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Original Article

Characteristic Numbers Whose Divisor Approaches Zero

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Abstract *- Historical records have examined the origins and significance of zero, yet a fundamental question remains unanswered: What is the value of a/0, where a can be either zero or a non-zero value? Brahmagupta and Bhaskara, two prominent Indian mathematicians, held opposing views on this issue, but neither offered a definitive solution. Various attempts to address this problem—such as introducing new symbols to expand the set of real numbers, applying proposed transreal analysis, utilizing stereographic projection, or employing the sgn function—have proven unsuccessful. The reality is that treating a/0 in isolation does not make sense. A meaningful natural process to elucidate this problem requires the quotients to appear as characteristic numbers. In conformity with the use of applied mathematics and real life, it is necessary to recognize that no natural process involves a zero denominator. This work, illustrated with experimental examples, shows that when the denominator of a quotient approaches zero, using characteristic numbers associated with the limit of that quotient yields finite results, intervening in the definition of infinity. This approach facilitates the management of potential hardware exceptions and helps programmers develop more efficient software to prevent division-by-zero errors.*

Keywords *- Aristotle's Theory on Natural Motion, Bhramagupta's and Bhaskara's 0/0, Characteristic Number, Division by Zero and Infinity, Solar Radiation Amplification using a Truncate Cone Mirror.*

1. Introduction

Numbers and letters are examples of symbols that are very important in our daily lives. They make our lives better in a very natural way from morning to night. Centuries of changes have made it easy for us to use and understand them now. Over time, semantic theories, corollaries, and principles developed to influence contemporary science and communication.

Amazingly, letters also behave as numbers. The Roman numerals, generally introduced in primary school and formed by the letters I, V, X, L, C, D, and M, have some particularities that are probably forgotten by the majority of students. Many, for example, may not remember that a crucial difference between Roman and Arabic numerals is that the latter are positional. Another exceptionality is that in rare cases where a null quantity needed representation, the Romans used the term "nulla," meaning "null," nestled between numerals.

The historical records documenting the concept or entity that assigns a fundamental role to the numeral zero are also intriguing. Pickover [1] notes that around AD 200, the Hindus, in conjunction with the Arabs, invented what is considered one of the most notorious mathematical elements: 0. He further states that 500 years before the Hindus, the Babylonians already used a special symbol to signify "absence." Many historians, such as Joseph [2], assert that this concept originated in diverse and distant regions. In the Maya and Indian cultures, zero was marked by the absence of magnitude or as a direction separator. Joseph [2] also observed that, while zero can be assigned a defined place in a numerical system, the Chinese are credited with introducing the concept of zero a few centuries before the Common Era (AD).

An immediate consequence of the concept of zero is the idea of infinity. If a positive integer N is repeatedly divided by another positive integer n, and each time n is chosen much smaller, the quotients grow larger and larger. This arithmetic process offers a preliminary sense of infinity, understood as a magnitude that is as far from zero as possible within the set of positive numbers—each representing opposing extremes on the number line.

The infinity symbol, or lemniscate (∞), appeared on Tarot cards in the 1700s and signifies endlessness. Philosophers and mathematicians from Aristotle to Pascal, as cited by Rucker [3], contributed to spreading this symbol and its meaning.

In fact, determining the value of $a/0$ has long been a challenge. However, $a/0$ will never represent the result of an operation among quantities and, therefore, will not show any possibility of evaluation. In treating real situations, what exists are quotients expressed in terms of a characteristic number, that is, a physical quantity without dimensions, often referred to as a dimensionless quantity, where the divisor could tend to zero. The fact is that characteristic numbers help clarify a process's behavior, as noted on iso.org [4]. Grasping the origins and implications of both the numerator and denominator, the result becomes meaningful by taking the limit even when the divisor approaches zero.

