

Original Article

# Almost $SC^*$ -Normal Spaces

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**Abstract** - This paper introduces and investigates a new class of almost normal spaces, referred to as almost  $SC^*$ -normal spaces, which are defined using  $SC^*$ -open sets. Building on the work of A. Chandrakala and K. Bala Deepa Arasi, we explore several properties of these spaces within the framework of topology. Moreover, we obtain some new characterizations and preservation theorems of almost  $SC^*$ -normal spaces.

**Keywords** -  $SC^*$ -open,  $SC^*$ -closed sets, Almost- $SC^*$ -irresolute functions, Almost- $SC^*$ -normal spaces.

## 1. Introduction

In this paper, we introduced the concept of almost  $SC^*$ -normal by using  $SC^*$ -open sets due to A. Chandrakala and K. Bala Deepa Arasi [9] obtained several properties of such a space. In 1970, Singal and Arya [8] introduced the concept of almost normal spaces as a generalization of normal spaces by using regularly closed sets and obtained several properties of such a space. Recently, Hamant Kumar and M. C. Sharma [2] introduced a new class of spaces, namely, almost  $\gamma$ -normal and mildly  $\gamma$ -normal spaces, which are a weaker form of  $\gamma$ -normal spaces. In this paper, we explore the properties of almost  $\gamma$ -normal and mildly  $\gamma$ -normal spaces, demonstrating that they are regularly open hereditary. We explore the connections between these spaces and certain weaker variants of almost normal and mildly normal spaces.

Motivated by these developments, the present study extends these ideas by introducing the notion of almost  $SC^*$ -normal spaces within the framework of  $SC^*$ -open sets.

In addition, several related concepts such as  $gSC^*$ -closed sets,  $rgSC^*$ -closed sets, regularly  $SC^*$ -closed sets,  $T-SC^*$ -closed sets,  $T-SC^*$ -open sets, and almost  $SC^*$ -irresolute functions are introduced and analyzed. Various characterizations and preservation theorems associated with these notions are also established, particularly in relation to almost  $SC^*$ -normal spaces and their interaction with other generalized forms of normality.

Throughout this work,  $(X, \tau)$  and  $(Y, \sigma)$  denote topological spaces, with no specific separation axioms assumed unless explicitly stated.

## 2. Preliminaries and Notations

In the following discussion, all spaces refer to topological spaces without any assumed separation axioms unless explicitly stated.

A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  (or simply  $f : X \rightarrow Y$ ) represents a mapping between two topological spaces  $(X, \tau)$  and  $(Y, \sigma)$ . For any subset  $A$  of a topological space  $X$ . The closure and the interior of  $A$  are denoted by  $\bar{A}$  and  $A^\circ$ , respectively.

**2.1. Definition:** A subset  $A$  of a topological space  $X$  is said to be

- (1) **regular open (r-open)** [9] if  $A = \text{int}(\text{cl}(A))$ .
- (2) **semi-open (s-open)** [3] if  $A \subset \text{cl}(\text{int}(A))$ .
- (3)  **$\alpha$ -open** [7] if  $A \subset (\text{int}(\text{cl}(\text{int}(A))))$ .
- (4)  **$ga$ -closed** [5] if  $\alpha\text{-cl}(A) \subset U$ , whenever  $A \subset U$ , and  $U$  is  $\alpha$ -open in  $X$ .
- (5)  **$rga$ -closed** [11] if  $\alpha\text{-cl}(A) \subset U$ , whenever  $A \subset U$ , and  $U$  is regularly  $\alpha$ -open in  $X$ .
- (6)  **$c^*$ -open** [6] if  $\text{int}(\text{cl}(A)) \subset A \subset \text{cl}(\text{int}(A))$ .



The complement of a r-open set is referred to as a r-closed set, while the complements of s-open and c\*-open sets are called s-closed and c\*-closed sets, respectively.

