

Original Article

Enhanced Solution of Transportation Problem using Pentagonal Fuzzy Numbers by Proposed Ranking Method

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Abstract - Fuzzy numbers figuring indispensable roles in problems in decision making, analysis of data, and socioeconomics arrangement. Finding the ranking of any fuzzy numbers is an inevitable step in many mathematical models. Many of the methods proposed produced the best solution to the transportation problems. This paper introduces a Proposed Ranking method that applies the same approach we used to transform the fuzzy transportation problem to an exquisite valued one, subsequently into a new proposed process to uncover the fuzzy realistic solution. The numerical illustration demonstrates that the new projected method offers an awesome means for managing the transportation problems using fuzzy algorithms.

Keywords - Transportation Problem, Optimal solution, Fuzzy set, Fuzzy Transportation Problem, Pentagonal Fuzzy Numbers.

1. Introduction

The transportation problem commonly arises in many real-world situations, especially within production and logistics systems. It holds significant importance in industries where goods must be efficiently distributed from multiple sources to various destinations. This problem is considered a special case of Linear Programming (LPP), which helps determine the most efficient allocation strategy while minimizing cost and maximizing profit.

By solving a transportation problem, one can identify the optimal quantity of goods to be transported from each origin to each destination. This ensures a feasible solution that reduces overall transportation cost, minimizes time consumption, and improves profit margins.

From the review of existing research papers and studies, a major challenge identified is obtaining the optimal solution with minimal computational effort. The key objective is to achieve cost-effective production and distribution in various industries.

In this work, we propose a new methodology aimed at improving the optimal solution value. Furthermore, the results are compared with well-known existing techniques such as the Northwest Corner Method (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM), to demonstrate their effectiveness.

A fuzzy transportation problem is a progressive method in that we can get the expenditure of the transportation. Demand and supply facts are fuzzy quantities—the first introduction of the fuzzy set concept by Zadeh [1]. Zimmerman [2] devised fuzzy linear programming. Panda and Pal [3] investigated exponent operation and arithmetic operations of Pentagonal Fuzzy numbers. Anitha and Parvathi [4] proposed a new method to find the expected crisp value of Pentagonal Fuzzy numbers. Helen and Uma [5] are defending the parametric representation of Pentagonal Fuzzy numbers. Siji and Kumari [6] have explored arithmetic operations and found the ranking of pentagonal fuzzy numbers. Annie Christi and Kasthuri [7] proposed a technique to crack the fuzzy transportation problem for PFN. Edward [8] introduces the simplex type method for solving the fuzzy transportation problem. Uma Maheswari [9] applied the Robust ranking technique to solve fully Fuzzy transportation problems using pentagonal fuzzy numbers. Sathya Geetha [10] explored the Range method to solve Fuzzy transportation problems using PFNs. R. Srinivasan and Karthikeyan, N [11] have explored a two-stage cost- minimizing fuzzy transportation problem where supply and demand are fuzzy numbers using a strict approach to reach a fully fuzzy solution. The proposed Ranking algorithm is to unravel a strong solution by using fuzzy transportation problems, taking into



account supply, demand, and item transportation price as pentagonal fuzzy numbers. R. Srinivasan and Karthikeyan [13] presented a proposed ranking method for solving a Transportation problem using pentagonal fuzzy numbers.

This manuscript proposes a novel and clear approach for ranking fuzzy pentagonal numbers in a simple and effective manner. To validate the proposed technique, an illustrative example is provided. The method is designed to be straightforward and easy to apply, enabling users to obtain the most suitable optimal solution for fuzzy transportation problems encountered in real-world situations.

2. Preliminaries

2.1. Fuzzy Transportation Problem

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply, and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem.

In a Fuzzy Transportation Problem, all parameters are fuzzy numbers, which may be normal or abnormal, triangular or trapezoidal. The mathematical representation of the fuzzy transportation problem is as follows:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \cdot x_{ij} \quad r = 1, 2, 3, \dots \dots k \tag{1}$$

Subject to

$$\left. \begin{aligned} \sum_{j=1}^n x_{ij} &= \tilde{a}_i, \quad i = 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &= \tilde{b}_j, \quad j = 1, 2, \dots, n \\ x_{ij} &\geq 0 \quad \forall i, j \end{aligned} \right\} \tag{2}$$

Where, \tilde{c}_{ij} , \tilde{a}_i and \tilde{b}_j are represented by the transportation cost, demand of product, and availability of product, respectively, in terms of fuzzy parameters.