Since Shri Bhramagupta [5], an Indian mathematician and astronomer born in 598, infinity has been associated with large numbers. He was the first to connect division by an arithmetic zero, or cipher, to certain mathematical results, as discussed by Colebrooke [6] and cited by Romig [7]. In a 1963 publication from the Indian Astronomical and Sanskrit Research Institute, Bhramagupta [5] was recognized as the pioneer in representing division by zero. His work, Brahmasphuta Siddhânta (c. 628), is the earliest known text to explore the arithmetic of zero and to present foundational rules for handling zero, of which some, as explained by Gorain [8], held true while others did not. For instance, the assertion $x/0 = 0$, where x is any number, is noted as inaccurate. Centuries later, in the 12th century, the renowned Indian mathematician Bhaskara [9] revisited Bhramagupta's [5] findings. While working on his influential text Līlāvatī (Slate Mathematics or Arithmetic), Bhaskara [9] not only reinforced the concept of division by zero but also introduced the idea that dividing any number, a, by zero yields infinity: $a/0 = \infty$.

Brahmagupta's [5] and Bhaskara's [9] work dates back many years, yet misconceptions about division by zero and the concept of infinity persist. In his discussion on division by zero, Czajko [10] appears to support Conventional Division by Zero (CDZ), which argues that division by zero results in infinity, defined as the inverse of zero (i.e., 1/0). On the other hand, Unconventional Division by Zero (UDZ) asserts that zero divided by zero equals zero. Czajko [10] notes that UDZ was favored by ancient Indian mathematicians and has recently been studied by Japanese mathematicians, as will be discussed later. It is clear that these theories challenge the rational boundaries of classical mathematics.

The extensive literature on division by zero illustrates numerous attempts to address this operation. The fact is that a quotient representing division by zero does not have any meaning in real life. However, by interpreting this ratio as a characteristic number in which the divisor tends to zero and evaluating the limit of the resulting function, one obtains a finite value. This work includes several practical examples that demonstrate the efficiency of the method, here proposed, to solve situations where the denominator approaches zero. Additionally, a key conclusion is that division by a number approaching zero does not yield infinity; rather, under certain circumstances, the result is a very small value. Resolving this issue can also help prevent common programming errors.

2. Materials and Methods

2.1. The Use of Characteristic Numbers Subsidized by a Limit Function Operation

2.1.1. Transforming Bhaskara's Division by Zero into a Conventional Operation Using Characteristic Numbers and Determining the Limit of the Result Near a Particular Input.

Among some operations involving zero, Bhaskara [9] announced that 0/0 = 1. To better understand this assertion, a mathematical problem was proposed in Līlāvatī [9], p. 23, and in the Colebrooke translation [11]. It consists of finding x in the following expression:

$$\frac{\left(x.0+\frac{1}{2}.\mathrm{x}\right).3}{0} = 63 \tag{1}$$

According to Bhaskara's [9] conception, Equation (1) should be written as:

$$\frac{9.x}{2}.\frac{0}{0} = 63 \tag{2}$$

or $\frac{9.\mathrm{x}}{2}.1 = 63$, thus $x = 14$.

Equations (1) and (2) involve division of zero by zero, where 0/0 is considered equal to 1. However, 0/0 = 1 is false because it allows any number to equal any other number, as Cajori [12] demonstrates. Fortunately, it is possible to write these equations in terms of characteristic numbers, thereby avoiding division by zero. Characteristic numbers are the numbers that capture the system's important features. So, equations that do not describe any process are merely statements that do not follow from any logical operation.

Any number can technically be considered as the answer for zero divided by zero; however, to assign a meaningful answer, there must be a clear process or operation behind it. Without this, the ratio becomes meaningless. For instance, Bhaskara [9] viewed division 0/0 = 1, while Brahmagupta [5] considered it to be 0. To avoid undefined scenarios, it can be helpful to redefine these problematic ratios when an expression lacks meaning. The 0/0 issue in Equations (1) and (2) can be better understood through simulations and limit solutions, illustrating the result as a variable approaches a specific value. To further clarify, let F(z) represent the quotient of two given expressions. To create situations comparable to Equations (1) and

(2), it is essential to account for all measured factors, expressed, e.g., in centimeters, thereby allowing a clearer expression of this relationship:

$$F(z) = \frac{(z+112)(z-x_0)(z-x_1)cm^3}{2(z-x_0)(z-x_1)cm^3} = 63 \quad (3)$$

Thus, evaluating the limit of F(z) in Equation (3), when $z \rightarrow x_o$, is possible to determine x_0:

$$\lim_{z \rightarrow x_0.cm} F(z) - 63 = \frac{x_0.cm}{2cm} - 7 = 0 \quad (4)$$

Solving Equation (4) yields $x_0 = 14$, without any mention of exceptional operation.

