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Original Article

Fuzzy Pentapartitioned Neutrosophic Soft Topological Space

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Abstract *- This study introduces the concept of fuzzy pentapartitioned Neutrosophic soft topological space. We define the interior and closure operators for fuzzy pentapartitioned Neutrosophic soft sets within this topological framework and examine their fundamental properties. Additionally, we introduce the notions of Fuzzy Pentapartitioned Neutrosophic Soft Pre-Open (FPNS-PO), semi-open (FPNS-SO), b-open (FPNS-b-O), and α-open (FPNS-α-O) sets. Suitable numerical examples are provided, and several basic results concerning these generalized open sets are established.*

Keywords *- Fuzzy pentapartitioned neutrosophic soft set, fuzzy pentapartitioned neutrosophic soft matrix, fpns-po, fpns-so, fpns-b-o, fpns-α-o.*

1. Introduction

The concept of fuzzy sets, introduced by Zadeh in 1965, laid the foundation for modeling vagueness in real-life problems. Atanassov (1986) further extended this idea by proposing intuitionistic fuzzy sets, which incorporate both membership and non-membership degrees. To handle parameters and uncertainties more flexibly, Molodtsov (1999) introduced the theory of soft sets. Smarandache (1998) unified these approaches by developing the concept of Neutrosophic sets, which independently assign truth, indeterminacy, and falsity memberships.

Building upon Neutrosophic sets, Salama and Alblowi (2012) pioneered the study of Neutrosophic topological spaces. Subsequently, numerous researchers investigated generalized open and closed sets in neutrosophic topological spaces. The hybridization of Neutrosophic sets with soft sets led to the emergence of Neutrosophic soft topological spaces (Bera, 2018; Akram et al., 2019; Mehmood et al., 2021). Various extensions, such as Neutrosophic soft bitopological spaces, tri-topological spaces, and n-topological spaces, have also been explored (Das &Tripathy, 2021b; Saeed et al., 2023; Al-Omari, 2024).

In parallel, refinements of Neutrosophic sets have been proposed to capture more nuanced indeterminacy. Chatterjee et al. (2016) introduced quadripartitioned Neutrosophic sets, and topology on quadripartitioned Neutrosophic sets was later studied by Das et al. (2021a). Mallick and Pramanik (2020) generalized this further by proposing pentapartitioned Neutrosophic sets, splitting indeterminacy into three distinct components (contradiction, ignorance, and unknown) in addition to truth and falsity. Topological structures on pentapartitioned Neutrosophic sets were first examined by Das and Tripathy (2021). Soft-set versions of pentapartitioned Neutrosophic sets have also appeared recently (Shil et al., 2024; Datta et al., 2025). Although topological structures have been extensively developed for Neutrosophic soft sets, Quadripartitioned Neutrosophic sets, and single-valued pentapartitioned Neutrosophic sets, no study has yet examined the topological properties (interior, closure, and generalized open sets) of the newly proposed fuzzy pentapartitioned Neutrosophic soft sets. The present work fills this gap by establishing the notion of fuzzy pentapartitioned Neutrosophic soft topological space.The topological framework and general proof techniques in this paper are motivated by and extend the classical approach of Das and Tripathy (2021) to the newly introduced fuzzy pentapartitioned Neutrosophic soft setting.The paper is organized as follows: Section 2 recalls necessary preliminaries. Section 3 develops the topological concepts and derives the main results. Section 4 concludes the study.

2. Preliminaries

In this section, we recall the basic concepts of fuzzy pentapartitined Neutrosophic soft sets and their algebraic operations that will be used throughout the paper.

Definition2.1:

Let X be a universe of discourse and $A \subseteq E$ a set of parameters. A fuzzy pentapartitioned Neutrosophic soft set F_A over X is a mapping $F: A \to FPNS(X)$ such that for each parameter $e \in A$ and each element. $x \in X$.

$$F(e)(x) = (T_{F_e}(x), C_{F_e}(x), U_{F_e}(x), H_{F_e}(x), F_{F_e}(x))$$

Where $T_{F_e}(x), C_{F_e}(x), U_{F_e}(x), H_{F_e}(x), F_{F_e}(x)$ are the truth, contradiction, ignorance, hesitation, and falsity membership degree, respectively, satisfying

$$0 \leq T_{F_e}(x) + C_{F_e}(x) + U_{F_e}(x) + H_{F_e}(x) + F_{F_e}(x) \leq 5$$

Definition 2.2:

The null fuzzy pentapartitioned Neutrosophic soft set $\varphi$and the absolute fuzzy pentapartitioned Neutrosophic soft set $\bar{X}$ over X are defined as follows

(i)$\varphi(e)(x) = (1,1,1,0,0) \forall e \in A, x \in X$

(ii)$\bar{X}(e)(x) = (0,0,0,1,1) \forall e \in A, x \in X$

Definition2.3:

Let F and G be two fuzzy pentapartitioned Neutrosophic soft sets over X.We say $F \subseteq G$ that iff $\forall e \in A$ and every $x \in X$.

$$T_F(e)(x) \leq T_G(e)(x), C_F(e)(x) \leq C_G(e)(x), U_F(e)(x) \leq U_G(e)(x),$$
$$H_F(e)(x) \geq H_G(e)(x), F_F(e)(x) \geq F_G(e)(x).$$

Definition 2.4:

The complement of F, denoted$F^c$by, is defined by

$$F^c(e)(x) = (1 - T_F(e)(x), 1 - C_F(e)(x), 1 - U_F(e)(x), 1 - H_F(e)(x), 1 - F_F(e)(x))$$

Definition 2.5:(sum)

Let F and G be two arbitrary FPNSS defined over the same universe X and the same parameter E; their sum can be defined as

$$(F + G)(e)(x) = min(T_F(e)(x), T_G(e)(x)), min(C_F(e)(x), C_G(e)(x)), min(U_F(e)(x),$$
$$U_G(e)(x)), max(H_F(e)(x), H_G(e)(x)), max(F_F(e)(x), F_G(e)(x))$$

Definition 2.5:(Difference)

Let F and G be two arbitrary FPNSS defined over the same universe X and the same parameter E; their difference can be defined as

$$(F - G)(e)(x) = max(T_F(e)(x), T_G(e)(x)), max(C_F(e)(x), C_G(e)(x)), max(U_F(e)(x), U_G(e)(x))$$
$$min(H_F(e)(x), H_G(e)(x)), min(F_F(e)(x), F_G(e)(x))$$

Definition2.7:(Average Product)

Let F and G be two arbitrary FPNSS defined over the same universe X and the same parameter E; their average product can be defined as

$$(F \nabla G)(x) = \left(\begin{array}{l} \frac{T_F(e)(x) + T_G(e)(x)}{2}, \frac{C_F(e)(x) + C_G(e)(x)}{2}, \frac{U_F(e)(x) + U_G(e)(x)}{2} \\ \frac{H_F(e)(x) + H_G(e)(x)}{2}, \frac{F_F(e)(x) + F_G(e)(x)}{2} \end{array} \right)$$

3. Fuzzy Pentapartitioned Neutrosophic Soft Topology

Definition 3.1:

Let X be a fixed universe. A family $\tau$of FPNSS over X is called a fuzzy pentapartitioned Neutrosophic soft topology(FPNS-T) on X if it satisfies.

(i)$\varphi, \bar{X} \in \tau$.

(ii)if $F_1, F_2 \in \tau$ then$F_1 \cap F_2 \in \tau$.

(iii) if $\{F_i : i \in \Delta\} \subseteq \tau$ then$U_{i\in\Delta} F_i \in \tau$.

The pair (X, τ) is then called a fuzzy pentapartitioned Neutrosophic soft topological space. Members of τ are called Fuzzy Pentapartitioned Neutrosophic Soft Open Sets(FPNS-OS), and their complements are called fuzzy pentapartitioned Neutrosophic soft closed sets.

Example 3.1:

Let $X = \{p, q\}$, and the parameter set $A = \{e\}$ consider the following FPNSS

$$F = \{(p, 0.1, 0.2, 0.3, 0.15, 0.1), (q, 0.2, 0.1, 0.2, 0.2, 0.15)\}$$

$$G = \{(p, 0.05, 0.15, 0.25, 0.2, 0.2). (q, 0.1, 0.2, 0.3, 0.1, 0.1)\}$$

$$H = \{(p, 0.3, 0.1, 0.2, 0.05, 0.05)(q, 0.25, 0.15, 0.25, 0.1, 0.05)\}$$

The collection $\tau = \{\varphi, \bar{X}, F, G, H\}$ satisfies the axioms of an FPNS-Topology on X.