2.2. Fuzzy Set

A Fuzzy set A is defined as the set of ordered pairs $(x, \mu_A(x))$ where x is an element of the universe of discourse U and $\mu_A(x)$ is the membership that attributes to each $x \in U$, a real number $\epsilon [0,1]$ describing the degree to which x belongs to the set.

2.3. Pentagonal Fuzzy Number

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \leq \tilde{a}_1 \\ \frac{\tilde{u}_1(x - \tilde{a}_2)}{(\tilde{a}_3 - \tilde{a}_2)} & \text{for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ (1 - (1 - u_1)) \frac{(x - \tilde{a}_2)}{(\tilde{a}_3 - \tilde{a}_2)}, & \text{for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 1, & \text{for } x = \tilde{a}_3 \\ (1 - (1 - u_2)) \frac{(\tilde{a}_4 - x)}{(\tilde{a}_4 - \tilde{a}_3)}, & \text{for } \tilde{a}_3 \leq x \leq \tilde{a}_4 \\ \frac{\tilde{u}_2(\tilde{a}_5 - x)}{(\tilde{a}_5 - \tilde{a}_4)} & \text{for } \tilde{a}_4 \leq x \leq \tilde{a}_5 \\ 0 & \text{for } x > a_5 \end{cases}$$

Here the midpoint \tilde{a}_3 has the grade of membership 1 and \tilde{a}_4, \tilde{a}_2 has the grades \tilde{u}_1, \tilde{u}_2 respectively. Note that every PFN is connected with two weights \tilde{u}_1, \tilde{u}_2 .

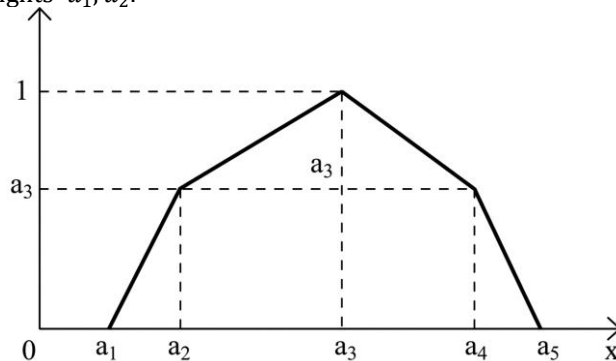


Fig.1 Pentagonal fuzzy number

2.4. Arithmetic Operations

Let $\tilde{\alpha}_A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5)$ and $\tilde{\alpha}_B = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5)$ are two fuzzy numbers where $\tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}_4 \leq \tilde{a}_5$ and $\tilde{b}_1 \leq \tilde{b}_2 \leq \tilde{b}_3 \leq \tilde{b}_4 \leq \tilde{b}_5$. Then the arithmetic operations are defined as follows.

Addition: $\tilde{\alpha}_A + \tilde{\alpha}_B = (\tilde{a}_1 + \tilde{b}_1, \tilde{a}_2 + \tilde{b}_2, \tilde{a}_3 + \tilde{b}_3, \tilde{a}_4 + \tilde{b}_4, \tilde{a}_5 + \tilde{b}_5)$

Subtraction: $\tilde{\alpha}_A - \tilde{\alpha}_B = (\tilde{a}_1 - \tilde{b}_1, \tilde{a}_2 - \tilde{b}_2, \tilde{a}_3 - \tilde{b}_3, \tilde{a}_4 - \tilde{b}_4, \tilde{a}_5 - \tilde{b}_5)$

Multiplication: $\tilde{\alpha}_A * \tilde{\alpha}_B = \left(\frac{\tilde{a}_1}{5} \tilde{\gamma}_B, \frac{\tilde{a}_2}{5} \tilde{\gamma}_B, \frac{\tilde{a}_3}{5} \tilde{\gamma}_B, \frac{\tilde{a}_4}{5} \tilde{\gamma}_B, \frac{\tilde{a}_5}{5} \tilde{\gamma}_B\right)$ where $\tilde{\gamma}_B = (\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_3 + \tilde{b}_4 + \tilde{b}_5)$