For any subset A of a topological space, the c\*-closure (resp. s-closure,  $\alpha$ -closure,  $g\alpha$ -closure,  $rg\alpha$ -closure) of A is defined as the intersection of all c\*-closed (resp. s-closed,  $\alpha$ -closed,  $g\alpha$ -closed,  $rg\alpha$ -closed) sets that contain A and is denoted by  $c^*-A$  (resp.  $s-\bar{A}$ ,  $\alpha-\bar{A}$ ,  $g\alpha-\bar{A}$ ,  $rg\alpha-\bar{A}$ ). Similarly, the c\*-interior (resp. s-interior,  $\alpha$ -interior,  $g\alpha$ -interior,  $rg\alpha$ -interior) of A is defined as the union of all c\*-open (resp. s-open,  $\alpha$ -open,  $g\alpha$ -open,  $rg\alpha$ -open) sets contained in A. This is denoted by  $c^*-A^\circ$  (resp.  $s-A^\circ$ ,  $\alpha-A^\circ$ ,  $g\alpha-A^\circ$ ,  $rg\alpha-A^\circ$ ).

**2.2. Definition.** A subset A of a space X is said to be SC\*-closed [1] if  $s-cl(A) \subset U$  whenever  $A \subset U$  and U is c\*-open in X. The complement of an SC\*-closed set is said to be SC\*-open. In a topological space, the SC\*-closure of a subset A is the smallest SC\*-closed set that includes A. It is represented as  $SC^*-\bar{A}$  and is obtained by taking the intersection of all SC\*-closed sets that contain A. Likewise, the SC\*-interior of a subset A, denoted as  $SC^*-A^\circ$ , is defined as the largest SC\*-open set contained within A. It is obtained by taking the union of all SC\*-open sets that are subsets of A.

The collection of all SC\*-open sets in a topological space X is represented by  $SC^*O(X)$ , while the families of SC\*-closed, r-open, r-closed, s-open, and s-closed sets are denoted by  $SC^*C(X)$  (resp.  $R-O(X)$ ,  $R-C(X)$ ,  $S-O(X)$ ,  $S-C(X)$ ).

**2.3. Definition.** A subset A of a topological space X is said to be  
 (1) **g-closed** [4] if  $cl(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ .  
 (2) **generalized SC\*-closed** [10] (briefly **gSC\*-closed**) if  $SC^*-cl(A) \subset U$  whenever  $A \subset U$  &  $U \in \tau$ .  
 (3) **SC\*generalized-closed** [10] (briefly **SC\*g-closed**) if  $SC^*-cl(A) \subset U$  whenever  $A \subset U$  &  $U \in SC^*O(X)$ .  
 (4) **regularly SC\*-open** if there is a regularly open set U such that  $U \subset A \subset SC^*-cl(U)$ .  
 (5) **rgSC\*-closed** if  $SC^*-cl(A) \subset U$ , whenever  $A \subset U$ , and U is regularly SC\*-open in X.

The complement of a g-closed set in a topological space is referred to as a g-open set. Similarly, the complements of gSC\*-closed, SC\*g-closed, & rgSC\*-closed sets are called gSC\*-open, SC\*g-open, & rgSC\*-open sets, respectively.

The complement of a regularly SC\*-open set is a regularly SC\*-closed set.

**2.4. Remark.** The following relationships hold for different types of subset properties in a topological space:

$$\begin{array}{c} \text{closed} \Rightarrow SC^*\text{-closed} \Leftrightarrow SC^*\text{g-closed} \Leftrightarrow gSC^*\text{-closed} \\ \Downarrow \\ \text{g-closed} \end{array}$$

This representation outlines the logical implications between these closure properties.

**2.5. Remark.** The following relationships hold for different types of subset properties in a topological space:

$$\begin{array}{ccccc} \text{closed} & \Rightarrow & \alpha\text{-closed} & \Rightarrow & g\alpha\text{-closed} & \Rightarrow & rg\alpha\text{-closed} \\ \Downarrow & & \Downarrow & & \Downarrow & & \\ SC^*\text{-closed} & & gSC^*\text{-closed} & & rgSC^*\text{-closed} & & \end{array}$$

This representation outlines the logical implications between these closure properties.