Using l'Hôpital's rule on Equation (3) gives, of course, the same result as in Equation (4).

2.1.2. Aristotle's Law of Falling Bodies Treated by Characteristic Numbers and a Limit Operation

Aristotle's principles [13] were never based on experimental evidence, as Mansion [14] noted, demonstrating Aristotle's exceptional skill. In the book Physics [13], he outlined the main factors that determine an object's velocity. It is possible to condense Aristotle's fundamental principle in the following words: "Everything that is in motion must be moved by something," - Aristotle [13]. He also stated that the time for a body to travel a certain distance is inversely proportional to the motive power of the mover, i.e., the rate of fall. In other words, it is directly proportional to the weight of the object and inversely proportional to the density of the medium through which it is moving, all under the influence of gravity. Thus, velocity v given by Equation (5), which is a characteristic number, expresses the resistance between two forces, that is

$$v \propto \frac{F}{R} = \frac{\rho o}{\rho f} \quad (5)$$

where,
F, the motive force, N;
R, resistance of medium, N
v is the speed of a moving object through a medium, m/s.
ρo is the density of the moving body, kg/m^3.
ρf the ambient density, where the body moves, kg/m^3.

Table 1. Density of Gases at 25 °C, 100 kPa Van Wylen, Sonntag, and Borgnakke [16]

Gas	Chemical Formula	Molecular Mass	Density kg/m³
Acetylene	C_2H_2	26.038	1.05
Ammonia	NH_3	17.031	0.694
Argon	Ar	39.948	1.613
Butane	C_4H_{10}	58.124	2.407
Carbon monoxide	CO	28.01	1.13
Carbon dioxide	CO_2	44.01	1.775
Ethane	C_2H_6	30.07	1.222
Ethanol	C_2H_5OH	46.069	1.883
Ethylene	C_2H_4	28.054	1.138
Helium	He	4.003	0.1615
Hydrogen	H_2	2.016	0.0813
Methane	CH_4	16.043	0.648
Methanol	CH_3OH	32.042	1.31
Neon	Ne	20.183	0.814
Nitric oxide	NO	30.006	1.21
Nitrogen	N_2	28.013	1.13
Nitrous oxide	N_2O	44.013	1.775
Oxygen	O_2	31.999	1.292
Propane	C_3H_8	44.094	1.808
R-12	CCl_2F_2	120.91	4.98
R-22	$CHClF_2$	86.469	3.54

In Equation (5), if a solid is moving in a vacuum, a division by zero may occur. Aristotle [13] most probably observed that his formula could yield unreal values, a fact interpreted by authors such as Barukčić [15] as related to division by zero in arithmetic. Others would wish that he had just figured out that a quotient like that could give you an undefined value. This indeterminacy can today be avoided with available technical data. It is to be noted that gas density is directly proportional to molecular mass. This relation is perceived in Table 1 of Van Wylen, Sonntag, and Borgnakke [16]. Some data were excluded from the analysis, such as air, steam, n-octane (gasoline), and R-134a (Freon). From Table 1, it is possible to obtain the following recursive formula of gas density with respect to molecular mass:

$$\rho(\mathrm{M}) = (0.0013 + 0.00399.\mathrm{M} + 2.2298\,\mathrm{x}10^{-5}.\mathrm{M}^2 - 9.6937\mathrm{x}10^{-8}.\mathrm{M}^3)\,\frac{\mathrm{kg}}{\mathrm{m}^3} \tag{6}$$

where
M is the molecular mass, amu;
ρf, is the gas specific mass, kg/m^3.

The coefficient of determination for the fitting curve given by Equation (6) is $R^2 = 0.9999$, and the standard error of the estimate is 0.0115. Therefore, when M goes to zero, ρf → 0.0013, physically, the explanation is that if ρo, in Equation (5), is finite, the body speed, v, should be conserved as finite, even when it is falling in a vacuum. Alternatively, it could be possible to have unlimited velocity by changing the medium's density.

