International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I5P111
Volume 72 Issue 5, 113-124, May 2026

Original Article

Neutrosophic Semi-Open and Semi-Closed SuperHyperSoft Sets with Application to MAGDM

A. Saranya[1,2]*, A. Kalavathi[1]

[1]*Department of Mathematics, Sri GVG Visalakshi College for Women, India.*
[2]*Department of Mathematics, Dr. Mahalingam College of Engineering and Technology, India.*

**Corresponding Author : saranyasaraswathy@gmail.com*

Received: 02 April 2026 Revised: 03 May 2026 Accepted: 18 May 2026 Published: 30 May 2026

Abstract *- Neutrosophic Semi-Open SuperHyperSoft Sets (NSOSHS) and Neutrosophic Semi-Closed SuperHyperSoft Sets (NSCSHS) are two new concepts in Neutrosophic SuperHyperSoft Topological Spaces that this paper introduces. Both of these concepts extend the classes of classical hypersoft and neutrosophic theories to include multi-parameterized semi-topological environments. The fundamental properties, characterizations, and structural relations of NSOSHS and NSCSHS are established, as well as their interactions under union, intersection, interior, and closure operations.*

Additionally, the authors define and study new operators (namely, neutrosophic semi-interior and neutrosophic semi-closure) and their algebraic and topological properties. A number of theorems are proven in this paper to illustrate the theoretical consistency and robustness of the proposed framework for decision-making. Finally, the authors demonstrate the practical application of the developed theory in a Multi-Attribute Group Decision Making (MAGDM) scenario for resource allocation in a smart hospital. Therefore, neutrosophic uncertainty combined with hierarchical parameterization creates a powerful and flexible tool for exploring undetermined and/or inconsistent information in complex decision-making systems.

Keywords *- Neutrosophic Sets, Soft Set Theory, Neutrosophic Semi-Open SuperHyperSoft Set (NSOSHS), Neutrosophic Semi-Closed SuperHyperSoft Set (NSCSHS), Neutrosophic SuperHyperSoft Topological Space (NSHSTS).*

1. Introduction

Currently, the need to represent uncertainty and vagueness in mathematical and decision-making systems, regardless of any constraints, is becoming increasingly important. Classical set theory and fuzzy sets generally prove inadequate for incomplete, inconsistent, or indeterminate data. Neutrosophic sets were developed to provide a more general way to model uncertainty by defining three separate membership functions for truth, indeterminacy, and falsity that exist independently from each other.

Simultaneously, hypersoft sets have evolved as a major extension of soft sets that allow for approximate functions with multiple arguments to be defined in terms of a set of parameters created from disjoint attribute-parameter value pairs. This will enable more precise modeling of complex systems, where the result of a decision depends on the combination of a number of attributes, rather than the single measurement of one attribute. Integration of neutrosophic logic with hypersoft structures has resulted in the creation of Neutrosophic Hypersoft Sets, thereby further improving our ability to manage uncertainty in multi-criteria environments.

A new concept, "SuperHyperSoft Sets," was recently proposed to include hierarchical and multilevel parameterization. This extension of hypersoft sets extends the representation of complex real-world problems requiring the consideration of multiple levels of attributes. The combination of neutrosophic logic with SuperHyperSoft Sets leads to the development of Neutrosophic SuperHyperSoft Sets, which constitute an integrated approach to solving uncertain problems in multi-dimensional decision spaces.

Despite their considerable developments, no one has really looked into the topological aspects of Neutrosophic SuperHyperSoft Sets, especially with regard to semi-open and semi-closed. Semi-open and semi-closed sets are significant in general topology because they are "between" the open and closed sets and can provide a finer examination of the characteristics of continuity, convergence, and boundaries.

Because of this lack of exploration, we define and study Neutrosophic Semi-Open SuperHyperSoft Sets (NSOSHS) and Neutrosophic Semi-Closed SuperHyperSoft Sets (NSCSHS) in the context of Neutrosophic SuperHyperSoft Topological Spaces, as well as identify their fundamental properties, establish key theorems, and examine their relationship with the standard interior and closure operators.

This paper also presents an application of NSOSHS and NSCSHS to a practical problem, specifically in the area of Multi-Attribute Group Decision Making (MAGDM) and their application in resource allocation in smart hospitals. To assist in identifying the best possible outcomes for resource allocation, our model uses a hierarchical attribute structure and neutrosophic evaluation to manage uncertainty and determine the optimal response in critical healthcare situations.

Our findings will advance knowledge in Neutrosophic Topology and pave the way for future research in intelligent decision systems, data analysis, and applied mathematical modelling under uncertainty.

1.1. Literature Review

In fuzzy set theory, Zadeh established the foundation of uncertainty modeling by creating a membership function to describe indeterminate information. Atanassov later expanded upon fuzzy sets to create intuitionistic fuzzy sets by using degree of membership and degree of non-membership.

The development of uncertainty theory received an important boost from the introduction of neutrosophic sets by Smarandache. In neutrosophic sets, memberships represent truth and falsehood are independent from each other, while both indeterminacy and membership can be defined with respect to a variable.

Subsequently, Wang et al. introduced single-valued neutrosophic sets and interval neutrosophic sets, which are potential models for the real world. The introduction of soft set theory by Molodtsov also created a framework for representing uncertain information based on parameters. After this, Maji et al. contributed by extending soft sets with fuzzy soft sets and intuitionistic fuzzy soft sets, while Deli and Broumi built upon these concepts to develop neutrosophic soft sets and neutrosophic soft relations.