Definition 3.2:

In a FPNS-TS(X, τ) for any FPNSS F, the fuzzy pentapartitionedneutrosophic soft interior of (denoted by FPNS-in(F)) is the union of all FPNS-OS contained in F, while the fuzzy pentapartitionedneutrosophic soft closure of F(denoted by FPNS-Cl(F)) is the intersection of all FPNS-CS containing F.

Theorem 3.1: Let Q and R be arbitrary FPNSS in an FPNS-TS (X, τ) , then(i)FPNS-int$(Q) \subseteq Q \subseteq$ FPNS-cl(Q)
(ii)$Q \subseteq$ R implies FPNS-cl$(Q) \subseteq$ FPNS-cl(R)
(iii)$Q \subseteq$ R implies FPNS-int$(Q) \subseteq$ FPNS-int(R)
(iv)FPNS-cl$(Q \cup R) =$ FPNS-cl$(Q) \cup$ FPNS-cl(R)
(v)FPNS-cl$(Q \cap R) \subseteq$ FPNS-cl$(Q) \cap$ FPNS-cl(R)
(vi)FPNS-int$(Q \cup R) \supseteq$ FPNS-int$(Q) \cup$ FPNS-int(R)
(vii)FPNS-int$(Q \cap R) \subseteq$ FPNS-int$(Q) \cap$ FPNS-int(R)

Proof:

(i)By definition 3.2 FPNS-int(Q) collects every open set lying inside Q. Because each open set is contained in Q, their union is also contained in Q.ThusFPNS-int$(Q) \subseteq Q$. On the other hand, FPNS-cl(Q) is formed by intersecting all closed sets that contain Q.Each of these closed sets includes Q, so the resulting intersection must also contain Q, hence $Q \subseteq FPNS - cl(Q)$.

(ii)Suppose $Q \subseteq R$ every closed set containing Q is automatically a closed set containing R.The intersection defining FPNS-cl(Q). therefore contains the intersection defining FPNS-cl(R),which yields FPNS-cl$(Q) \subseteq$ FPNS-cl(R).

(iii) Suppose $Q \subseteq R$,every open set contained in Q is also contained in R. Consequently, the union forming FPNS-int(Q) is contained in the union forming FPNS-int(R).

(iv) Both Q and R are contained in $Q \cup R$, so $FPNS - cl(Q) \subseteq FPNS - cl(Q \cup R)$ and $FPNS - cl(R) \subseteq FPNS - cl(Q \cup R)$. This shows that their union is contained in FPNS-cl$(Q \cup R)$.

Conversely $Q \cup R$, it is contained in $FPNS - cl(Q) \cup FPNS - cl(R)$, and the latter union is itself a closed set. Since $FPNS - cl(Q \cup R)$ is the smallest set containing $Q \cup R$, we obtain the reverse inclusion equality.

(v-vii) The remaining inclusions are established by applying the containment relations for intersections and unions together with the monotonicity of the interior and closure operators.

Theorem 3.2:
For any FPNSS Q in an FPNS-TS,(X, τ) we have

(i) $(FPNS - int(Q))^c = FPNS - cl(Q^c)$.
(ii) $(FPNS - cl(Q))^c = FPNS - int(Q^c)$

Proof:
(i) The interior of Q is the union of all open sets inside Q. Taking the complement of their union produces the intersection of the corresponding closed sets. Because of the way complement reserves the five membership degrees. this intersection exactly equals the closure of the complement of Q^c

(ii) A symmetric argument, interchanging the roles of interior and closure while using the complement operation, yields the second equality.

Definition3.3:
An FPNSS X in an FPNS-TS(X, τ) is called

(i) a fuzzy pentapartitioned neutrosophic soft semi-open set(FPNS-SO) if $X \subseteq FPNS - cl(FPNS - int(X))$.
(ii) a fuzzy pentapartitioned neutrosophic soft pre-open set(FPNS-PO) if $X \subseteq FPNS - int(FPNS - cl(X))$.

Theorem 3.4:
Every FPNS-OS is both an FPNS-SO set and an FPNS-PO set.