3. Results

The primary benefit of using characteristic numbers combined with the limit function operation is that they help eliminate indeterminacy, particularly in situations where the denominator of a quotient approaches zero. Additionally, analyzing Aristotle's solution to the problem mentioned in section 2.1.1, the division of a finite numerator by a quantity that approaches zero, the outcome remains finite. However, the representation of this case is often the lemniscate, that is, infinity.

3.1. Physically, Infinity May Not Be Understood as a Quantity of Unlimited Magnitude

3.1.1. The Potential Flow Cases

The term "infinity" often refers to something vast or limitless and stems from the classical idea of the inverse of zero. In practical contexts, infinity can represent a boundary. For instance, when detecting sounds or light, "infinity" describes the point where a sensor no longer perceives these stimuli. For example, a person on a sidewalk can hear a car passing by, but the sound fades after a certain distance, which may vary from person to person. This illustrates that infinity is not an absolute concept. In scientific contexts, as mentioned in Blasius [17], the standard distance from a flat plate at which the flow becomes potential is 5.0, leading to a specific boundary condition

$$\text{at } \eta = \eta_\infty \qquad \frac{df}{d\eta} = 1$$

where $\eta_\infty = 5$, η is a dimensionless position variable, and f(η) is the dimensionless stream function, as shown in Bennett and Myers [18] on p. 140 or in Rohsenow and Choi [19] on p. 38.

3.1.2. The Extended Surface Cases

Setting L = ∞ can simplify the Equation and its solution in certain cases, as demonstrated in Holman's analysis [20] of one of the extended surface scenarios (see page 30). If the fin is very long and the temperature at its end matches that of the surrounding fluid, the general solution is:

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \tag{7}$$

where,
$\theta = T - T_\infty$
T_∞ is the temperature of the surrounding fluid.
Moreover, the boundary conditions are:
at x = 0, $\theta = \theta_o$; at x = L, θ = 0. So, at x = 0, it comes that $C_1 + C_2 = 0$, and at x = L, the boundary condition gives $C_2 = 0$.
The oddity of linking infinite fin lengths is that most fins are just centimeters or even millimeters long.

3.2. The Supposedly Impossible Result Related to The Quotient $\frac{a}{n}$, Where $a \neq 0$ and n → 0

Similar results obtained from Aristotle's problem also arise in a more complex example related to determining the solar radiation concentration factor, $\frac{E}{E_o}$ That is a characteristic number. (It is important to note that the lemniscate symbol should never appear as a result of a practical problem.) Here, E_o represents the amount of solar power (irradiance), measured in watts per square meter, that strikes a sun-tracking plane mirror. This mirror reflects sunlight onto a truncated cone-shaped mirror with a vertex angle of π/2 (see Figure 1). The conical mirror redirects the rays from each section of radius R, converging and amplifying the energy to the power E onto the walls of a tubular, transparent, concentric reactor of radius r, aligned with its axis (see Figure 2).

Fig. 1 Truncated cone mirror with a tubular reactor on its axis, receiving sunlight reflected by a flat plane mirror.

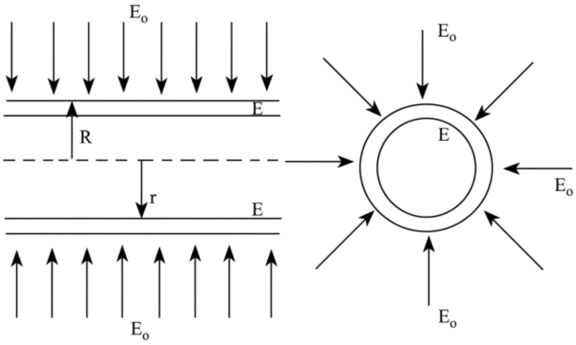

Fig. 2 Sketch of the tubular reactor with transparent walls, receiving solar radiation perpendicularly to its walls.

