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Original Article

A Novel Pythagorean Neutrosophic Cubic Correlation Measure for Multi-Criteria Group Decision Making: An Approach to Consumer Electronic Selection

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Abstract *- The Pythagorean Neutrosophic Cubic Sets (PNCS) are a hybrid approach that is of the Interval Valued Pythagorean Neutrosophic Set (IVPNS) and the Pythagorean Neutrosophic Set (PNS). To induce the Multi-Criteria Decision-Making (MCDM) in high unreliability, a unique PNCS Correlation measure* $\xi^{\cdot}(H_{PNCS}, Z_{PNCS})$ *has been introduced. Robust and adaptable decision-making methods are required due to the rising growth of mobile devices, more complicated feature sets, and subjective user preferences. The conventional Multi-Criteria Decision-Making (MCDM) methodologies often fail to adequately reflect the inherent unpredictability, indefiniteness, and overlapping perceptions encountered in real-world evaluation procedures. To bridge this gap, this research proposes an experimental integrated MCDM mechanism based on PNCS. The findings demonstrate that this sophisticated correlation-based method offers a more solid and trustworthy ranking, providing a potent instrument for collective decision-making in the consumer electronics industry and other extremely unpredictable fields.*

Keywords *- Correlation measure, Multi-Criteria Decision-Making, Pythagorean Neutrosophic Cubic set.*

1. Introduction

Zadeh[11] coined the term Fuzzy Sets (FSs) in 1965. Zadeh [12] implemented another variant of FSs: FSs with interval-valued membership functions. The Neutrosophic Set (NS) is an extension of FSs that takes falsity, indeterminacy, and truth into account in 1995 by Florentin Smarandache[7]. In 2012, Jun et al. formulated the revolutionary framework for Cubic Sets (CS) [4]. Neutrosophic Cubic Sets (NCS) are a new term that Chang Su Kim et el. [5] implemented in 2017. Yagar[9] put forward the developmental process of the Pythagorean Fuzzy Set (PFS). In 2019, the Pythagorean Cubic Fuzzy Sets (PCFs) phrase was proposed by F. Khana. et.el., [6]. PNS were successfully presented by R. Jhansi and K. Mohana [3]. Stephy et al. spoke about the interval-valued Neutrosophic Pythagorean Sets[8]. Berna Joyce and Elvina Mary[1] brought up an intriguing new idea for Pythagorean Neutrosophic Cubic Sets (PNCS). In 2018, Jun Ye[10] presented a MADM method for Neutrosophic cubic numbers. Hanafy et al. [2] suggested the correlation coefficients of NS and investigated their features. [3] R. Jhansi & K. Mohana discussed the correlation measure of PNS. The primary limitation of existing Pythagorean Neutrosophic Sets is their inability to capture the dual-layered internal and external uncertainty inherent in real-world complex data. While Neutrosophic Cubic Sets offer a framework for interval and single-valued components, a dedicated correlation measure for their Pythagorean extension has been missing. This research bridges the gap by introducing a novel PNCS Correlation Measure, $\xi_{1^{\cdot}}(\mathrm{H}_{PNCS}, \mathrm{Z}_{PNCS})$ and $\xi_{2^{\cdot}}(\mathrm{H}_{PNCS}, \mathrm{Z}_{PNCS})$ specifically designed to handle dual-layered information with higher precision. In this paper, the Correlation measure of PNCS is discussed, and some of its properties are proved. Then, using the correlation measure of PNCS, an application based on the MADM problem has been introduced and discussed.

2. Preliminaries

Definition 2.1[1] Let $\hbar \neq \varphi$ a PNCS, having a form, $\widehat{H}_{PNCS} = \{\hat{h}, T_{PNIVS}(\hat{h}), \varsigma_{PNS}(\hat{h}): \hat{h}: \hbar\} \rightarrow (1)$ where $T_{PNIVS}(\hat{h})$ represents the PNIVS and $\varsigma_{PNS}(\hat{h})$ represents the PNS. PNCS can be denoted as a pair $\widehat{H}_{PNCS} = (T_{PNIVS}(\hat{h}), \varsigma_{PNS}(\hat{h}))$.