Division: $\frac{\tilde{\alpha}_A}{\tilde{\alpha}_B} = \left(\frac{5\tilde{a}_1}{\tilde{\gamma}_B}, \frac{5\tilde{a}_2}{\tilde{\gamma}_B}, \frac{5\tilde{a}_3}{\tilde{\gamma}_B}, \frac{5\tilde{a}_4}{\tilde{\gamma}_B}, \frac{5\tilde{a}_5}{\tilde{\gamma}_B}\right)$ if $\tilde{\gamma}_B \neq 0$ where $\tilde{\gamma}_B = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5)$

Scalar Multiplication: $k\tilde{\alpha}_A = \begin{cases} k\tilde{a}_1, k\tilde{a}_2, k\tilde{a}_3, k\tilde{a}_4, k\tilde{a}_5, & \text{if } k > 0 \\ k\tilde{a}_5, k\tilde{a}_4, k\tilde{a}_3, k\tilde{a}_2, k\tilde{a}_1 & \text{if } k < 0 \end{cases}$

2.5. Fuzzy Transportation Problem by means of Mathematical Formulation

The mathematical formulation of the pentagonal fuzzy numbers under the case that the total supply is equivalent to the total demand is given as follows

$$\text{Min } Z = \sum_{i=1}^s \sum_{j=1}^t \tilde{a}_{ij} \tilde{x}_{ij}$$

Subject to the constraints

$$\sum_{j=1}^t \tilde{x}_{ij} = \tilde{a}_i, \quad j = 1, 2, \dots, \dots, t$$

$$\sum_{i=1}^s \tilde{x}_{ij} = \tilde{b}_j, \quad i = 1, 2, \dots, \dots, s$$

$$\sum_{i=1}^s \tilde{a}_i = \sum_{j=1}^t \tilde{b}_j, \quad i = 1, 2, \dots, \dots, s, j = 1, 2, \dots, \dots, t$$

and

$$\tilde{x}_{ij} \geq 0, \quad i = 1, 2, \dots, \dots, s, j = 1, 2, \dots, \dots, t$$

The fuzzy transportation problem is explicitly represented by the fuzzy transportation table:

	1	...	t	supply
1	\tilde{a}_{11}	\tilde{a}_{1t}	\tilde{a}_1
.
.
.
s	\tilde{a}_{s1}	...	\tilde{a}_{st}	\tilde{a}_s
Demand	\tilde{b}_1	...	\tilde{b}_t	

2.6. Ranking Function

In this manuscript, we proposed a centroid ranking technique. Let $\tilde{\alpha}_A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5)$ be a Pentagonal fuzzy number by using the centroid Ranking technique for PFNs, here the ranking was introduced.

$$R(\tilde{\alpha}_A) = \left(\frac{\tilde{a}_5^2 + \tilde{a}_4^2 + \tilde{a}_5\tilde{a}_4 - \tilde{a}_2^2 - \tilde{a}_1^2 - \tilde{a}_2\tilde{a}_1}{3(\tilde{a}_5 + \tilde{a}_4 - \tilde{a}_2 - \tilde{a}_1)}\right)$$

2.7. Proposed Method Algorithm

Step 1: Verify whether the given problem is stable or not. i.e.

$$\sum_{i=1}^s \tilde{a}_i = \sum_{j=1}^t \tilde{b}_j$$

If unstable, change into a stable one by introducing a model source or model destination utilizing zero fuzzy item transportation expenses.

Step 2: The Ranking value is imparted to transform both demand and supply.

Step 3: The row-wise multiple between the greatest and least values of each row, and is divided by the product of the rows and columns of the cost matrix.

Step 4: The column-wise multiple between the greatest and least values of each column, and is divided by the multiple of the rows and columns of the cost matrix.

Step 5: We find the maximum of the resultant value and do the allocation of that particular cell of the given matrix. Suppose we have more than one maximum consequent value. We can select anyone.