Admitting that linear rays transmit solar energy, it is possible to write the following:

$$\frac{E}{E_o} = \frac{R}{r} \tag{8}$$

Therefore, when r approaches 0, the concentration factor approaches infinity. Equation (8) illustrates that a quotient with a non-zero numerator and a denominator approaching zero does not indicate a mathematical impossibility. However, the factor approaching infinity is not valid because E_o, the incoming power, is finite. This contradiction reveals that the theoretical framework previously used was inadequate to describe solar radiation propagation. Instead of being transmitted by straight rays, solar energy is transmitted by conical rays. These conical rays represent the solid angle subtended by the spherical sun as observed from a point on Earth's surface. Consequently, as demonstrated by Jaguaribe [33], the ratio of concentration must be reevaluated, leading to the following expression:

$$\frac{E}{E_o} = \frac{2}{\pi\alpha}\left\{\sin(\theta_L)\sqrt{1-\left[\frac{\lambda\sin(\theta_L)}{\sin(\alpha)}\right]^2} + \frac{\alpha}{\lambda}\mathrm{asin}\left[\frac{\lambda\sin(\theta_L)}{\sin(\alpha)}\right]\right\} \tag{9}$$

where,
α is the semi-vertical angle of a cone emitted by any radiant source, thus

$$\sin(\alpha) = \lambda\sin(\theta_L) \tag{10}$$

$\lambda = d/R$.

Figure 3 shows a section of a truncated conical mirror with radius R. A light source illuminates the mirror surface, as shown by the point labeled M. This light reflects off the mirror and reaches a point P inside the section, located at a distance d from the circle's center.

It is essential to note that, when the light source is the sun, δ replaces α in Equation (9). This adjustment accounts for half the apparent diameter of the sun. The distance from the sun to the Earth, divided by two times the sun's diameter, gives the parameter δ:

$$\delta = \frac{1400000\ \mathrm{km}}{2\mathrm{x}150000000\ \mathrm{km}} = 0.00467 \tag{11}$$

By replacing α with δ, in Equation (9) and substituting sin (δ) with $\lambda\sin(\theta_L)$, —where $\theta_L$is the external angle formed by the lines $\overline{M_oP}$ and $\overline{PM_\alpha}$ As shown in Figure 3, there is a determination of the point (0.00467, 214.2857*).* This point indicates the limit of superposition between the curves defined by Equations (8) and (9).

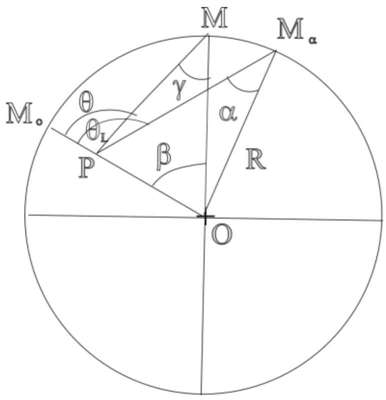

Fig. 3 Section of the truncated conical mirror, showing parameters figuring in Equation (9).

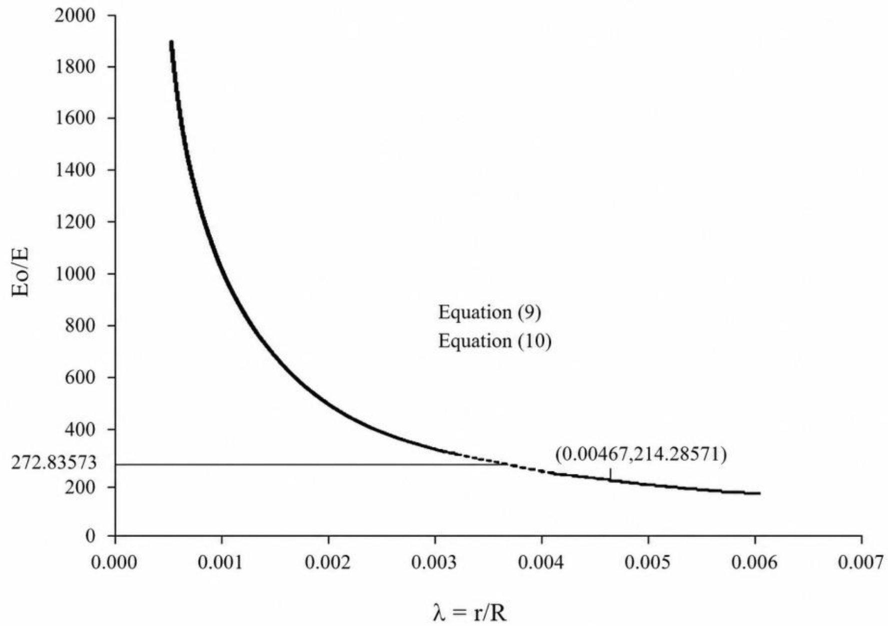

Fig. 4 Curves representing the solar radiation amplification as a function of r/R, given by Equations (8) and (9).