Definition 2.1[3] Let $\hbar \neq \varphi$. Let $\widehat{\mathrm{H}}_{PNS}$ and $\hat{\mathrm{Z}}_{PNS}$ be two PNSs. Then the CC of $\widehat{H}_{PNS}$ and $\hat{\mathrm{Z}}_{PNS}$ is

$$\rho(\widehat{H}_{PNS}, \hat{Z}_{PNS}) = \frac{C_{PNS}(\widehat{H}_{PNS}, \hat{Z}_{PNS})}{\sqrt{C_{PNS}(\widehat{H}_{PNS}, \widehat{H}_{PNS}) . C_{PNS}(\hat{Z}_{PNS}, \hat{Z}_{PNS})}} \rightarrow \quad (2)$$

3. Correlation Measure for Pythagorean Neutrosophic Cubic Sets:

Definition 3.1 Let $\hbar \neq \varphi$ A PNCS $\widehat{H}_{PNCS} = \{\hat{h}, T_{PNIVS}(\hat{h}), \varsigma_{PNS}(\hat{h}) : \hat{h} : \hbar\}$ and $\widehat{Z}_{PNCS} = \{\hat{h}, Z_{PNIVS}(\hat{h}), \upsilon_{PNS}(\hat{h}) : \hat{h} : \hbar\}$. Then the Correlation Measure (CRM) of $\widehat{H}_{PNCS}$ and $\widehat{Z}_{PNCS}$

$$\boldsymbol{\xi_1(\widehat{H}_{PNCS}, \widehat{Z}_{PNCS}) = \frac{T_{crm}(\widehat{H}_{PNCS}, \widehat{Z}_{PNCS})}{\sqrt{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}) \cdot T_{crm}(\widehat{Z}_{PNCS}, \widehat{Z}_{PNCS})}} \rightarrow (3)}$$

where

$$T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \sum_{i=1}^{n} \begin{pmatrix} (T_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + \\ (F_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (T_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (I_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_T}(\hat{h}_i))^2.(\nu_{PNS_T}(\hat{h}_i))^2 + (\varsigma_{PNS_I}(\hat{h}_i))^2.(\nu_{PNS_I}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_F}(\hat{h}_i))^2.(\nu_{PNS_F}(\hat{h}_i))^2 \end{pmatrix}$$

$$T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}) = \sum_{i=1}^{n} \begin{pmatrix} (T_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(T_{PNIVS_H}{}^{-}(\hat{h}_i))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(I_{PNIVS_H}{}^{-}(\hat{h}_i))^2 + \\ (F_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(F_{PNIVS_H}{}^{-}(\hat{h}_i))^2 + (T_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(T_{PNIVS_H}{}^{+}(\hat{h}_i))^2 + \\ (I_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(I_{PNIVS_H}{}^{+}(\hat{h}_i))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(F_{PNIVS_H}{}^{+}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_T}(\hat{h}_i))^2.(\varsigma_{PNS_T}(\hat{h}_i))^2 + (\varsigma_{PNS_I}(\hat{h}_i))^2.(\varsigma_{PNS_I}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_F}(\hat{h}_i))^2.(\varsigma_{PNS_F}(\hat{h}_i))^2 \end{pmatrix}$$

and

$$T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS}) = \sum_{i=1}^{n} \begin{pmatrix} (T_{PNIVS_Z}{}^{-}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (I_{PNIVS_Z}{}^{-}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + \\ (F_{PNIVS_Z}{}^{-}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (T_{PNIVS_Z}{}^{+}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (I_{PNIVS_Z}{}^{+}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + (F_{PNIVS_Z}{}^{+}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (\nu_{PNS_T}(\hat{h}_i))^2.(\nu_{PNS_T}(\hat{h}_i))^2 + (\nu_{PNS_I}(\hat{h}_i))^2.(\nu_{PNS_I}(\hat{h}_i))^2 + \\ (\nu_{PNS_F}(\hat{h}_i))^2.(\nu_{PNS_F}(\hat{h}_i))^2 \end{pmatrix}$$

Theorem 3.2 *The CRM between PNCS* $\widehat{H}_{PNCS}$ *and* $\widehat{Z}_{PNCS}$ *fulfills the traits preceding*

i. $0 \leq \xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$

ii. $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1 \text{iff } \widehat{H}_{PNCS} = \hat{Z}_{PNCS}$

iii. $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \xi_2(\hat{Z}_{PNCS}, \widehat{H}_{PNCS})$

Proof:

i. To Prove: $0 \leq \xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$.