In addition, the extension of soft set theory known as hypersoft set theory provides ways to describe multidimensional and parametric information. Hypersoft set theory is an extension of soft set theory and was first introduced by Smarandache, who has proposed the further development of hypersoft set theory with plithogenic hypersoft structures as a representation for very complex uncertain systems. Ali and Smarandache introduced Neutrosophic Hypersoft Sets and researched their uses in solving decision-making problems.

Finally, Ali, Deli, and Smarandache pioneered the concept of SuperHyperSoft Sets, which represent hierarchical extensions of hypersoft sets that can be used to model multi-scale, parameterized environments. These developments have significantly improved the representation of very complex uncertain systems that have multiple layers of attributes and can also assist in evaluating alternative choices.

Various researchers have studied neutrosophic topology, soft and hypersoft varieties of topological structures, and generalized open/closed sets. Classic/topological approaches to the topic of semi-open/closed sets have been widely applied, as they are important for analyzing continuity and compactness.

Ajay et al. introduced the concept of neutrosophic semi-open hypersoft sets and demonstrated their use in MAGDM-related applications during COVID-19. However, their work is limited to exploring hypersoft structures and did not explore hierarchical SuperHyperSoft parameterizations.

This creates a large research gap on the development of Neutrosophic Semi-Open and Semi-Closed SuperHyperSoft Sets, along with their application within intelligent decision-making systems. The current study fills this gap through rigorous definitions, detailed algebraic and topological properties, illustrative examples and proofs, and an application to MAGDM-smart hospitals' resource allocations.

2. Preliminaries

Definition:2.1

Let (X,τ) be a topological space and $A \subseteq X$

Then A is called a semi-open set if $A \subseteq cl\big(int(A)\big)$.

Definition:2.2

Let (X,τ) be a topological space and $A \subseteq X$

Then A is called a semi-closed set if $cl\big(int(A)\big) \subseteq A$.

Definition 3.3

Let us define a Universal Set called $\mathbb{U}$, and the Power Set of $\mathbb{U}$ is represented by the notation $P(\mathbb{U})$, and for $n \geq 1$, there are n distinct attributes such as $k_1, k_2, \ldots, k_n$ and$K_1, K_2, \ldots, K_n$ are sets for corresponding values, attributes respectively, with conditions such as $K_i \cap K_j = \emptyset$ $(i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair (F, Λ) is NHSS over $\mathbb{U}$ if there exists a relation. $K_1 \times K_2 \times \ldots \times K_n) = \Lambda$. F maps from $K_1 \times K_2 \times \ldots \times K_n)$ to $P(\mathbb{U})$ and $F(K_1 \times K_2 \times \ldots \times K_n)) = \{< u, u_A(u), v_A(u), w_A(u) >: u \in \mathbb{U}\}$ where u , v , w are membership values that represent truth, unclear, and false, respectively. Thus u, v, w: $\mathbb{U} \rightarrow]0^-, 1^+[$ and $0^- \leq u_A(u), +v_A(u) + w_A(u) \leq 3^+$.

Definition 3.4

Let Δ be a universe of discourse, P($\mathbb{U}$) the powerset of $\mathbb{U}$, and $k_1, k_2, \ldots \ldots, k_n$ (n ≥ 1) distinct attributes with disjoint corresponding sets $K_1, K_2, \ldots \ldots, K_n$. Let P($K_i$) denote the powerset of K_i for i ∈ {1, 2, . . . , n}.

A Neutrosophic SuperHyperSoft Set (NSHSS) is defined as a pair (S, P(K_1) × P(K_2) × · · · × P(K_n)), over the universe of discourse U, with S is a mapping from P(K_1) × P(K_2) × · · · × P(K_n)→ P($\mathbb{U}$), and

S = { $\alpha, \langle x, T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x) \rangle$: x ∈ $\mathbb{U}$, α ∈ P(K_1) × P(K_2) × · · · × P(K_n)}.

The definitions, $T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x) : \mathbb{U} \rightarrow [0, 1]$ represent the membership, indeterminacy, and non-membership degrees respectively, of x ∈ $\mathbb{U}$ for each α, satisfying $0 \leq \Big(T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x)\Big) \leq 3$.

3. Neutrosophic Semi-Open SuperHyperSoft Set – NSOSHS

Definition: 3.1 Let Δ be a universe of discourse, P($\mathbb{U}$) the powerset of $\mathbb{U}$, and $k_1, k_2, \ldots \ldots, k_n$ (n ≥ 1) distinct attributes with disjoint corresponding sets $K_1, K_2, \ldots \ldots, K_n$. Let *P($K_i$)* denote the powerset of K_i for i ∈ {1, 2, . . . , n}.

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space ($NSHSTS$) and let $(O,J) \in NSHSS(X)$, where a Neutrosophic SuperHyperSoft Set is defined as

$(O,J) \in (S, P(K_1) \times P(K_2) \times \cdots \times P(K_n))$ with $S : P(K_1) \times P(K_2) \times \cdots \times P(K_n) \rightarrow P(\mathbb{U})$. Then (O,J) is called a Neutrosophic Semiopen SuperHyperSoft Set $NSOSHS(X)$ if

$(O,J) \subseteq NSHcl\big(NSHint(O,J)\big)$. The collection of all such sets is denoted by $NSOSHS(X)$.

