m}| - 1 + 2j) & ; if\ j = 1,2,\dots,m\ and\ i = \left\lceil\frac{n+1}{2}\right\rceil + 1,\dots,n \end{cases}$$

Case-2: $n \equiv 0(mod 2)$

$$f(u_{i,j}) = \begin{cases} u_1 - 2j & ; if\ j = 1,2,\dots,m\ and\ i = 1 \\ (-1)^{i+1}(|u_{i-1,m}| + 1 - 2j) & ; if\ j = 1,2,\dots,m\ and\ i = 2,3,\dots,\left\lceil\frac{n+1}{2}\right\rceil \\ (-1)^i(|u_{i-1,m}| - 1 + 2j) & ; if\ j = 1,2,\dots,m\ and\ i = \left\lceil\frac{n+1}{2}\right\rceil + 1,\dots,n \end{cases}$$

The labeling function f defined as above is one- one, as there are no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective. Thus, f is an absolute mean graceful labeling for the given graph.

Therefore, the graph $P_n \odot \overline{K_m}$ is an absolute mean graceful graph.

Illustration 2.14. Absolute mean graceful labeling for $P_6 \odot \overline{K_4}$ is shown in the following Figure 7.

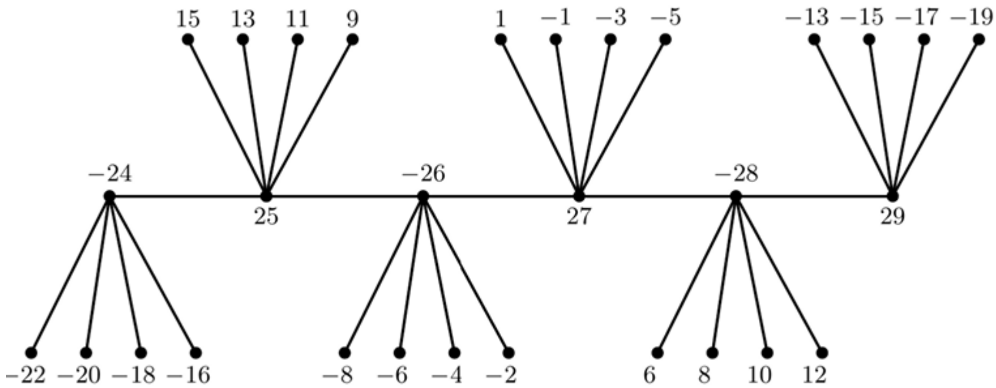

Fig. 7 Absolute mean graceful labeling of $P_6 \odot \overline{K_4}$

Corollary 2.15. The comb graph $P_n \odot K_1$ Corona product of path P_n and a complete graph K_1 is an absolute mean graceful graph.
Proof. Let $G = P_n \odot K_1$ be any comb graph.
In the above theorem, if we take m=1 in the corona product of the path P_n with the compliment of a complete graph $\overline{K_4}$, then we obtain the comb graph $P_n \odot K_1$.

Hence, the proof follows directly from the above theorem.

3. Conclusion

In this work, we establish that several graph structures can admit absolute mean graceful labeling. For different operations, we determine whether the resulting graph admits such a labeling, thereby extending the scope of mean labeling theory to new structural contexts.

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