**2.6. Example.** Consider the set  $X = \{i, j, k, l\}$  with the topology  $\tau = \{\phi, \{i\}, \{j\}, \{i, j\}, \{i, j, k\}, X\}$ . The subset  $A = \{k\}$  is  $\alpha$ -closed and SC\*-closed but may not necessarily be closed in the topological space X.

**2.7. Example.** Consider  $X = \{i, j, k, l\}$  with the topology  $\tau = \{\phi, \{i\}, \{j\}, \{i, j\}, \{i, j, k\}, X\}$ . The set  $A = \{k\}$  is both  $\alpha$ -closed and gSC\*-closed, but it is not a closed set in X.

**2.8. Example.** Consider the set  $X = \{i, j, k, l\}$  with the topology

$$\tau = \{\phi, \{j, l\}, \{i, j, l\}, \{j, k, l\}, X\}.$$

Then

- (1) closed sets in  $X$  are  $\phi, X, \{i\}, \{k\}, \{i, k\}$ .
- (2)  $g$ -closed set in  $X$  are  $\phi, X, \{i\}, \{k\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}$ .
- (3)  $SC^*$ -closed set in  $X$  are  $\phi, X, \{i\}, \{j\}, \{k\}, \{l\}, \{i, j\}, \{i, k\}, \{i, l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{i, j, k\}, \{i, j, l\}, \{i, k, l\}, \{j, k, l\}$ .
- (4)  $gSC^*$ -closed set in  $X$  are  $\phi, X, \{i\}, \{j\}, \{k\}, \{l\}, \{i, j\}, \{i, k\}, \{i, l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{i, j, k\}, \{i, j, l\}, \{i, k, l\}, \{j, k, l\}$ .
- (5)  $SC^*g$ -closed set in  $X$  are  $\phi, X, \{i\}, \{j\}, \{k\}, \{l\}, \{i, j\}, \{i, k\}, \{i, l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{i, j, k\}, \{i, j, l\}, \{i, k, l\}, \{j, k, l\}$ .

### 3. Almost $SC^*$ -Normal Spaces

**3.1. Definition.** A topological space  $X$  is said to be **almost normal** [8] (resp. **almost  $SC^*$ -normal**) if for every pair of disjoint closed sets  $A$  &  $B$ , one of which is closed and the other is regularly closed, there exist disjoint open (resp.  $SC^*$ -open) sets  $U$  &  $V$  of  $X$  s.t.

$$A \subset U \text{ \& } B \subset V.$$

**3.2. Example.** Let  $X = \{i, j, k, l\}$  and  $\tau = \{\phi, \{i\}, \{j\}, \{i, j\}, \{k, l\}, \{i, k, l\}, \{j, k, l\}, X\}$ . In this space, the subset  $A = \{j\}$  is closed, while the subset  $B = \{i\}$  is  $r$ -closed. There are separate open sets  $U = \{j, k, l\}$  &  $V = \{i\}$  where  $A$  is contained in  $U$  &  $B$  is contained in  $V$ , ensuring they remain disjoint. Thus,  $X$  satisfies the conditions for being almost normal. Moreover, since every open set in  $X$  is also  $SC^*$ -open, the space is almost  $SC^*$ -normal.

A normal space implies an almost normal space, which in turn implies an almost  $SC^*$ -normal space. This relationship can be represented as follows:

$$\text{normal} \Rightarrow \text{almost-normal} \Rightarrow \text{almost } SC^*\text{-normal}$$

**3.3. Lemma.** If  $A$  is a subset of a topological space  $X$  is  $rgSC^*$ -open iff for every regularly closed set  $F$  with  $F \subset A$ , it follows that  $F \subset SC^*-A^\circ$ .