Step 6: Follow the Third, fourth, and fifth steps until $(s + t - 1)$ groups are allocated. Suppose the allocated cell does not achieve the application of the Modi method to find optimality

3. Numerical Example

A resolution that we affirm to the fuzzy Transportation Problem, which involves transportation cost, customer needs and demands, and the existence of products using Pentagonal Fuzzy figures

	R_a	R_b	R_c	R_d	Inventory
I_a	(5,1,3,4,9)	(1,3,2,4,8)	(4,1,8,6,2)	(1,4,5,8,2)	35
I_b	(2,4,7,11,5)	(2,8,4,5,7)	(1,4,3,7,10)	(2,5,8,7,11)	15
I_c	(2,3,6,4,8)	(2,4,9,7,10)	(2,4,5,1,6)	(3,4,5,6,2)	50
I_d	(2,3,8,5,6)	(1,2,4,3,6)	(1,3,6,7,10)	(1,3,7,6,2)	40
Requirement	20	10	65	45	140

By using the Ranking Technique, we have to convert Fuzzy Trapezoidal numbers into a crisp value.

$$R(\tilde{\alpha}_A) = \left(\frac{\tilde{a}_5^2 + \tilde{a}_4^2 + \tilde{a}_5 \tilde{a}_4 - \tilde{a}_2^2 - \tilde{a}_1^2 - \tilde{a}_2 \tilde{a}_1}{3(\tilde{a}_5 + \tilde{a}_4 - \tilde{a}_2 - \tilde{a}_1)} \right)$$

$$R(5,1,3,4,9) = \left(\frac{9^2 + 4^2 + 9 \times 4 - 1^2 - 5^2 - 1 \times 5}{3(9 + 4 - 1 - 5)} \right) = \left(\frac{81 + 16 + 36 - 1 - 25 - 5}{3(9 + 4 - 1 - 5)} \right) = \frac{102}{21} = 4.8571$$

$$R(1,3,2,4,8) = \left(\frac{8^2 + 4^2 + 8 \times 4 - 3^2 - 1^2 - 1 \times 3}{3(8 + 4 - 3 - 1)} \right) = \frac{99}{24} = 4.1250$$

$$R(4,1,8,6,2) = \left(\frac{2^2 + 6^2 + 2 \times 6 - 1^2 - 4^2 - 1 \times 4}{3(2 + 6 - 1 - 4)} \right) = \frac{31}{9} = 3.4444$$

$$R(1,4,5,8,2) = \left(\frac{2^2 + 8^2 + 2 \times 8 - 4^2 - 1^2 - 1 \times 4}{3(2 + 8 - 4 - 1)} \right) = \frac{63}{15} = 4.2000$$

$$R(2,4,7,11,5) = \left(\frac{5^2 + 11^2 + 5 \times 11 - 4^2 - 2^2 - 2 \times 4}{3(5 + 11 - 4 - 2)} \right) = \frac{173}{30} = 5.7666$$

$$R(2,8,4,5,7) = \left(\frac{7^2 + 5^2 + 7 \times 5 - 8^2 - 2^2 - 8 \times 2}{3(7 + 5 - 8 - 2)} \right) = \frac{25}{6} = 4.1666$$

$$R(1,4,3,7,10) = \left(\frac{10^2 + 7^2 + 10 \times 7 - 4^2 - 1^2 - 4 \times 1}{3(10 + 7 - 4 - 1)} \right) = \frac{198}{36} = 5.5000$$

$$R(2,5,8,7,11) = \left(\frac{11^2 + 7^2 + 11 \times 7 - 5^2 - 2^2 - 5 \times 2}{3(11 + 7 - 5 - 2)} \right) = \frac{208}{33} = 6.3030$$

$$R(2,3,6,4,8) = \left(\frac{8^2 + 4^2 + 8 \times 4 - 3^2 - 2^2 - 3 \times 2}{3(8 + 4 - 3 - 2)} \right) = \frac{93}{21} = 4.4285$$

$$R(2,4,9,7,10) = \left(\frac{10^2 + 7^2 + 10 \times 7 - 4^2 - 2^2 - 4 \times 2}{3(10 + 7 - 4 - 2)} \right) = \frac{191}{33} = 5.7878$$

$$R(2,4,5,1,6) = \left(\frac{6^2 + 1^2 + 6 \times 1 - 4^2 - 2^2 - 4 \times 2}{3(6 + 1 - 4 - 2)} \right) = \frac{15}{3} = 5.0000$$

$$R(3,4,5,6,2) = \left(\frac{2^2 + 6^2 + 2 \times 6 - 4^2 - 3^2 - 4 \times 3}{3(2 + 6 - 4 - 3)} \right) = \frac{15}{3} = 5.0000$$