Examining the triangles PMO and $PM_\alpha O$ in Figure 3, the conclusion is that the angles γ and) are identical until $\theta_L$exceeds π/2. Then, it follows that λ must be less than 1/214.2857, resulting in $\gamma < \alpha$. In such situations, Equation (12) replaces Equation (10), that is,

$$\sin(\gamma) = \lambda\sin.(\theta_L) \tag{12}$$

As illustrated in Figure 4, the curves begin to diverge at the point (0.00467, 214.2857). Specifically, the curve represented by Equation (8) approaches infinity, which contradicts real-world observations. Conversely, the curve produced by Equation (9), naturally represented by characteristic numbers, follows a more consistent and reliable trajectory.

As r approaches 0, the factor $\frac{E}{E_o}$, due to the half part of the truncated conical mirror, approaches 272.836. This indicates that the entire mirror permits a factor equal to 545.672.

In the same installation, filling the reactor with a liquid that has an extinction coefficient μ (see Figure 1), the ratio $\frac{E}{E_o}$ Given by Equation (9) is reduced by the factor expressed by

$$\frac{1}{2}\left[e^{-\mu(R-r)} + e^{-\mu(R+r)}\right] \tag{13}$$

Using Equation (9) and the factor (13), it follows that the rate influenced by the liquid absorbance is given by

$$\frac{E}{E_o} = \frac{1}{\pi\alpha}\left\{\sin(\theta_L)\sqrt{1-\left[\frac{\lambda\sin(\theta_L)}{\sin(\alpha)}\right]^2} + \frac{\alpha}{\lambda}\text{asin}\left[\frac{\lambda\sin(\theta_L)}{\sin(\alpha)}\right]\right\}\left[e^{-\mu(R-r)} + e^{-\mu(R+r)}\right] \tag{14}$$

The magnitude of $\frac{E}{E_0}$ can vary significantly depending on the value of μ. For instance, if using a similar method to the one in section 3.1.2 with L set to infinity, then by choosing μ as infinity as well, E becomes 0, regardless of the values of r and R. Conversely, if r approaches zero, applying the Mathcad 15 solve function to determine μ to ensure that E approaches 0—specifically, E = 10^{-15} —the outcome is as follows:

$$\mu = \frac{43.143378}{R} \tag{15}$$

This procedure seems much more realistic than μ = ∞, since it yields a finite quotient that depends on R.

4. Discussion

4.1. Contemporaneous Views on Division by Zero, 0/0.

Surprisingly, many mathematicians still consider the possibility of division by zero. As previously noted, following Czajko [10], several Japanese mathematicians have revisited Brahmagupta's interpretation of 0/0. For instance, Okumura [21] chose to explore division by zero and division-by-zero calculus through plane geometry. He proposed that

$$\frac{z}{0} = 0 \tag{16}$$

for any z in a field and referred to Equation (16) as a mesomorphic function. However, this classification is incorrect, as a mesomorphic function is defined as the quotient of two well-behaved (holomorphic) functions, and it should be nice everywhere except at points where it has singularities. Furthermore, he deduced from Equation (16) that tan(π/2) = 0, which is absurd. In contrast, Pinelas and Saitoh [22] illustrated this perspective by assuming that 1/0 = 0, providing examples in the context of solving initial value problems for a function y(x) and its derivatives. However, when solving one of these problems conventionally, one obtains $\lim_{x \to 0} y(x) < 0$, while they concluded that y(0) = $\frac{3}{4}$. Additionally, Saitoh [23], another mathematician, claims that 0/0 = 0. He offers examples supporting his view, suggesting that this conclusion arises from his perspective on new mathematics and that it will naturally evolve over time. Consequently, he defines *P*(*A*/*B*) as the probability of *A* given *B*, representing it as:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \tag{17},$$

assuming that either A or B could be a zero vector, which makes the inner product undefined. Consequently, this reasoning obstructs efforts to demonstrate that 0 = 0/0. Unfortunately, a similar argument is presented by Okumura and Saburou [24].