Example: 3.1

Let $\Delta = \{x_1, x_2\}$, be the universe of discourse. Let the attribute sets be $K_1 = \{a_1\}$ and $K_2 = \{b_1\}$. Then

$P(K_1) = \{\emptyset, \{a_1\}\}, P(K_2) = \{\emptyset, \{b_1\}\}$ and the parameter space is given by $J = P(K_1) \times P(K_2)$

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space($NSHSTS$) where, $\tau = \{\emptyset, X, A\}$

Define a neutrosophic SuperHyperSoft open set A as $A = \begin{cases} x_1 \rightarrow (0.7, 0.2, 0.1) \\ x_2 \rightarrow (0.5, 0.3, 0.2) \end{cases}$

Define the set (O,J)

$$(O,J) = \begin{cases} x_1 \rightarrow (0.6, 0.2, 0.1) \\ x_2 \rightarrow (0.3, 0.3, 0.2) \end{cases}$$

The neutrosophic SuperHyperSoft interior of (O,J) being the largest open set contained in (O,J) is given by

$$NSHint(O,J) = \begin{cases} x_1 \to (0.6, 0.2, 0.1) \\ x_2 \to (0.3, 0.3, 0.2) \end{cases}$$

The neutrosophic SuperHyperSoft closure of the interior is

$$NSHcl\big(NSHint(O,J)\big) = \begin{cases} x_1 \to (0.7, 0.2, 0.1) \\ x_2 \to (0.5, 0.3, 0.2) \end{cases}$$

Therefore $(O,J) \subseteq NSHcl\big(NSHint(O,J)\big)$.

Definition: 3.2 *Neutrosophic Semi-Closed SuperHyperSoft Set – NSCSHS*
Let Δ be a universe of discourse, P(𝕌) the powerset of 𝕌 , and $k_1, k_2, \dots\dots, k_n$ (n ≥ 1) distinct attributes with disjoint corresponding sets $K_1, K_2, \dots\dots, K_n$. Let $P(K_i)$ denote the powerset of K_i for i ∈ {1, 2, . . . , n}.
Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHSTS)$. Let $(O,J) \in NSHSS(X)$, where a Neutrosophic SuperHyperSoft Set is defined as
$(O,J) \in (S, P(K_1) \times P(K_2) \times \cdots \times P(K_n))$ with $S : P(K_1) \times P(K_2) \times \cdots \times P(K_n) \to P(\mathbb{U})$. Then (O,J) is called a Neutrosophic Semiclosed SuperHyperSoft Set $NSCSHS(X)$ if
$NSHcl\big(NSHint(O,J)\big) \subseteq (O,J)$. The collection of all such sets is denoted by $NSCSHS(X)$.

Example: 3.2 Let $\Delta = \{x_1, x_2\}$, be the universe of discourse. Let the attribute sets be $K_1 = \{a_1\}$ and $K_2 = \{b_1\}$. Then $P(K_1) = \{\emptyset, \{a_1\}\}$, $P(K_2) = \{\emptyset, \{b_1\}\}$ and the parameter space is given by $J = P(K_1) \times P(K_2)$

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space$(NSHSTS)$ where, $\tau = \{\emptyset, X, A\}$
Define a neutrosophic SuperHyperSoft open set A as $A = \begin{cases} x_1 \to (0.7, 0.2, 0.1) \\ x_2 \to (0.5, 0.3, 0.2) \end{cases}$
Define the set (O,J)

$$(O,J) = \begin{cases} x_1 \to (0.8, 0.2, 0.1) \\ x_2 \to (0.6, 0.3, 0.2) \end{cases}$$

The interior is the biggest neutrosophic open set contained in (O,J)

$$NSHint(O,J) = \begin{cases} x_1 \to (0.7, 0.2, 0.1) \\ x_2 \to (0.5, 0.3, 0.2) \end{cases}$$

$$NSHcl\big(NSHint(O,J)\big) = \begin{cases} x_1 \to (0.7, 0.2, 0.1) \\ x_2 \to (0.5, 0.3, 0.2) \end{cases}$$

Therefore $NSHcl\big(NSHint(O,J)\big) \subseteq (O,J)$.

Theorem:3.1
Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHTS)$and $(O,J) \in NSOSHS(X)$. Then
(i) The arbitrary union of Neutrosophic SuperHyperSoft open sets is also a Neutrosophic SuperHyperSoft open set.
(ii) The arbitrary intersection of Neutrosophic SuperHyperSoft closed sets is also a Neutrosophic SuperHyperSoft closed set.

Proof:
(i) Let $\{(O,J)_j : j \in \mathcal{J}\} \subseteq NSOSHS(X)$

Then for each $j \in \mathcal{J}$,

$(O,J)_j \subseteq NSHcl\left(NSHint\big((O,J)_j\big)\right)$

Taking a union on both sides

$\cup_j (O,J)_j \subseteq \cup_j NSHcl\left(NSHint\big((O,J)_j\big)\right)$

Hence, $\cup_j NSHcl\left(NSHint\big((O,J)_j\big)\right) \subseteq \cup_j NSHcl\left(NSHint\big(\cup_j (O,J)_j\big)\right)$

Thus $\cup_j (O,J)_j \subseteq \cup_j NSHcl\left(NSHint\big(\cup_j (O,J)_j\big)\right)$

Hence $\cup_j (O,J)_j \in NSOSHS(X)$.

(ii) Let $\{(O,J)_j : j \in \mathcal{J}\} \subseteq NSCSHS(X)$
Then $NSHcl\big((O,J)_j\big) = (O,J)_j$

Now $NSHcl(\cap_j (O,J)_j) = \cap_j NSHcl((O,J)_j) = \cap_j (O,J)_j$
Thus, $\cap_j (O,J)_j \in NSCSHS(X)$.