**3.4. Theorem.** The following conditions are equivalent for any topological space  $X$ :

- (1)  $X$  is almost  $SC^*$ -normal,
- (2) If  $I$  is a closed set and  $J$  is a regularly closed set, then there exist disjoint  $gSC^*$ -open sets  $M$  &  $N$  such that  $I \subset M$  &  $J \subset N$ .
- (3) If  $I$  is a closed set and  $J$  is a regularly closed set, then there exist disjoint  $rgSC^*$ -open sets  $M$  &  $N$  such that  $I \subset M$  &  $J \subset N$ .
- (4) If  $I$  is a closed set and  $J$  is a regularly open set containing  $I$ , then there exists a  $gSC^*$ -open set  $M$  satisfying  $I \subset M \subset SC^*-\bar{I} \subset J$ .
- (5) If  $I$  is a closed set and  $J$  is a regularly open set containing  $I$ , then there exists an  $rgSC^*$ -open set  $M$  satisfying  $I \subset U \subset SC^*-\bar{I} \subset J$ .
- (6) If  $I$  &  $J$  are disjoint sets, where one is closed, and the other is regularly closed, then there exist disjoint  $SC^*$ -open sets  $M$  &  $N$  such that  $I \subset M$  &  $J \subset N$  and  $M \cap N = \phi$ .

**Proof.** We will prove the implications step by step.

(1)  $\Rightarrow$  (2), (2)  $\Rightarrow$  (3), (4)  $\Rightarrow$  (5), (3)  $\Rightarrow$  (4), (5)  $\Rightarrow$  (6), (6)  $\Rightarrow$  (1).

(1)  $\Rightarrow$  (2). Assume that  $X$  is almost  $SC^*$ -normal. Let  $I$  be a closed set, and  $J$  be a regularly closed set in  $X$ . By the definition of almost  $SC^*$ -normality, there exist disjoint  $SC^*$ -open sets  $M$  &  $N$  such that  $I \subset M$  &  $J \subset N$ . Since every  $SC^*$ -open set is also  $gSC^*$ -open, it follows that  $M$  &  $N$  are  $gSC^*$ -open sets, satisfying the required condition  $I \subset M$  &  $J \subset N$ .

(2)  $\Rightarrow$  (3). Let  $I$  be a closed set, and  $J$  be a regularly closed set in  $X$ . By the assumption in (2), there exist disjoint  $gSC^*$ -open sets  $M$  &  $N$  such that  $I \subset M$  &  $J \subset N$ . Since every  $gSC^*$ -open set is also an  $rgSC^*$ -open set, it follows that  $M$  and  $N$  are  $rgSC^*$ -open sets. Thus, we still have,  $I \subset M$  &  $J \subset N$ , satisfying the condition in (3).

(4)  $\Rightarrow$  (5). Let  $I$  be a closed set, and  $J$  be a regularly open set such that  $I \subset J$ . By the assumption in (4), there exists a  $gSC^*$ -open set  $M$  in  $N$  satisfying:  $I \subset M \subset SC^*-\bar{I} \subset J$ . Since every  $gSC^*$ -open set is also an  $rgSC^*$ -open set, the set  $M$  retains the same containment properties while being  $rgSC^*$ -open. Thus, there exists an  $rgSC^*$ -open set  $M$  such that:

$$I \subset M \subset SC^*-\bar{I} \subset J. \text{ This confirms the validity of (5).}$$

(3)  $\Rightarrow$  (4). Let  $I$  be a closed set, and  $J$  be a regularly open set such that  $I \subset J$ . By the assumption in (3), there exist disjoint  $rgSC^*$ -

open sets  $M$  &  $N$  s.t.  $I \subset M$  and  $X - J \subset N$ . From Lemma 3.3, we know that:  $X - J \subset SC^* - N^\circ$ . Additionally, the property

$$SC^* - \bar{M} \cap SC^* - N^\circ = \phi$$

holds. Using these facts, we conclude:  $I \subset M \subset SC^* - \bar{M} \subset X - SC^* - N^\circ \subset J$  Thus, the statement in (4) follows.

(5)  $\Rightarrow$  (6). Consider a closed set  $I$  and a regularly open set  $J$  that contains  $I$ . This implies that

the complement  $X - J$  is a regularly closed set. By the assumption in (5), there exists an rgSC<sup>\*</sup>-open set  $K$  in  $X$  such that:

$I \subset K \subset SC^* - \bar{K} \subset X - J$ . Define:  $M = SC^* - K^\circ$ ,  $N = X - SC^* - \bar{K}$ , Since  $M$  &  $N$  are disjoint SC<sup>\*</sup>-open sets in  $X$ , it follows that  $I \subset M \& J \subset N$ , as required.

