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Original Article

A Comparative Study of Max-Min, Max-Product and Max-Average Compositions in Reflexive Fuzzy Relation

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Abstract *- This paper presents a comparative study of three important composition operators in fuzzy relations, such as max-min, max-product, and max-average compositions. The study demonstrates that these operators hold for a reflexive fuzzy relation. Furthermore, an illustrative example is provided for each type of composition, using matrices of order* 3×3.

Keywords *- Composition of fuzzy relations, Fuzzy relations, Fuzzy sets, Reflexive, Order of matrix.*

1. Introduction

Many real-world systems are inherently ambiguous and uncertain, where binary or crisp logic is unable to adequately express information. To overcome these limitations, Zadeh [3] introduced the concept of fuzzy set theory in 1965, which has since become a cornerstone in the modeling of imprecise and ambiguous data. Building upon this foundation, fuzzy relations extend the classical notion of relations by allowing degrees of association between elements, making them a powerful tool in areas such as decision-making [4], control systems [5], pattern recognition [6], and artificial intelligence.

A fuzzy relation represents a mapping that captures the strength or degree of relationship between elements of two or more fuzzy sets. Among the various operations defined on fuzzy relations, the binary composition plays a pivotal role in modeling multi-stage or interconnected systems. Fang-Fang et al. [7] studied the post-inverse of fuzzy relation matrices using addition-min composition. They derived conditions for uniqueness and proposed an algorithm by converting the problem into a system of linear equations.

The diversity of applications and operator characteristics, a comparative study of composition operators, is essential to understand their advantages, limitations, and suitability for various problem domains. This paper aims to analyze and compare the performance of selected composition operators in a reflexive fuzzy relation. The study examines their mathematical formulations, computational complexity, and logical behavior, supported by examples.

2. Preliminaries

First, some definitions and preliminary concepts related to fuzzy sets, fuzzy relations, and composition of fuzzy relations are discussed.

Definition 2.1. [1] If U is a universe of discourse and $u$is any particular element of U, then a fuzzy set $\tilde{A}$ defined on U may be written as a collection of ordered pairs $\tilde{A} = \{(u, \mu_{\tilde{A}}(u)) : u \in U\}$ where, each pair $(u, \mu_{\tilde{A}}(u))$ is called a singleton.

Definition 2.2. [2] Let $U$and Vbe nonempty sets. Then$\tilde{R} = \{((u, v), \mu_{\tilde{R}}(u, v)) : (u, v) \in U \times V\}$it is called a fuzzy relation in $U \times V$.

Definition 2.3. [2] Let $U, V$and Wbe nonempty sets, $\tilde{A}$ and be a fuzzy relation in $U \times V$, and $\tilde{B}$be a fuzzy relation in $V \times W$. Then$\tilde{A} \circ \tilde{B} = \{(u, w), \max_{v \in V}\{\min\{\mu_{\tilde{A}}(u, v), \mu_{\tilde{B}}(v, w)\} : u \in U, \quad w \in W\}$ is called the max-min Composition of $\tilde{A}$ and $\tilde{B}$.

Definition 2.4. [1] Let $\tilde{A}$and$\tilde{B}$ be two fuzzy relations on (U, V) and (V, W) respectively. Then, the max-prod composition is denoted as $\tilde{A} o \tilde{B}$ and is defined as $\tilde{A} o \tilde{B} = \{[(u, w), \max\{\mu_{\tilde{A}}(u, v)\mu_{\tilde{B}}(v, w)\}]\}, \forall u \in U, v \in V, w \in W$.

Definition 2.5. [1] Let $\tilde{A}$and$\tilde{B}$ be two fuzzy relations on (U,V) and (V,W) respectively. Then, the max-av composition is denoted as $\tilde{A} \underset{av}{o} \tilde{B}$ and is defined as $\tilde{A} \underset{av}{o} \tilde{B} = \{[(u,w), \frac{1}{2}\max\{\mu_{\tilde{A}}(u,v) + \mu_{\tilde{B}}(v,w)\}]\}, \forall u \in U, v \in V, w \in W.$

Definition 2.6. [8] Let $\tilde{R}$ be a fuzzy relation in $U \times U$. Then $\tilde{R}$ is said to be reflexive if

$$\mu_{\tilde{R}}(u,u) = 1 \forall u \in U.$$

Definition 2.7. [8] A fuzzy relation $\tilde{R}$ is called symmetric if

$$\tilde{R}(u,v) = \tilde{R}(v,u) \forall u, v \in U.$$

Definition 2.8. [8] A fuzzy relation $\tilde{R}$ is called (max-min) transitive if

$$\tilde{R} o \tilde{R} \subseteq \tilde{R}$$

3. Comparative Analysis of Reflexivity in Different Composition Operators

Now, reflexivity under max-min, max-product, and max-average compositions will be discussed.

3.1. Reflexivity Under Each Composition Operator

Let$\tilde{R}$ be a fuzzy relation on $U \times U$. Reflexivity is investigated when $\tilde{R}$ is composed with itself using each operator.

3.1.1. Max-Min Composition

Proof: Assume that $\tilde{R}$ is reflexive, i.e., $\mu_{\tilde{R}}(u,u) = 1$ for all$u \in U$.
Consider the composition $\tilde{R} o \tilde{R}$.
Then,

$$\mu_{\tilde{R}o\tilde{R}}(u,u) = \max_{v \in U} min(\mu_{\tilde{R}}(u,v), \mu_{\tilde{R}}(v,u))$$

For$v = u$, This gives

$$min(\mu_{\tilde{R}}(u,u), \mu_{\tilde{R}}(u,u)) = min(1,1) = 1$$

Therefore,

$$\mu_{\tilde{R}o\tilde{R}}(u,u) \geq 1$$

Since membership values cannot exceed 1, it follows that

$$\mu_{\tilde{R}o\tilde{R}}(u,u) = 1$$

Hence,$\tilde{R}o\tilde{R}$ is also reflexive.