Theorem:3.2
Let (X,τ) be **a** Neutrosophic SuperHyperSoft Topological Space ($NSHTS$)and $(O,J) \in NSOSHS(X)$. Then
(i) $(O,J) \in NSOSHS(X)$ if and only if there exists $(\aleph,\mathcal{B}) \in \tau$ such that $(\aleph,\mathcal{B}) \subseteq (O,J) \subseteq NSHcl(\aleph,\mathcal{B})$.
(ii) $(O,J) \in NSOSHS(X)$ and $(O,J) \subseteq (\mathfrak{H},\mathcal{C}) \subseteq NSHcl(O,J)$ then $(\mathfrak{H},\mathcal{C}) \in NSOSHS(X)$.

Proof:
(i) Let $(O,J) \in NSOSHS(X)$ then $(O,J) \subseteq NSHcl(NSHint(O,J))$

We know that $NSHint(O,J) \subseteq (O,J)$
Then $NSHint(O,J) \subseteq (O,J) \subseteq NSHcl(NSHint(O,J))$
Let $(\aleph,\mathcal{B}) = NSHint(O,J)$
Then $(\aleph,\mathcal{B}) \in \tau$ and hence $(\aleph,\mathcal{B}) \subseteq (O,J) \subseteq NSHcl(\aleph,\mathcal{B})$.
Conversely, assume there exists $(\aleph,\mathcal{B}) \in \tau$ such that $(\aleph,\mathcal{B}) \subseteq (O,J) \subseteq NSHcl(\aleph,\mathcal{B})$
Since $(\aleph,\mathcal{B})$ is open $(\aleph,\mathcal{B}) \subseteq (O,J) \subseteq NSHint(O,J)$
Thus, $NSHcl(\aleph,\mathcal{B}) \subseteq NSHcl(NSHint(O,J))$
Hence $(O,J) \subseteq NSHcl(NSHint(O,J))$
Therefore $(O,J) \in NSOSHS(X)$.

(ii) Let $(O,J) \in NSOSHS(X)$ Then there exists $(\aleph,\mathcal{B}) \in \tau$ such that $(\aleph,\mathcal{B}) \subseteq (O,J) \subseteq NSHcl(\aleph,\mathcal{B})$
Given $(O,J) \subseteq (\mathfrak{H},\mathcal{C}) \subseteq NSHcl(O,J)$
Then $(\aleph,\mathcal{B}) \subseteq (\mathfrak{H},\mathcal{C}) \subseteq NSHcl(O,J) \subseteq NSHcl(\aleph,\mathcal{B})$
Thus, $(\aleph,\mathcal{B}) \subseteq (\mathfrak{H},\mathcal{C}) \subseteq NSHcl(\aleph,\mathcal{B})$
By part (i) $(\mathfrak{H},\mathcal{C}) \in NSOSHS(X)$.

Definition: 3.3 Let Δ be a universe of discourse, P($\mathbb{U}$) the powerset of $\mathbb{U}$, and $k_1, k_2, \dots\dots, k_n$ (n ≥ 1) distinct attributes with disjoint corresponding sets $K_1, K_2, \dots\dots, K_n$. Let *P($K_i$)* denote the powerset of K_i for i ∈ {1, 2, . . . , n}.
Let (X,τ) be **a** Neutrosophic SuperHyperSoft Topological Space ($NSHSTS$) and let $(O,J) \in NSHSS(X)$, where a Neutrosophic SuperHyperSoft Set is defined as
$(O,J) \in (S, P(K_1) \times P(K_2) \times \cdots \times P(K_n))$ with $S : P(K_1) \times P(K_2) \times \cdots \times P(K_n) \rightarrow P(\mathbb{U})$.

Then the largest Neutrosophic Semi-Open SuperHyperSoft set contained in (O,J) s called the Neutrosophic Semi-Interior of (O,J), represented by $NSHSint(O,J)$ and is defined as

$$NSHSint(O,J) = \bigcup \{(\aleph,\mathcal{B}) : (\aleph,\mathcal{B}) \subseteq (O,J), (\aleph,\mathcal{B}) \in NSOSHS(X)\}$$

The smallest Neutrosophic Semi-Closed SuperHyperSoft set containing. (O,J) is called the Neutrosophic Semi-Closure of (O,J) is symbolized by

$$NSHScl(O,J) = \bigcap \{(\mathfrak{H},\mathcal{C}) : (\mathfrak{H},\mathcal{C}) \subseteq (O,J), (\mathfrak{H},\mathcal{C}) \in NSCSHS(X)\}.$$

Theorem:3.3
Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space ($NSHSTS$) and $(O,J), (\aleph,\mathcal{B}) \in NSHSS(X)$. Then the following properties hold:
(i) $NSHSint(\emptyset_X) = \emptyset_X$ and $NSHSint(X) = X$
(ii) $NSHSint(O,J) \subseteq (O,J)$
(iii) $NSHSint(O,J)$ is the largest neutrosophic semiopen superhypersoft set contained in (O,J)
(iv) If $(O,J) \subseteq (\aleph,\mathcal{B})$, then $NSHSint(O,J) \subseteq NSHSint(\aleph,\mathcal{B})$
(v) $NSHSint(NSHSint(O,J)) = NSHSint(O,J)$
(vi) $NSHSint(O,J) \cup NSHSint(\aleph,\mathcal{B}) \subseteq NSHSint[(O,J) \cup (\aleph,\mathcal{B})]$

(vii) $NSHSint[(O,J) \cap (\aleph,\mathcal{B})] \subseteq NSHSint(O,J) \cap NSHSint(\aleph,\mathcal{B})$.