The inequality $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \geq 0$is forthright; then it is necessary to show that $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$. To prove this, let us consider,

$$T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \sum_{i=1}^{n} \begin{pmatrix} (T_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + \\ (F_{PNIVS_H}{}^{-}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_i))^2 + (T_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (I_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_i))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_T}(\hat{h}_i))^2.(\nu_{PNS_T}(\hat{h}_i))^2 + (\varsigma_{PNS_I}(\hat{h}_i))^2.(\nu_{PNS_I}(\hat{h}_i))^2 + \\ (\varsigma_{PNS_F}(\hat{h}_i))^2.(\nu_{PNS_F}(\hat{h}_i))^2 \end{pmatrix}$$

Now expanding the i terms from 1 to n,

$$\leq \left(\begin{array}{l}
(T_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + \\
(F_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + (T_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + \\
(I_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + \\
(\varsigma_{PNS_T}(\hat{h}_1))^2.(\nu_{PNS_T}(\hat{h}_1))^2 + (\varsigma_{PNS_I}(\hat{h}_1))^2.(\nu_{PNS_I}(\hat{h}_1))^2 + (\varsigma_{PNS_F}(\hat{h}_1))^2.(\nu_{PNS_F}(\hat{h}_1))^2 + \ldots + \\
(T_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + (F_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + \\
(T_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + (I_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + \\
(\varsigma_{PNS_T}(\hat{h}_n))^2.(\nu_{PNS_T}(\hat{h}_n))^2 + (\varsigma_{PNS_I}(\hat{h}_n))^2.(\nu_{PNS_I}(\hat{h}_n))^2 + (\varsigma_{PNS_F}(\hat{h}_n))^2.(\nu_{PNS_F}(\hat{h}_n))^2
\end{array}\right)$$