$$R(2,3,8,5,6) = \left(\frac{6^2 + 5^2 + 6 \times 5 - 3^2 - 2^2 - 3 \times 2}{3(6 + 5 - 3 - 2)} \right) = \frac{72}{18} = 4.0000$$

$$R(1,2,4,3,6) = \left(\frac{6^2 + 3^2 + 6 \times 3 - 2^2 - 1^2 - 2 \times 1}{3(6 + 3 - 2 - 1)} \right) = \frac{56}{18} = 3.1111$$

$$R(1,3,6,7,10) = \left(\frac{10^2 + 7^2 + 10 \times 7 - 3^2 - 1^2 - 3 \times 1}{3(10 + 7 - 3 - 1)} \right) = \frac{206}{39} = 5.2820$$

$$R(1,3,7,6,2) = \left(\frac{2^2 + 6^2 + 2 \times 6 - 3^2 - 1^2 - 3 \times 1}{3(2 + 6 - 3 - 1)} \right) = \frac{39}{12} = 3.2500$$

Table 1

	R_a	R_b	R_c	R_d	Inventory
I_a	4.8571	4.1250	3.4444	4.2000	35
I_b	5.7666	4.1666	5.5000	6.3030	15
I_c	4.4285	5.7878	5.0000	5.0000	50
I_d	4.0000	3.1111	5.2820	3.2500	40
Requirement	20	10	65	45	140

The given problem is balanced. Then choose the maximum of the penalty values and find the corresponding minimum cost value, and allocate the particular cost cell of the given problem. If we have more than one maximum resultant value, we can choose any one.

Table 2

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.8571	4.1250	3.4444	4.2000	35	$\frac{4.8571 \times 3.4444}{16} = 1.04$ 56
I_b	5.7666	(10) 4.1666	5.5000	6.3030	15	$\frac{6.3030 \times 4.1666}{16} = 1.6413$ (Max)
I_c	4.4285	5.7878	5.0000	5.0000	50	$\frac{5.7878 \times 4.4285}{16} = 1.60$ 19

I_d	4.0000	3.1111	5.2820	3.2500	40	$\frac{5.2820 \times 3.1111}{70} = 1.02$
Requirement	20	10	65	45		
$\frac{\min \times \max}{\text{row} \times \text{column}}$	$\frac{5.7666 \times 4}{16} = 1.44$	$\frac{5.7878 \times 3.1111}{16} = 1.12$	$\frac{5.2820 \times 3.4444}{16} = 1.13$	$\frac{6.3030 \times 3.2500}{16} = 1.28$		

We choose the maximum of the penalty values and find the corresponding minimum cost value, and allocate the particular cost cell of the given problem. If we have more than one maximum resultant value, we can choose any one.

Table 3

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.8571	4.1250	3.4444	4.2000	35	1.3993
I_b	5.7666	(10) 4.1666	(5) 5.5000	6.3030	15	2.8888(Max)
I_c	4.4285	5.7878	5.0000	5.0000	50	1.8452
I_d	4.0000	3.1111	5.2820	3.2500	40	1.4305
Requirement	20	10	65	45		
$\frac{\min \times \max}{\text{row} \times \text{column}}$	1.9222		1.5786	1.7070		

We choose the maximum of the penalty values and find the corresponding minimum cost value, and allocate the particular cost cell of the given problem. If we have more than one maximum resultant value, we can choose any one.

Table 4

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.8571	4.1250	3.4444	4.2000	35	1.8657
I_b	5.7666	(10) 4.1666	(5) 5.5000	6.3030	15	
I_c	(20) 4.4285	5.7878	5.0000	5.0000	50	2.4602(Max)
I_d	4.0000	3.1111	5.2820	3.2500	40	1.9073
Requirement	20	10	65	45		
$\frac{\min \times \max}{\text{row} \times \text{column}}$	2.1587		2.0214	1.8055		

We choose the maximum of the penalty values and find the corresponding minimum cost value, and allocate the particular cost cell of the given problem. If we have more than one maximum resultant value, we can choose any one.