Theorem:3.4

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHSTS)$. Let $(O,J),(\aleph,\mathcal{B})$ be in $NSHSS(X)$. Then the following properties are true:

(i) $NSHScl(\emptyset_X) = \emptyset_X$ and $NSHScl(X) = X$

(ii) $NSHScl(O,J) \subseteq (O,J)$

(iii) $NSHScl(O,J)$ is the largest neutrosophic semiopen superhypersoft set contained in (O,J)

(iv) If $(O,J) \subseteq (\aleph,\mathcal{B})$, then $NSHScl(O,J) \subseteq NSHScl(\aleph,\mathcal{B})$

(v) $NSHScl(NSHScl(O,J)) = NSHScl(O,J)$

(vi) $NSHScl(O,J) \cup NSHScl(\aleph,\mathcal{B}) \subseteq NSHScl[(O,J) \cup (\aleph,\mathcal{B})]$

(vii) $NSHScl[(O,J) \cap (\aleph,\mathcal{B})] \subseteq NSHScl(O,J) \cap NSHScl(\aleph,\mathcal{B})$.

Theorem:3.5

Every neutrosophic open (closed) superhypersoft set in a Neutrosophic SuperHyperSoft Topological Space (X,τ) is a neutrosophic semiopen (semiclosed) superhypersoft set.

Proof:

Let (O,J) be a neutrosophic open superhypersoft set. Then

$$NSHSint(O,J) = (O,J)$$

Since $(O,J) \subseteq NSHScl(O,J)$, we have

$$(O,J) \subseteq NSHScl(NSHScl(O,J))$$

Thus, (O,J) is a neutrosophic semiopen superhypersoft set, that is $(O,J) \in NSOSHS(X)$.

Similarly, every neutrosophic closed superhypersoft set is a neutrosophic semiclosed superhypersoft set.

Theorem:3.6

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHSTS)$ and let $(O,J),(\aleph,\mathcal{B}) \in NSHSS(X)$. If either $(O,J) \in NSHSS(X)$ or $(\aleph,\mathcal{B}) \in NSHSS(X)$, then

$$NSHSclNSHSint((O,J) \cap (\aleph,\mathcal{B})) \subseteq NSHScl(NSHSint(O,J) \cap NSHScl(NSHSint(\aleph,\mathcal{B}))).$$

Proof:

Let $(O,J),(\aleph,\mathcal{B}) \in NSHSS(X)$.Then we have

$$NSHSclNSHSint((O,J) \cap (\aleph,\mathcal{B})) \subseteq NSHScl(NSHSint(O,J) \cap NSHScl(NSHSint(\aleph,\mathcal{B})))$$

Also

$$\begin{aligned} &NSHScl\left(NSHSint(O,J) \cap NSHScl(NSHSint(\aleph,\mathcal{B}))\right) \\ &\subseteq NSHScl\left(NSHScl(NSHSint(O,J)) \cap NSHScl(NSHSint(\aleph,\mathcal{B}))\right) \\ &\subseteq NSHScl(NSHScl(NSHSint(O,J) \cap NSHSint(\aleph,\mathcal{B}))) \\ &= NSHScl(NSHSint(O,J) \cap NSHSint(\aleph,\mathcal{B}) \\ &\subseteq NSHSclNSHSint((O,J) \cap (\aleph,\mathcal{B})) \end{aligned}$$

Hence $NSHScl(NSHSint(O,J)) \cap NSHScl(NSHSint(\aleph,\mathcal{B})) \subseteq NSHSclNSHSint((O,J) \cap (\aleph,\mathcal{B}))$

Thus

$$NSHSclNSHSint((O,J) \cap (\aleph,\mathcal{B})) \subseteq NSHScl(NSHSint(O,J)) \cap NSHScl(NSHSint(\aleph,\mathcal{B})).$$

Theorem:3.7

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHSTS)$ and let (O,J) be a Neutrosophic SuperHyperSoft Open Set and $(\aleph,\mathcal{B}) \in NSHOS(X)$. Then $(O,J) \cap (\aleph,\mathcal{B}) \in NSOSHS(X)$.

Proof:

Let (O,J) be a Neutrosophic Open SuperHyperSoft Set and $(\aleph,\mathcal{B})$ be a neutrosophic semiopen superhypersoft set. Then

$NSHSint\big((O,J)\ \cap\ (\aleph,\mathcal{B})\big) \subseteq\ (O,J)\ \cap\ (\aleph,\mathcal{B})$

Also,

$(O,J)\ \cap\ (\aleph,\mathcal{B}) \subseteq NSHScl\left(NSHSint\big((O,J)\ \cap\ (\aleph,\mathcal{B})\big)\right)$

Thus,

$NSHSint\big((O,J)\ \cap\ (\aleph,\mathcal{B})\big) \subseteq (O,J)\ \cap\ (\aleph,\mathcal{B}) \subseteq\ NSHScl\left(NSHSint\big((O,J)\ \cap\ (\aleph,\mathcal{B})\big)\right)$

Hence, $(O,J)\ \cap\ (\aleph,\mathcal{B}) \in\ NSOSHS(X)$.

Preposition:3.1

Let (O,J) be a neutrosophic superhypersoft set in (X,τ) is a neutrosophic semiclosed superhypersoft set $NSCSHS$ if and only if there exists a neutrosophic closed superhypersoft set $(\mathfrak{H},\mathcal{C})$ such that:

$NSHSint(\mathfrak{H},\mathcal{C}) \subseteq (O,J) \subseteq (\mathfrak{H},\mathcal{C})$.

Preposition:3.2

Every neutrosophic superhypersoft closed set is a $NSHSTS(X,\tau)$ is a $NSCSHS(X)$, but the converse need not be true.

Theorem:3.8

Let (X,τ) be a Neutrosophic SuperHyperSoft Topological Space $(NSHSTS)$ and $(O,J)\ \in NSHSS(X)$. Then $(O,J)\ \in NSCSHS(X)$ if and only if $NSHScl\big(NSHSint(O,J)\big) \subseteq (O,J)$.