Proof:
Let X be an open set; then FPNS-int(X)=X. Since X is contained in its own closure, it satisfies the condition for being semi-open. Similarly, substituting FPNS-int(X)=X into the pre-open condition shows that X is pre-open.

Remark 3.4:
The converse statements do not hold in general, as demonstrated by the following example.

Example 3.2:
Consider the FPNS-TS of example 3.1 and FPNSS

$Q = \{(p, 0.3,0.7,0.2,0.05,0.05), (q, 0.3,0.7,0.5,0.05,0.05)\}$Direct verification shows$Q \subseteq FPNS - cl(FPNS - int(Q))$. that So Q is semi-open, yet $Q \notin \tau$. likewise the set $P = \{(p, 0.3,0.7,0.2,0.05,0.05), (q, 0.3,0.7,0.5,0.05,0.05)\}$satisfies the pre-open condition, but it is not open.

Definition 3.4:
An FPNSS X is called a fuzzy pentapartitioned Neutrosophic soft αopen set (FPNS-αopen) if $X \subseteq FPNS - int(FPNS - cl(FPNS - int(X))$.

Definition 3.5:
An FPNSS X is called a fuzzy pentapartitioned Neutrosophic soft b-open set(FPNS-b open) if $X \subseteq FPNS - int(FPNS - cl(X)) \cup FPNS - cl(FPNS - int(X))$.

Preposition 3.1:
Every FPNS-OS is an FPNS-α open set.

Proof:
Let F be an arbitrary fuzzy pentapartitioned Neutrosophic soft open set in (X, τ) by the definition of open set $F \in \tau$.

By the definition (3.2), the interior of an open set is the set itself, FPNS-int(F)=F.

Now, substituting this into the definition of αan open set. The condition for F to be the FPNS-α -Open is $F \subseteq FPNS - int(\ FPNS - cl(FPNS - int(\ F))$.

Replacing FPNS-int(F) with F, the RHS becomes

FPNS-int (FPNS-cl(F))

Thus, the α -open condition simplifies to $F \subseteq FPNS - int(\ FPNS - cl(F))$

But we already know that for any set Y

$$\text{FPNS-int}(Y) \subseteq Y$$

Taking $Y = FPNS - cl(F)$ we obtain

$$FPNS - int(\ FPNS - cl(F)) \subseteq FPNS - cl(F)$$

Since F is open, it is also contained in its own closure.

$$F \subseteq FPNS - cl(F)$$

Since F is open, it is also contained in its own closure. $F \subseteq FPNS - cl(F)$

$$\therefore F \subseteq FPNS - cl(F) \supseteq FPNS - int(\ FPNS - cl(F))$$

This shows that $F \subseteq FPNS - int(\ FPNS - cl(F))$ which is precisely the definition of a fuzzy pentapartitioned Neutrosophic soft α -open set.

Hence, every FPNS-OS is an FPNS-αopen set.

Remark 3.2:
The converse of this proposition does not hold.

Theorem 3.7
Every fuzzy pentapartitioned Neutrosophic soft α -open set is a fuzzy pentapartitioned Neutrosophic soft semi-open set.

Proof:
Let F be any fuzzy pentapartitioned Neutrosophic soft α -open set in the FPNS-TS, by definition. $F \subseteq FPNS - int(\ FPNS - cl(FPNS - int(\ F))$

For any fuzzy pentapartitioned Neutrosophic soft set Y, we always have

$$\text{FPNS-int}(Y) \subseteq Y \subseteq \text{FPNS-cl}(Y)$$

Taking $Y = FPNS - cl(FPNS - int(F)) \subseteq FPNS - cl(FPNS - int(F))$ thus $F \subseteq FPNS - cl(FPNS - int(F))$

Which is exactly the definition of a fuzzy pentapartitioned Neutrosophic soft semi-open (FPNS-SO) set.

∴Every FPNS-α open set is an FPNS-semi-open set.

Theorem 3.5:
Every fuzzy pentapartitioned Neutrosophic soft α -open set is a fuzzy pentapartitioned Neutrosophic soft pre-open set.