Table 5

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.8571	4.1250	3.4444	4.2000	35	2.4108
I_b	5.7666	(10) 4.1666	(5) 5.5000	6.3030	15	
I_c	(20) 4.4285	5.7878	(30) 5.0000	5.0000	50	4.1666(Max)
I_d	4.0000	3.1111	5.2820	3.2500	40	2.8610
Requirement	20	10	65	45		
$\frac{\min \times \max}{\text{row} \times \text{column}}$			3.0322	2.275		

The same procedure will be followed again and again until we reach the final allocation. Finally, using the new proposed ranking algorithm, the best possible resolutions are as follows.

Table 6

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.8571	4.1250	(35) 3.4444	4.2000	35	3.6166
I_b	5.7666	(10) 4.1666	(5) 5.5000	6.3030	15	
I_c	(20) 4.4285	5.7878	5.0000	(30) 5.0000	50	
I_d	4.0000	3.1111	(25) 5.2820	(15) 3.2500	40	4.2916
Requirement	20	10	65	45		
$\frac{\min \times \max}{\text{row} \times \text{column}}$			4.5478 (Max)	3.4125		

Here, we get a total of 7 allocation cells.

The optimal solution is $(3.4444 \times 35) + (4.1666 \times 10) + (5.5000 \times 10) + (4.4285 \times 20) + (5.0000 \times 30) + (5.2820 \times 25) + (3.2500 \times 15) = 120.554 + 41.666 + 55 + 88.57 + 150 + 132.05 + 48.75 = 636.77$

For Unbalanced Transportation Problem:

Table 7

	R_a	R_b	R_c	R_d	Inventory
I_a	(1,3,4,5,9)	(2,3,1,4,8)	(2,4,6,1,8)	(1,4,2,5,8)	35
I_b	(11,7,4,2,5)	(7,5,4,2,8)	(10,7,3,1,4)	(11,7,8,2,5)	15
I_c	(8,6,4,2,3)	(10,9,7,2,4)	(6,5,4,1,2)	(6,5,4,2,3)	75
I_d	(8,6,3,2,5)	(6,4,1,2,3)	(10,7,3,1,6)	(7,6,2,3,1)	25
Requirement	30	40	50	60	

By using the Ranking Technique, we have to convert Fuzzy Trapezoidal numbers into a crisp value.

$$R(\tilde{\alpha}_A) = \left(\frac{\tilde{\alpha}_5^2 + \tilde{\alpha}_4^2 + \tilde{\alpha}_5 \tilde{\alpha}_4 - \tilde{\alpha}_2^2 - \tilde{\alpha}_1^2 - \tilde{\alpha}_2 \tilde{\alpha}_1}{3(\tilde{\alpha}_5 + \tilde{\alpha}_4 - \tilde{\alpha}_2 - \tilde{\alpha}_1)} \right)$$

$$R(1,3,4,5,9) = \left(\frac{9^2 + 5^2 + 9 \times 5 - 3^2 - 1^2 - 1 \times 3}{3(9 + 5 - 3 - 1)} \right) = \frac{138}{30} = 4.6000$$

$$R(2,3,1,4,8) = \left(\frac{8^2 + 4^2 + 8 \times 4 - 3^2 - 2^2 - 3 \times 2}{3(8 + 4 - 3 - 2)} \right) = \frac{93}{21} = 4.4285$$

$$R(2,4,6,1,8) = \frac{45}{9} = 5.0000$$

$$R(1,4,2,5,8) = \frac{108}{24} = 4.5000$$

$$R(11,7,4,2,5) = \frac{208}{33} = 6.3030$$

$$R(7,5,4,2,8) = \frac{25}{6} = 4.1666$$

$$R(10,7,3,1,4) = \frac{198}{36} = 5.5000$$

$$R(11,7,8,2,5) = \frac{208}{33} = 6.3030$$

$$R(8,6,4,2,3) = \frac{129}{27} = 4.7777$$

$$R(10,9,7,2,4) = \frac{243}{39} = 6.2307$$

$$R(6,5,4,1,2) = \frac{84}{24} = 3.5000$$

$$R(6,5,4,2,3) = \frac{72}{18} = 4.0000$$

$$R(8,6,3,2,5) = \frac{109}{21} = 5.1904$$

$$R(6,4,1,2,3) = \frac{57}{15} = 3.8000$$

$$R(10,7,3,1,6) = \frac{176}{30} = 5.8666$$

$$R(7,6,2,3,1) = \frac{114}{27} = 4.22222$$

Table 8

	R_a	R_b	R_c	R_d	Inventory
I_a	4.6000	4.4285	5.0000	4.5	35
I_b	6.3030	4.1666	5.5000	6.3030	15
I_c	4.7777	6.2307	3.5000	4.0000	75
I_d	5.1904	3.8000	5.8666	4.2222	25
I_e	0	0	0	0	30
Requirement	30	40	50	60	180