Czajko [25], who accepts all forms of division by zero, believes that the challenges faced by mathematicians arise from their reliance on the single-space reality (SSR) paradigm. He argues that a shift to the Multispatial Reality (MSR) paradigm is necessary for a deeper understanding of these issues. As a result, he concludes that the concept of degeneracy is relevant only within the SSR paradigm, but this is not actually the case.

In an effort to explain and provide operational credibility to the concept of division by zero, various researchers have employed a number of techniques. For instance, Potts [26], Carlström [27], and Peter and Wahid [28] have introduced additional symbols to the set of real numbers, leading to the creation of expanded mathematical structures such as the "wheel" and the "semi-structured complex. Other methods include using stereographic projection or the sgn function to rotate vectors by 90°. Additionally, some researchers proposed transreal analysis as a generalization of real analysis. James Anderson [29] has noted this approach, and Meyenburg [30] has suggested using the binary set [0, 1] to define Boolean operations, which offers a different perspective on division and introduces the issue of indeterminacy. Notably, all these authors agree that 0/0 is assigned the value of 1 and concur with the solution that 1/0 equals 0.

Ernest [31] argues that division by zero is undefined, claiming that any attempt to resolve such an expression leads to contradictions. He is particularly critical of the views expressed by Saitoh and Saitoh [32], who assert that 1/0 = 0/0 = 0. While Ernest [31] appears to believe that zero can serve as a divisor, he remains opposed to these assertions.

4.2. The Distinct Work of Sen and Agarwal [34]

Sen and Agarwal [34] are critical of the Japanese group that attempts to revive Brahmagupta's old concepts. It is important to recognize that these ancient ideas are not applicable in our modern technological world, which relies heavily on computers. Over the centuries, mathematics has advanced, learning from past mistakes. Sen and Agarwal [34] state, "Division by zero is mathematically illegal and amounts to a violation of a law of nature." To bolster their argument, they point out that "in natural mathematics (the mathematics conducted by nature), division by zero will never occur, as nature does not violate her laws under any circumstances or at any time." They illustrate this reasoning by referring to Newton's law of universal gravitation:

$$F = c\frac{m_1 m_2}{d^2} \tag{18}$$

Building on this, they further explain that the condition $d \neq 0$ is assumed because, if $d = 0$, there would not be two distinct masses to consider.

5. Conclusion

1. If there is a zero in the denominator, the expression is artificial. Such cases do not arise from real-world problems and, for sure, do not represent a real situation.
2. In nature, continuous functions that increase as the variable approaches zero do not become undefined.
3. The reciprocal of zero is not infinity because zero cannot be in the denominator. Cantor's use of set theory to define infinity is the classical approach that makes sense in math. Another pragmatic possibility is to view it as a limit value of a parameter, such as distance, volume, mass, and so on, linked to a variable that can be detected by a receptor or sensor within its accuracy, as exemplified in section 3. Of course, the result will not always be the same size.
4. Focusing on sets of numbers, transreal analysis, the use of the function sgn, paradigm considerations, and so on have led to ineffective mechanisms for explaining division by zero, as shown in our discussions.
5. In some instances, in day-to-day idealizations, infinity—a value greater than any real number—can be meant to describe very small quantities. While this may seem unusual, it occurs in certain cases.
6. If the answer to a real problem is infinity, one should look very carefully into the physics and math. This attention helps to ensure that the answers are connected with real situations.
7. The group of Japanese mathematicians who developed division-by-zero calculus proposes that there are situations in which zero can be a divisor, which is not admissible.
8. The use of the lemniscate symbol in mathematical expressions does not, by itself, indicate valid or meaningful results; its presence alone does not guarantee mathematical credibility. A correlation that uses zero as a divisor is insufficient to describe any natural process accurately.
9. This paper presents practical examples to illustrate that, using characteristic numbers associated with a limit function operation, it is possible to obtain definitive solutions even when the denominator approaches zero.
10. The discussions presented may be beneficial for those interested in defining exception-handling mechanisms for computer hardware, as well as for programmers who need to set commands to avoid division by zero.

Data Availability

All data supporting the discussions and conclusions of this paper are found in the References.

Conflicting Interests

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