$$\leq \left(\begin{array}{l}
(T_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(T_{PNIVS_H}{}^{-}(\hat{h}_1))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(I_{PNIVS_H}{}^{-}(\hat{h}_1))^2 + (F_{PNIVS_H}{}^{-}(\hat{h}_1))^2.(F_{PNIVS_H}{}^{-}(\hat{h}_1))^2 + \\
(T_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(T_{PNIVS_H}{}^{+}(\hat{h}_1))^2 + (I_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(I_{PNIVS_H}{}^{+}(\hat{h}_1))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_1))^2.(F_{PNIVS_H}{}^{+}(\hat{h}_1))^2 + \\
(\varsigma_{PNS_T}(\hat{h}_1))^2.(\varsigma_{PNS_T}(\hat{h}_1))^2 + (\varsigma_{PNS_I}(\hat{h}_1))^2.(\varsigma_{PNS_I}(\hat{h}_1))^2 + (\varsigma_{PNS_F}(\hat{h}_1))^2.(\varsigma_{PNS_F}(\hat{h}_1))^2 + \ldots + \\
(T_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(T_{PNIVS_H}{}^{-}(\hat{h}_n))^2 + (I_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(I_{PNIVS_H}{}^{-}(\hat{h}_n))^2 + (F_{PNIVS_H}{}^{-}(\hat{h}_n))^2.(F_{PNIVS_H}{}^{-}(\hat{h}_n))^2 + \\
(T_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(T_{PNIVS_H}{}^{+}(\hat{h}_n))^2 + (I_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(I_{PNIVS_H}{}^{+}(\hat{h}_n))^2 + (F_{PNIVS_H}{}^{+}(\hat{h}_n))^2.(F_{PNIVS_H}{}^{+}(\hat{h}_n))^2 + \\
(\varsigma_{PNS_T}(\hat{h}_n))^2.(\varsigma_{PNS_T}(\hat{h}_n))^2 + (\varsigma_{PNS_I}(\hat{h}_n))^2.(\varsigma_{PNS_I}(\hat{h}_n))^2 + (\varsigma_{PNS_F}(\hat{h}_n))^2.(\varsigma_{PNS_F}(\hat{h}_n))^2 \times \ldots \\
(T_{PNIVS_Z}{}^{-}(\hat{h}_1))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + (I_{PNIVS_Z}{}^{-}(\hat{h}_1))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + (F_{PNIVS_Z}{}^{-}(\hat{h}_1))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_1))^2 + \\
(T_{PNIVS_Z}{}^{+}(\hat{h}_1))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + (I_{PNIVS_Z}{}^{+}(\hat{h}_1))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + (F_{PNIVS_Z}{}^{+}(\hat{h}_1))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_1))^2 + \\
(\nu_{PNS_T}(\hat{h}_1))^2.(\nu_{PNS_T}(\hat{h}_1))^2 + (\nu_{PNS_I}(\hat{h}_1))^2.(\nu_{PNS_I}(\hat{h}_1))^2 + (\nu_{PNS_F}(\hat{h}_1))^2.(\nu_{PNS_F}(\hat{h}_1))^2 + \ldots + \\
(T_{PNIVS_Z}{}^{-}(\hat{h}_n))^2.(T_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + (I_{PNIVS_Z}{}^{-}(\hat{h}_n))^2.(I_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + (F_{PNIVS_Z}{}^{-}(\hat{h}_n))^2.(F_{PNIVS_Z}{}^{-}(\hat{h}_n))^2 + \\
(T_{PNIVS_Z}{}^{+}(\hat{h}_n))^2.(T_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + (I_{PNIVS_Z}{}^{+}(\hat{h}_n))^2.(I_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + (F_{PNIVS_Z}{}^{+}(\hat{h}_n))^2.(F_{PNIVS_Z}{}^{+}(\hat{h}_n))^2 + \\
(\nu_{PNS_T}(\hat{h}_n))^2.(\nu_{PNS_T}(\hat{h}_n))^2 + (\nu_{PNS_I}(\hat{h}_n))^2.(\nu_{PNS_I}(\hat{h}_n))^2 + (\nu_{PNS_F}(\hat{h}_n))^2.(\nu_{PNS_F}(\hat{h}_n))^2
\end{array}\right)$$

By using Cauchy-Schwarz inequality, $\left(T_{crm}(\hat{H}_{PNCS}, \hat{Z}_{PNCS})\right)^2 \leq \left(T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS})\right) \times \left(T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\right)$ and taking the square root on both sides, $\left(T_{crm}(\hat{H}_{PNCS}, \hat{Z}_{PNCS})\right) \leq \sqrt{\left(T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS})\right) \times \left(T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\right)}$ which implies $\frac{(T_{crm}(\hat{H}_{PNCS}, \hat{Z}_{PNCS}))}{\sqrt{(T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS})) \times (T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS}))}} \leq 1$. Now, $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$therefore, the property $0 \leq \xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$ is proved.

ii. $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$iff $\hat{H}_{PNCS} = \hat{Z}_{PNCS}$

Let us assume $\hat{H}_{PNCS} = \hat{Z}_{PNCS}$ and prove $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$

Now, considering $\hat{H}_{PNCS} = \hat{Z}_{PNCS}$then $T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS}) = T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})$. Since$\hat{H}_{PNCS} = \hat{Z}_{PNCS}$then $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = \frac{T_{crm}(\hat{H}_{PNCS}, \hat{Z}_{PNCS})}{\sqrt{T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS}) \cdot T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}} = \frac{T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}{\sqrt{T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS}) \cdot T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}}$

$$\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = \frac{T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}{T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})} = 1$$

Therefore when $\hat{H}_{PNCS} = \hat{Z}_{PNCS}$ then $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$

Now, to prove conversely, let us assume $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$ and prove $\hat{H}_{PNCS} = \hat{Z}_{PNCS}$

Let $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$ then $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS}) = \frac{T_{crm}(\hat{H}_{PNCS}, \hat{Z}_{PNCS})}{\sqrt{T_{crm}(\hat{H}_{PNCS}, \hat{H}_{PNCS}) \cdot T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}} = 1$.