Proof:

Suppose $(O,J)\ \in NSCSHS(X)$ Then, by definition, there exists a neutrosophic closed superhypersoft set $(\mathfrak{H},\mathcal{C})$ such that

$$NSHSint(O,J) \subseteq (O,J) \subseteq (\mathfrak{H},\mathcal{C})$$

Since $(O,J) \subseteq (\mathfrak{H},\mathcal{C})$, we have

$$NSHScl(O,J) \subseteq\ NSHScl(\mathfrak{H},\mathcal{C}) = (\mathfrak{H},\mathcal{C})$$

Thus $NSHSint(O,J) \subseteq (O,J) \subseteq (\mathfrak{H},\mathcal{C})$

Taking closure of interior, we get $NSHScl\big(NSHSint(O,J)\big) \subseteq\ NSHScl(\mathfrak{H},\mathcal{C}) = (\mathfrak{H},\mathcal{C})$

hence $NSHScl\big(NSHSint\ (O,J)\big)\ \subseteq (O,J)$

Conversely,

$NSHScl\big(NSHSint\ (O,J)\big)\ \subseteq (O,J)$

Let $(\mathfrak{H},\mathcal{C}) = NSHScl(O,J)$

Then clearly $(O,J) \subseteq (\mathfrak{H},\mathcal{C})$

Also

$$NSHSint(\mathfrak{H},\mathcal{C}) = NSHSint\big(NSHScl(O,J)\big) \subseteq (O,J)$$

Thus $NSHSint(\mathfrak{H},\mathcal{C}) \subseteq (O,J) \subseteq (\mathfrak{H},\mathcal{C})$

Therefore $(O,J)\ \in NSCSHS(X)$.

Theorem:3.9

Let $\{(O,J)_\beta$ for $\beta \in I\}$ be a family of Neutrosophic SuperHyperSoft Closed Sets $NSCSHS$ n a Neutrosophic HyperTopological Space (X,τ). Then the intersection $\bigcap_{\beta\in I}(O,J)_\beta$ is also a Neutrosophic SuperHyperSoft Closed Set $NSCSHS$ in (X,τ).

Proof:

Since each $(O,J)_\beta$ is a $NSCSHS$, by definition, there exists a Neutrosophic Closed HyperSoft Set $(NCHS)$ $(\mathfrak{H},\mathcal{C})_\beta$ such that

$NHSint\big((\mathfrak{H},\mathcal{C})_\beta\big) \subseteq (O,J)_\beta \subseteq (\mathfrak{H},\mathcal{C})_\beta$

Taking the intersection over all $\beta \in I$, we get

$$\bigcap_{\beta\in I} NHSint\big((\mathfrak{H},\mathcal{C})_\beta\big) \subseteq \bigcap_{\beta\in I}(O,J)_\beta \subseteq \bigcap_{\beta\in I}(\mathfrak{H},\mathcal{C})_\beta$$

Let $(\mathfrak{H}, \mathcal{C}) = \bigcap_{\beta \in I} (\mathfrak{H}, \mathcal{C})_\beta$

Since the intersection of $NCHS$s is again an $NCHS$, it follows that $(\mathfrak{H}, \mathcal{C})$ is an $NCHS$.

Also, $NHSint(\mathfrak{H}, \mathcal{C}) \subseteq \bigcap_{\beta \in I} (O, J)_\beta \subseteq (\mathfrak{H}, \mathcal{C})$

Hence by definition $\bigcap_{\beta \in I} (O, J)_\beta$ is a Neutrosophic SuperHyperSoft Closed Set $NSCSHS$.

Theorem:3.10

Let (O, J) be a Neutrosophic SuperHyperSoft Closed Set $(NSCSHS)$ and (v, J) be a Neutrosophic Closed HyperSoft Set $(NCHS)$ in (X, τ). If $NHSint(O, J) \subseteq (v, J) \subseteq (O, J)$, then (v, J) is a Neutrosophic SuperHyperSoft Closed Set $(NSCSHS)$.

Proof:

Since (O, J) is a $NSCSHS$, there exists a $NCHS$ $(\mathfrak{H}, \mathcal{C})$ such that
$NHSint(\mathfrak{H}, \mathcal{C}) \subseteq (O, J) \subseteq (\mathfrak{H}, \mathcal{C})$
Given
$NHSint(O, J) \subseteq (v, J) \subseteq (O, J)$
We combine both inclusions
from $(O, J) \subseteq (\mathfrak{H}, \mathcal{C})$, we get $(v, J) \subseteq (\mathfrak{H}, \mathcal{C})$
Also, $NHSint(\mathfrak{H}, \mathcal{C}) \subseteq NHSint(O, J) \subseteq (v, J)$
Thus $NHSint(\mathfrak{H}, \mathcal{C}) \subseteq (v, J) \subseteq (\mathfrak{H}, \mathcal{C})$
Hence by definition (v, J) is $(NSCSHS)$.