(6)  $\Rightarrow$  (1). This follows directly from the definition of almost SC<sup>\*</sup>-normality, making the implication straightforward.

**3.5. Definition.** A function  $f: X \rightarrow Y$  is called **rc-continuous** [10] if for each regular closed set  $F$  in  $Y$ ,  $f^{-1}(F)$  is regularly closed in  $X$ .

**2.6. Definition.** A function  $f: X \rightarrow Y$  is termed **T-SC<sup>\*</sup>-open** (resp. **T-SC<sup>\*</sup>-closed**) if the image of every SC<sup>\*</sup>-open (resp. SC<sup>\*</sup>-closed) set in  $X$  remains SC<sup>\*</sup>-open (resp. SC<sup>\*</sup>-closed) in  $Y$ .

**2.7. Definition.** A function  $f: X \rightarrow Y$  is said to be **almost SC<sup>\*</sup>-irresolute** if, for every point  $x \in X$  and for any SC<sup>\*</sup>-neighborhood  $N$  of  $f(x)$ , the SC<sup>\*</sup>-closure of the preimage  $f^{-1}(N)$  serves as an SC<sup>\*</sup>-neighborhood of  $x$ .

## 4. Preservation Theorems

**4.1. Theorem.** Let  $f: X \rightarrow Y$  be a surjective function that is continuous, T-SC<sup>\*</sup>-open, rc-continuous, and almost SC<sup>\*</sup>-irresolute. If  $X$  is an almost SC<sup>\*</sup>-normal space, then  $Y$  is almost-SC<sup>\*</sup>-normal.

**Proof.** Let  $I$  be a closed set in  $Y$  and  $J$  be a regularly open set s.t.  $I \subset J$ . Since  $f$  is rc-continuous, the preimage  $f^{-1}(I)$  is a closed set in  $X$ , and it is contained in the regularly open set  $f^{-1}(J)$ . Since  $X$  is almost-SC<sup>\*</sup>-normal, Theorem 2.4, ensures the existence of an SC<sup>\*</sup>-open set  $N$  in  $X$  s.t.  $f^{-1}(I) \subset N \subset SC^* - cl(N) \subset f^{-1}(J)$  Applying  $f$  to both sides, we obtain:

$$f(f^{-1}(I)) \subset f(N) \subset f(SC^* - cl(N)) \subset f(f^{-1}(J)).$$

Since  $f$  is a T-SC<sup>\*</sup>-open and almost-SC<sup>\*</sup>-irresolute surjection, it follows that

$$f(N) \in SC^*O(Y), \text{ Thus, we conclude:}$$

$$I \subset f(N) \subset SC^* - cl(f(N)) \subset J.$$

By **Theorem 3.4**, this confirms that  $Y$  is almost SC<sup>\*</sup>-normal, completing the proof.

**4.2. Theorem.** If  $f: X \rightarrow Y$  is an rc-continuous and T-SC<sup>\*</sup>-closed mapping from an almost-SC<sup>\*</sup>-normal space  $X$  onto  $Y$ , then  $Y$  is also almost-SC<sup>\*</sup>-normal.

**Proof.** The proof follows directly from the definitions and properties of rc-continuity, T-SC<sup>\*</sup>-closed maps, and almost-SC<sup>\*</sup>-normal spaces. By applying these properties, it is straightforward to verify that  $Y$  inherits the almost SC<sup>\*</sup>-normality from  $X$ .

## 5. Conclusion

The concept of almost SC<sup>\*</sup>-normal spaces has been introduced as a generalization of almost normal spaces using SC<sup>\*</sup>-open sets. Fundamental properties, characterizations, and preservation results have been established. The study also examines related notions such as generalized SC<sup>\*</sup>-closed sets and their interactions with the proposed class. The results extend existing work in generalized topology and provide a broader framework for separation axioms. Further research may explore additional properties and applications of SC<sup>\*</sup>-based structures.

## Conflicts of Interest

The authors declare that there is a potential conflict of interest related to this study, which has been fully disclosed.

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