Then $T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \sqrt{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}) \cdot T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})}$. Squaring on both sides, we get

$\left(T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS})\right)^2 = T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}) \cdot T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})$. By using the Cauchy-Schwarz inequality, we prove that $\widehat{H}_{PNCS} = \hat{Z}_{PNCS}$if and only if $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$. Therefore $\widehat{H}_{PNCS} = \hat{Z}_{PNCS}$

iii. To prove: $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \xi_2(\hat{Z}_{PNCS}, \widehat{H}_{PNCS})$

The proof is obvious.

Definition 3.3 Let $\widehat{H}_{PNCS}$ and $\hat{Z}_{PNCS}$be two PNCS, then the CRM can be stated as

$$\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \frac{T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS})}{max\{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}), T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\}} \rightarrow (2)$$

Theorem 3.4 The CRM between PNCS $\widehat{H}_{PNCS}$ and $\hat{Z}_{PNCS}$ fulfills the traits preceding

i. $0 \leq \xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$

ii. $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$iff $\widehat{H}_{PNCS} = \hat{Z}_{PNCS}$

iii. $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \xi_2(\hat{Z}_{PNCS}, \widehat{H}_{PNCS})$

Proof

The traits (ii) and (iii) are obvious, so do not go into depth on them. Also $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \geq 0$, it is observable. Let us try to demonstrate for $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$. Since Theorem 3.2, it is necessary to have $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = \frac{T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS})}{max\{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}), T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\}}$. When $\xi_2(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) = 1$then $\left(T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS})\right) \leq max\{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}), T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\}$Then squaring on both sides we get, $\frac{(T_{crm}(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}))}{max\{T_{crm}(\widehat{H}_{PNCS}, \widehat{H}_{PNCS}), T_{crm}(\hat{Z}_{PNCS}, \hat{Z}_{PNCS})\}} \leq 1$. Now, $\xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$therefore, the property $0 \leq \xi_1(\widehat{H}_{PNCS}, \hat{Z}_{PNCS}) \leq 1$ is proved.

4. Application on Consumer Electronic Selection using PNCS Correlation Measure - MADM approach

This study provides an array of PNCS applications in using MADM that use the CRM in this section.

4.1. Decision-Making Problem

A business offers to put forth an important financial commitment in the mobile sector. Let $R_l = \{IR_1, IR_2, IR_3\}$represents a group of investors, $V_{SM} = \{V_{SM_1}, V_{SM_2}, V_{SM_3}, V_{SM_4}\}$ represents multiple smartphone manufacturers, and $M_{AT} = \{M_{AT_1}, M_{AT_2}, M_{AT_3}, M_{AT_4}\} = \{Battery, Storage, Display, Cameraquality\}$ represents a combination set of attributes.

Table 1. The Connection of attributes & customers ($R_l$and M_{AT})

T	M_{AT_1}	M_{AT_2}	M_{AT_3}	M_{AT_4}
IR_1	[0.2, 0.4], [0.1, 0.4], [0.1, 0.3], (0.3, 0.2, 0.2)	[0.4, 0.6], [0.1, 0.4], [0.2, 0.3], (0.6, 0.2, 0.3)	[0.4, 0.7], [0.7, 0.9], [0.1, 0.4], (0.5, 0.8, 0.3)	[0.1, 0.3], [0.2, 0.6], [0.1, 0.2], (0.2, 0.4, 0.1)
IR_2	[0.1, 0.4], [0.3, 0.5], [0.2, 0.6], (0.2, 0.4, 0.3)	[0.1, 0.3], [0.2, 0.3], [0.5, 0.6], (0.2, 0.2, 0.5)	[0.1, 0.4], [0.5, 0.6], [0.1, 0.4], (0.4, 0.6, 0.3)	[0.1, 0.2], [0.4, 0.8], [0.2, 0.4], (0.1, 0.8, 0.2)
IR_3	[0.6, 0.7], [0.3, 0.4], [0.7, 0.8], (0.6, 0.4, 0.7)	[0.5, 0.6], [0.2, 0.4], [0.1, 0.4], (0.5, 0.2, 0.1)	[0.2, 0.5], [0.6, 0.9], [0.8, 0.9], (0.2, 0.5, 0.9)	[0.2, 0.4], [0.5, 0.7], [0.3, 0.5], (0.3, 0.6, 0.4)