Theorem:3.11

Let (O, J) be a Neutrosophic SuperHyperSoft Set $(NSHS)$ in a Neutrosophic HyperTopological Space (X, τ). Then

(i) $\big(NSHSint(O, J)\big)^C = NSNHcl((O, J)^C)$
(ii) $\big(NSHScl(O, J)\big)^C = NSNHint((O, J)^C)$
(iii) $NSHSint\big(NHSint(O, J)\big) \subseteq NHSint\big(NSHSint(O, J)\big) \subseteq NHSint(O, J)$
(iv) $NSHScl\big(NHScl(O, J)\big) \subseteq NHScl\big(NSHScl(O, J)\big) \subseteq NHScl(O, J)$

Proof:

(i) Since $NSHSint(O, J) \subseteq (O, J)$ taking complements gives

$(O, J)^C \subseteq \big(NSHSint(O, J)\big)^C$

Now, $\big(NSHSint(O, J)\big)^C$ is a $NSCSHS$ hence by minimality of closure

$NSNHcl((O, J)^C) \subseteq \big(NSHSint(O, J)\big)^C$

Conversely, $(O, J)^C \subseteq NSNHcl((O, J)^C)$

$\Rightarrow \Big(\big(NSHScl(O, J)\big)^C\Big)^C \subseteq (O, J)$

Since $\Big(\big(NSHScl(O, J)\big)^C\Big)^C$ is $NSOSHS$, it follows that

$\Big(\big(NSHScl(O, J)\big)^C\Big)^C \subseteq NSHSint(O, J)$

Taking a compliment again

$\big(NSHSint(O, J)\big)^C \subseteq NSNHcl((O, J)^C)$

Thus

$\big(NSHSint(O, J)\big)^C = NSNHcl((O, J)^C)$

(ii) Follows directly from (i) by the duality of interior and closure with respect to complement.

(iii) Since $NHSint(O, J)$ is a $NSOSHS$

$NSHSint\big(NHSint(O, J)\big) \subseteq NHSint(O, J)$

Also,

$NHSint(O, J) \subseteq \big(NSHSint(O, J)\big) \subseteq (O, J)$

Applying $NHSint$

$$NHSint\big(NSHSint(O,J)\big) \subseteq NHSint(O,J)$$

Thus $NSHSint\big(NHSint(O,J)\big) \subseteq NHSint\big(NSHSint(O,J)\big) \subseteq NHSint(O,J)$

(iv) Since $NHScl(O,J)$ is a $NSCSHS$

$$NSHScl\big(NHScl(O,J)\big) \subseteq NHScl(O,J)$$

Also,

$(O,J) \subseteq \big(NSHScl(O,J)\big) \subseteq NHScl(O,J)$

Applying $NHScl$

$$NHScl\big(NSHScl(O,J)\big) \subseteq NHScl(O,J)$$

Thus $NSHScl\big(NHScl(O,J)\big) \subseteq NHScl\big(NSHScl(O,J)\big) \subseteq NHScl(O,J)$.

4. Application: Smart Hospital Resource Allocation using NSOSHS/NSCSHS–MAGDM

Step 1: Universe and Attributes

Let $X = \{p_1, p_2, p_3, p_4, p_5\}$ be the set of patients admitted to a hospital.

The decision is based on multiple attributes:

- E_1: Age $= \{\text{young}, \text{middle}, \text{old}\}$
- E_2: Illness $= \{\text{mild}, \text{moderate}, \text{severe}\}$
- E_3: Oxygen level $= \{\text{low}, \text{medium}, \text{high}\}$
- E_4: Infection risk $= \{\text{low}, \text{medium}, \text{high}\}$

The super-parameter space is

$$\mathcal{J} = P(E_1) \times P(E_2) \times P(E_3) \times P(E_4)$$

We consider specific super-parameters

$$\mathcal{I}_1 = (\{\text{young}\}, \{\text{severe}\}, \{\text{low}\}, \{\text{high}\})$$
$$\mathcal{I}_2 = (\{\text{middle}\}, \{\text{moderate}\}, \{\text{medium}\}, \{\text{medium}\})$$
$$\mathcal{I}_3 = (\{\text{old}\}, \{\text{severe}\}, \{\text{low}\}, \{\text{high}\})$$

These represent different clinical scenarios.

Let (O,J) be a Neutrosophic Semi-Open SuperHyperSoft Set (NSOSHS) representing patients satisfying partially acceptable emergency conditions.

Let (C,J) be a Neutrosophic Semi-Closed SuperHyperSoft Set (NSCSHS) representing patients associated with uncertain or restricted medical conditions.

Define mappings:

$$f, g: \mathcal{J} \to NS(X)$$

where each mapping assigns neutrosophic values (T, I, F) $\forall x \in X$

f: High-risk evaluation under NSOSHS

g: Medium-risk evaluation under NSCSHS

Step 2: High-Risk NSOSHS Table

patients	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$
p_1	(0.5,0.4,0.1)	(0.5,0.3,0.2)	(0.8,0.2,0.1)
p_2	(0.7,0.2,0.2)	(0.6,0.4,0.2)	(0.6,0.1,0.2)
p_3	(0.1,0.4,0.4)	(0.8,0.1,0.1)	(0.5,0.2,0.1)
p_4	(0.1,0.4,0.1)	(0.1,0.2,0.1)	(0.3,0.2,0.1)
p_5	(0.8,0.3,0.1)	(0.6,0.2,0.1)	(0.4,0.1,0.1)

The above table represents the NSOSHS structure because the patients satisfy partially admissible emergency conditions.

Step 3: Medium-Risk NSCSHS Table

patients	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$
p_1	(0.8,0.2,0.1)	(0.5,0.3,0.2)	(0.9,0.1,0.1)
p_2	(0.9,0.2,0.1)	(0.6,0.4,0.2)	(0.8,0.2,0.2)
p_3	(0.5,0.3,0.4)	(0.8,0.1,0.1)	(0.6,0.4,0.2)
p_4	(0.6,0.4,0.2)	(0.1,0.2,0.1)	(0.5,0.3,0.2)
p_5	(0.3,0.2,0.1)	(0.6,0.2,0.1)	(0.6,0.3,0.2)

The above table represents the NSCSHS structure because it includes uncertain and semi-restricted medical conditions.