Proof:
Let F be any fuzzy pentapartitioned Neutrosophic soft α open sets by definition.

$$F \subseteq FPNS - int(FPNS - cl(FPNS - int(F))$$

Since the interior of any set is contained in the set itself.$FPNS - int(F) \subseteq F$

Applying the closure operator $FPNS - cl(FPNS - int(F)) \subseteq FPNS - cl(F)$

Applying the interior operator $FPNS - int(FPNS - cl(FPNS - int(F)) \subseteq FPNS - int(FPNS - cl(F))$

Combining this with the αopen condition yields

$$F \subseteq FPNS - int(FPNS - cl(FPNS - int(F))) \subseteq FPNS - int(FPNS - cl(F))$$

Thus $F \subseteq FPNS - int(FPNS - cl(F))$

Which means F is a fuzzy pentapartitioned Neutrosophic soft pre-open set.

Theorem 3.6:

Every fuzzy pentapartitioned Neutrosophic soft pre-open set and every fuzzy pentapartitioned Neutrosophic soft semi-open set is a fuzzy pentapartitioned Neutrosophic soft b-open set.

Proof:
Let F be a fuzzy pentapartitioned Neutrosophic soft pre-open set by definition,$F \subseteq FPNS - int(FPNS - cl(F)$

Since the union of any set with another set that contains it remains a superset, we have

$$F \subseteq FPNS - int(FPNS - cl(F)) \subseteq FPNS - int(FPNS - cl(F) \cup FPNS - cl(FPNS - int(F))$$

Hence, F satisfies the definition of an open set.

Similarly, if F is a fuzzy pentapartitioned Neutrosophic soft semiopen set, then

$$F \subseteq FPNS - cl(FPNS - int(F)) \subseteq FPNS - int(FPNS - cl(F) \cup FPNS - cl(FPNS - int(F))$$

Thus, every FPNS-pre open set and every FPNS-semi open set is an FPNS-b open set.

Theorem 3.7

The union of any two fuzzy pentapartitioned Neutrosophic soft b-open sets is again a fuzzy pentapartitioned Neutrosophic soft b –open set.

Proof:

Let F &G be two FPNS-b open sets in the FPNS-TS(X, τ), then.

$$F \subseteq FPNS - int(FPNS - cl(F)) \cup FPNS - cl(FPNS - int(F))$$
$$G \subseteq FPNS - int(FPNS - cl(G)) \cup FPNS - cl(FPNS - int(G))$$

Since $F \subseteq F \cup G \& G \subseteq F \cup G$the monotonicity of interior and closure operators gives

$$FPNS - int(FPNS - cl(F) \subseteq FPNS - int(FPNS - cl(F \cup G))$$
$$FPNS - int(FPNS - cl(F) \subseteq FPNS - cl(FPNS - int(F \cup G))$$

And similarly for G.

$$\therefore F \cup G \subseteq FPNS\, int(FPNS - cl(F)) \cup FPNS - cl(FPNS - int(F)) \cup$$
$$(FPNS - int(FPNS - cl(G)) \cup FPNS - cl(FPNS - int(G)) \subseteq$$
$$FPNS - int(FPNS - cl(F \cup G)) \cup FPNS - cl(FPNS - int(F \cup G)$$

Hence $F \cup G$, it is an FPNS-b open set.

4. Conclusion

In this paper, we have introduced the novel concept of fuzzy pentapartitioned Neutrosophic soft topological space by extending the recently proposed fuzzy pentapartitioned Neutrosophic soft set. The notions of interior and closure operators are defined, and their fundamental properties are established. Furthermore, four important classes of generalized open sets — namely, Fuzzy Pentapartitioned Neutrosophic Soft Pre-Open (FPNS-PO), semi-open (FPNS-SO), b-open (FPNS-b-O), and α-open (FPNS-α-O) sets — are introduced and studied in detail. Several illustrative numerical examples are provided to demonstrate the concepts, and key results are proved using the algebraic operations defined on these sets.

The proposed topological framework offers a more refined and flexible tool for handling highly uncertain, inconsistent, and indeterminate information in real-world decision-making problems. This work fills an important research gap in the literature and opens several promising directions for future research, including the study of interval-valued and type-2 fuzzy extensions, compactness and connectedness in these spaces, and applications in multi-criteria decision-making, medical diagnosis, pattern recognition, and risk assessment under complex uncertain environments.

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