3.1.2. Max-Product Composition

Proof: Let$\tilde{R}$ be reflexive, so that $\mu_{\tilde{R}}(u,u) = 1$for all$u \in U$.
Then,

$$\mu_{\tilde{R}\,o\,\tilde{R}}(u,u) = \max_{v \in U}(\mu_{\tilde{R}}(u,v).\mu_{\tilde{R}}(v,u))$$

For$v = u$, This gives

$$\mu_{\tilde{R}}(u,u).\mu_{\tilde{R}}(u,u) = 1.1 = 1$$

Hence,

$$\mu_{\tilde{R}\,o\,\tilde{R}}(u,u) \geq 1$$

Which implies

$$\mu_{\tilde{R}o\tilde{R}}(u,u) = 1$$

Thus,$\tilde{R}\,o\,\tilde{R}$ is reflexive.

3.1.3. Max-Average Composition

Proof: Let $\tilde{R}$ be reflexive, i.e., $\mu_{\tilde{R}}(u,u) = 1$ for all$u \in U$.
Then,

$$\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u,u) = \max_{v \in U}\left(\frac{\mu_{\tilde{R}}(u,v) + \mu_{\tilde{R}}(v,u)}{2}\right)$$

For$v = u$, This gives

$$\frac{\mu_{\tilde{R}}(u,u)+\mu_{\tilde{R}}(u,u)}{2} = \frac{1+1}{2} = 1$$

Therefore,

$$\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u,u) \geq 1$$

and hence $\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u,u) = 1$

Thus, $\tilde{R} \underset{av}{o} \tilde{R}$ is also reflexive.

3.2. Example

To illustrate the reflexive behaviour of the three composition operators, i.e., max-min, max-product, and max-average, consider the fuzzy relation $\tilde{R}$ on the universe $U = \{u_1, u_2, u_3\}$, represented by the membership matrix:

$$\tilde{R} = \begin{bmatrix} 1.0 & 0.7 & 0.5 \\ 0.6 & 1.0 & 0.8 \\ 0.4 & 0.9 & 1.0 \end{bmatrix}$$

It is clear that $\mu_{\tilde{R}}(u_i, u_i) = 1$ for all, $i = 1,2,3$, so $\tilde{R}$ is reflexive.
Next, $\tilde{R} o \tilde{R}$ is computed using the three composition operators and verifies whether the diagonal elements remain 1.

3.2.1. Max-Min Composition

$$\mu_{\tilde{R} o \tilde{R}}(u_i, u_j) = \max_{k \in \{1,2,3\}} min(\mu_{\tilde{R}}(u_i, u_k), \mu_{\tilde{R}}(u_k, u_j))$$

Example Calculation:

$$\mu_{\tilde{R} o \tilde{R}}(u_1, u_1) = max\{min(1,1), min(0.7,0.6), min(0.5,0.4)\} = max\{1,0.6.0.4\} = 1$$

Similarly, for u_2 and u_3:

$$\mu_{\tilde{R} o \tilde{R}}(u_2, u_2) = 1, \quad \mu_{\tilde{R} o \tilde{R}}(u_3, u_3) = 1$$

All diagonal entries remain 1. Reflexivity is preserved under max-min.

3.2.2. Max-Product Composition

$$\mu_{\tilde{R} o \tilde{R}}(u_i, u_j) = \max_{k \in \{1,2,3\}} (\mu_{\tilde{R}}(u_i, u_k). \mu_{\tilde{R}}(u_k, u_j))$$

Example Calculation:

$$\mu_{\tilde{R} o \tilde{R}}(u_1, u_1) = max\{1.0 \times 1.0, 0.7 \times 0.6, 0.5 \times 0.4\} = max\{1,0.42.0.20\} = 1.0$$

Similarly,

$$\mu_{\tilde{R} o \tilde{R}}(u_2, u_2) = 1.0, \quad \mu_{\tilde{R} o \tilde{R}}(u_3, u_3) = 1.0$$

Thus, reflexivity is preserved under max-product composition.

3.2.3. Max-Average Composition

$$\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u_i, u_j) = \max_{k \in \{1,2,3\}} \left(\frac{\mu_{\tilde{R}}(u_i, u_k) + \mu_{\tilde{R}}(u_k, u_j)}{2}\right)$$

Example Calculation:

$$\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u_1, u_1) = max\left\{\frac{1.0+1.0}{2}, \frac{0.7+0.6}{2}, \frac{0.5+0.4}{2}\right\} = max\{1,0.65.0.45\} = 1.0$$

Similarly,

$$\mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u_2, u_2) = 1.0, \quad \mu_{\tilde{R} \underset{av}{o} \tilde{R}}(u_3, u_3) = 1.0$$

Hence, reflexivity is preserved under max-average composition.

4. Conclusion

This paper presented a comparative study of three important composition operators in fuzzy relations, such as the max-min, max-product, and max-average compositions. Each operator was examined in terms of its fundamental relational properties, namely reflexivity. The analytical results were supported by illustrative examples using fuzzy relations of $3 \times 3$the order of the matrix.

Table 1. The comparison of the properties is summarized as follows

Property	Max-Min	Max-Product	Max-Average
Reflexive	Yes	Yes	Yes

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