The neutrosophic semi-interior operator identifies strongly admissible patients:

$$NSHSint(O,J) = \{p_1, p_2, p_5\}$$

This indicates p_1, p_2 and p_5 strongly satisfy the emergency admission criteria.

The neutrosophic semi-closure operator identifies extended uncertain-risk patients:

$$NSHScl(C,J) = \{p_2, p_3, p_4\}$$

This indicates p_2, p_3 and p_4 belong to the uncertain or extended-risk medical group.

Step 4: Score Function
To convert neutrosophic values into scalar form, the score function is defined as:

$$S_{ij} = \frac{T_{ij} + (1 - I_{ij}) + (1 - F_{ij})}{3}$$

Step 5: High-Risk NSHS Table

patients	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$
p_1	0.67	0.67	0.83
p_2	0.77	0.67	0.77
p_3	0.43	0.87	0.73
p_4	0.53	0.60	0.67
p_5	0.80	0.77	0.73

Step 6: Medium-Risk NSHS Table

patients	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$
p_1	0.83	0.67	0.90
p_2	0.87	0.67	0.80
p_3	0.60	0.87	0.67
p_4	0.67	0.60	0.67
p_5	0.67	0.77	0.70

Step 7: Decision Function
The decision value for each patient is computed as:

$$d_i = \frac{1}{n}\sum_{j=1}^{n} S_{ij}$$

Similarly, e_i is computed for medium risk.

Step 8: High-Risk Decision Values (d_i)-

patients	d_i
p_1	0.72
p_2	0.74
p_3	0.68
p_4	0.60
p_5	0.77

These values represent the degree to which each patient belongs to the NSOSHS high-risk category.

Step 9: Medium-Risk Decision Values(e_i)

patients	e_i
p_1	0.80
p_2	0.78
p_3	0.71
p_4	0.65
p_5	0.71

These values represent the contribution of each patient under the NSCSHS medium-risk category.

Step 10: Final Decision Score
The overall decision score combines both high-risk and medium-risk contributions:
$D_i = d_i + e_i$

Step 11: Computed Final Scores

patients	D_i
p_1	1.52
p_2	1.52
p_3	1.39
p_4	1.25
p_5	1.48

Step 12: Ranking of Patients
Based on the descending order of D_i

$$p_1 \approx p_2 > p_5 > p_3 > p_4$$

The results indicate that the patient. p_1 and p_2 have the highest priority due to consistently high evaluation scores across all parameter scenarios.

5. Conclusion

In this paper, we introduced and studied Neutrosophic Semi-Open SuperHyperSoft Sets (NSOSHS) and Neutrosophic Semi-Closed SuperHyperSoft Sets (NSCSHS) in Neutrosophic SuperHyperSoft Topological Spaces. To prove the consistency and effectiveness of the proposed framework, several basic properties, theorems, and operators like semi-interior and semi-closure were introduced. The developed theory extends existing neutrosophic and hypersoft models by integrating hierarchical multi-parameterization and semi-topological concepts for more effective handling of uncertainty. Moreover, the applicability of the proposed approach was illustrated by a MAGDM-based smart hospital resource allocation problem, indicating its effectiveness in real-world uncertain decision-making environments.

References

[1] Mumtaz Ali, and Florentin Smarandache, "Complex Neutrosophic Set," *Neural Computing and Applications*, vol. 28, pp. 1817-1834, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[2] Mumtaz Ali, Irfan Deli, and Florentin Smarandache, "The Theory of Neutrosophic Cubic Sets and their Applications in Pattern Recognition," *Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology*, vol. 30, no. 4, pp.1957-1963, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[3] Krassimir T. Atanassov, "Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, 1986. [CrossRef] [Google Scholar] [Publisher Link]
[4] D. Ajay et al., "Neutrosophic Semiopen Hypersoft Sets with an Application to MAGDM under the COVID-19 Scenario," *Journal of Mathematics*, vol. 2021, pp. 1-16, 2021. [CrossRef] [Google Scholar] [Publisher Link]
[5] Said Broumi, and Florentin Smarandache, "Intuitionistic Neutrosophic Soft Sets," *Journal of Information & Computational Science*, vol. 8, no. 2, pp. 130-140, 2013. [Google Scholar] [Publisher Link]
[6] Irfan Deli, and Said Broumi, "Neutrosophic Soft Relations and Some Properties," *Annals of Fuzzy Mathematics and Informatics*, vol. 9, no. 1, pp. 169-182, 2015. [Google Scholar] [Publisher Link]
[7] P.K. Maji, R. Biswas, and A.R. Roy, "Soft Set Theory," *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555-562, 2003. [CrossRef] [Google Scholar] [Publisher Link]
[8] A.R. Roy, and P.K. Maji, "A Fuzzy Soft Set Theoretic Approach to Decision Making Problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412-418, 2007. [CrossRef] [Google Scholar] [Publisher Link]
[9] D. Molodtsov, "Soft Set Theory—First Results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, 1999. [CrossRef] [Google Scholar] [Publisher Link]
[10] Florentin Smarandache, *Neutrosophy: Neutrosophic Probability, Set, and Logic*, American Research Press, 1998. [Google Scholar] [Publisher Link]
[11] Florentin Smarandache, "Extension of Soft Set to Hypersoft Set, and Then to Plithogenic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 22, pp. 168-170, 2018. [Google Scholar] [Publisher Link]
[12] Haibin Wan et al., *Interval Neutrosophic Sets and Logic,Theory and Applications*, Hexis, 2010. [Google Scholar] [Publisher Link]
[13] L.A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965. [CrossRef] [Google Scholar] [Publisher Link]