Table 9

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.6000	4.4285	5.0000	4.5	35	$\frac{5 \times 4.4285}{20} = 1.1071$
I_b	6.3030	(15) 4.1666	5.5000	6.3030	15	$\frac{6.3030 \times 4.1666}{20} = 1.3131(\text{Max})$
I_c	4.7777	6.2307	3.5000	4.0000	75	$\frac{6.2307 \times 3.5}{20} = 1.0903$
I_d	5.1904	3.8000	5.8666	4.2222	25	$\frac{5.8666 \times 3.8}{20} = 1.1146$
I_e	0	0	0	0	30	
Requirement	30	40	50	60	180	
$\frac{\min \times \max}{\text{row} \times \text{column}}$	0	0	0	0		

Table 10

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.6000	4.4285	5.0000	4.500	35	$\frac{5 \times 4.4285}{16} = 1.3839$
I_b	6.3030	(15) 4.1666	5.5000	6.3030	15	
I_c	4.7777	6.2307	3.5000	4.0000	75	$\frac{6.2307 \times 3.5}{16} = 1.3629$
I_d	5.1904	(25) 3.8000	5.8666	4.2222	25	$\frac{5.8666 \times 3.8}{16} = 1.3933(\text{Max})$
I_e	0	0	0	0	30	
Requirement	30	40	50	60	180	
$\frac{\min \times \max}{\text{row} \times \text{column}}$	0	0	0	0		

Table 11

	R_a	R_b	R_c	R_d	Inventory	$\frac{\min \times \max}{\text{row} \times \text{column}}$
I_a	4.6000	4.4285	5.0000	(35) 4.5	35	$\frac{5 \times 4.5}{9} = 2.5(\text{Max})$
I_b	6.3030	(15) 4.1666	5.5000	6.3030	15	
I_c	4.7777	6.2307	3.5000	4.0000	75	$\frac{4.7777 \times 3.5}{9} = 1.8579$
I_d		(25)			25	

	5.1904	3.8000	5.8666	4.2222		
I_e	0	0	0	0	30	
Requirement	30	40	50	60	180	
$\frac{\text{min} \times \text{max}}{\text{row} \times \text{column}}$	0	0	0	0		

Table 12

	R_a	R_b	R_c	R_d	Inventory	$\frac{\text{min} \times \text{max}}{\text{row} \times \text{column}}$
I_a	4.6000	4.4285	5.0000	(35) 4.5	35	
I_b	6.3030	(15) 4.1666	5.5000	6.3030	15	
I_c	4.7777	6.2307	(50) 3.5000	(25) 4.0000	75	$\frac{4.7777 \times 3.5}{6} =$ 2.7869(Max)
I_d	5.1904	(25) 3.8000	5.8666	4.2222	25	
I_e	(30) 0	0	0	0	30	0
Requirement	30	40	50	60	180	
$\frac{\text{min} \times \text{max}}{\text{row} \times \text{column}}$	0	0	0	0		

The optimal solution is $(4.5 \times 35) + (4.1666 \times 15) + (50 \times 3.5) + (25 \times 4) + (25 \times 3.8) + (30 \times 0) = 589.999 \approx 590$.

4. Conclusion

The primary contribution of this manuscript is to develop and derive the most effective optimal solution for a fuzzy transportation problem, where uncertainties in transportation costs, supply, or demand are modeled using fuzzy logic for Pentagonal fuzzy numbers using the newly proposed Ranking algorithm method. This practice can be used for all types of fuzzy transportation problems. The recently proposed Ranking technique is a regularized practice, simple to relate to, and able to be operated for all types of transportation problems, either to capitalize on or play down an intended function. It offers a simple yet powerful tool for optimizing transportation solutions under fuzzy conditions.

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