Table 2. The Connection of mobile phones & attributes (V_{SM} and M_{AT})

S	V_{SM_1}	V_{SM_2}	V_{SM_3}	V_{SM_4}
M_{AT_1}	[0.5, 0.9], [0.1, 0.6], [0.2, 0.5], (0.8, 0.6, 0.3)	[0.1, 0.4], [0.2, 0.5], [0.4, 0.6], (0.3, 0.4, 0.5)	[0.2, 0.4], [0.6, 0.7], [0.8, 0.9], (0.3, 0.7, 0.9)	[0.1, 0.3], [0.3, 0.4], [0.6, 0.8], (0.2, 0.3, 0.6)
M_{AT_2}	[0.1, 0.2], [0.2, 0.6], [0.3, 0.7], (0.2, 0.4, 0.7)	[0.2, 0.5], [0.3, 0.7], [0.6, 0.8], (0.4, 0.7, 0.6)	[0.1, 0.5], [0.2, 0.4], [0.8, 0.9], (0.2, 0.3, 0.8)	[0.1, 0.6], [0.2, 0.4], [0.5, 0.7], (0.1, 0.3, 0.6)
M_{AT_3}	[0.2, 0.4], [0.5, 0.7], [0.8, 0.9], (0.3, 0.7, 0.9)	[0.8, 0.9], [0.3, 0.4], [0.7, 0.8], (0.9, 0.4, 0.7)	[0.1, 0.3], [0.4, 0.6], [0.7, 0.8], (0.1, 0.5, 0.6)	[0.4, 0.5], [0.6, 0.7], [0.1, 0.5], (0.4, 0.6, 0.3)
M_{AT_4}	[0.5, 0.6], [0.7, 0.9], [0.2, 0.5], (0.6, 0.8, 0.3)	[0.4, 0.5], [0.8, 0.9], [0.6, 0.7], (0.4, 0.9, 0.6)	[0.5, 0.6], [0.1, 0.4], [0.8, 0.9], (0.5, 0.3, 0.8)	[0.1, 0.4], [0.2, 0.5], [0.6, 0.8], (0.3, 0.4, 0.7)

Table 3. Correlation Measure $\xi_1(\hat{H}_{PNCS}, \hat{Z}_{PNCS})$

	V_{SM_1}	V_{SM_2}	V_{SM_3}	V_{SM_4}
IR_1	0.5695	0.5506	0.3665	**0.6214**
IR_2	0.7454	**0.7543**	0.6435	0.7606
IR_3	**0.8252**	0.6611	0.6837	0.6829

Table 4. Correlation Measure $\xi_2(\hat{H}_{PNCS}, \hat{Z}_{PNCS})$

	V_{SM_1}	V_{SM_2}	V_{SM_3}	V_{SM_4}
IR_1	0.1815	0.1667	0.1170	**0.1995**
IR_2	0.2046	**0.2163**	0.1716	0.2092
IR_3	**0.3914**	0.2978	0.3245	0.2486

The optimal solution for the MADM problem is provided by the CRM from Tables 3 and 4, which results in a decision to buy a V_{SM_4} brand. IR_2 makes a decision to buy V_{SM_2} a brand. IR_3 makes a decision to V_{SM_1} brand. Therefore, it is evident from the two types of CRM mentioned above that the end results are the same. Comparing all the existing correlation measures of cubic sets, this CRM of PNCS is the most generalized method to handle the data very easily.

5. Conclusion

In this paper, the CRM of PNCS were established with a few of their fundamental attributes. In accordance with this, the CRM theory has transformed from PNS to PNCS in the present article. An example has been demonstrated for the correlation coefficient of PNCS and compared with a decision-making problem.

Conflicts of Interest

The author(s) declare that there is no conflict of interest regarding the publication of this paper.

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