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Original Article

Johan Coloring of Edge Corona Product of Graphs

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Abstract - *Graph coloring has been a fundamental area of study in graph theory, with applications in various fields. The Johan Coloring of a graph G is a proper coloring of G if every vertex of G has a rainbow neighbourhood. Given two graphs with vertices $v_1, v_2, v_3, \ldots v_n$ and edges $e_1, e_2, e_3, \ldots e_n$. The edge corona of G and H is denoted as G ⋄H, and is obtained by taking one copy of G and |E(G)| copies of H and joining each end vertex of the i-th edge of G to every vertex in the i-th copy of H. In this paper, the Johan Coloring of the edge corona product of two graphs is determined.*

Keywords - *Rainbow Neighbourhood, Johan Coloring, Edge Corona Product.*

1. Introduction

In this paper, we focus on simple connected graphs. A graph is denoted by G = (V, E), where V(G) represents the set of vertices and E(G) signifies the set of edges [1,2]. The degree of a vertex v within the vertex set V(G) is determined by the count of edges in G that are connected to V, and it is represented as d(v). Furthermore, the graphs are characterized by an order of n and a size of p, with their degrees ranging from a minimum of δ to a maximum of Δ.

A proper vertex coloring of a graph G is a mapping that assigns colors to vertices such that no two adjacent vertices share the same color. The study of graph coloring has become important theoretically as well as because of its applications to network design, scheduling, coding theory, and communication systems.

Graph operations are important in graph theory, since they are used to construct new graphs from existing graphs. These operations combine structural properties of two or more graphs. They produce complex graph classes that are useful in both theoretical and applied research. The standard graph operations are the Cartesian product, corona product, edge corona product, tensor product, and lexicographic product. These operations are used to study many graph parameters such as coloring, domination, connectivity, and topological indices. Operations on graphs also provide a useful framework for modeling networks, communication systems, and combinatorial structures.

The corona product of two graphs was introduced by Frucht and Harary in 1970. The edge corona product was introduced by Haynes and Lawson [7]. In 2020, Wang et al. proposed the Edge corona product to model complex simplicial networks[8]. Lu et al. in 2020 studied the acyclic chromatic numbers corresponding to the Edge Corona product of paths or cycles and any simple graph.[9].

Hou et al. presented the edge corona product, another type of corona product, in [6]. It is readily understood how edge corona products of graphs can be used in routing functions, in transportation networks that depend on automobiles, trains, planes, and ships to reach their destinations, and by information seekers navigating multiple paths and Hamiltonian paths within information networks.

Johan Kok introduced Johan coloring. Johan coloring has received a lot of attention among the different coloring concepts due to its strict neighborhood coloring conditions and relation to the structural properties of graphs.

We have investigated the Johan Chromatic Number for the edge corona product of the path graph with path, cycle, complete graph, star graph, and wheel graph.

2. Preliminaries

2.1. Definition [4]

A maximal proper coloring of a graph G is a Johan colouring of G, denoted by J-coloring, if and only if every vertex of G belongs to a rainbow neighbourhood of G. The maximum number of colors in a J-coloring is called the J-chromatic number of G, denoted by J(G).

2.2. Definition [3]

Let G be a graph with a chromatic coloring C defined on it. The rainbow neighbourhood in G is the closed neighbourhood N[v] of a vertex v belonging to V(G) which contains at least one coloured vertex of each color in 1 the chromatic coloring C of G.

2.3. Definition [6]

The edge corona of two graphs G and H, denoted by G ⋄ H, is obtained by taking one copy of G and |E(G)| copies of H and joining each end vertex of the i-th edge of G to every vertex in the i-th copy of H.

3. Johan Coloring of Edge Corona Product of Graphs

3.1. Theorem

Edge corona product of the path graph with the path graph admits Johan Coloring.

Proof:

Let us consider a path graph P_n, n≥2, and a path graph P_m, m≥2.

Let the vertices and edges be

$V(P_n) = \{u_i: 1 \le i \le n\}$

$V(P_m) = \{v_i: 1 \le i \le m\}$

$E(P_n) = \{u_i u_{i+1}: 1 \le i \le n-1\}$

$V(P_m) = \{v_i v_{i+1}: 1 \le i \le m-1\}$

The edge corona product of Pn and Pm, denoted as $P_n \diamond P_m$, is obtained by taking one copy of P_n and $|E(P_n)|$ copies of P_m and joining each end vertex of the i-th edge of P_n to every vertex in the i-th copy of P_m.

By the definition of Johan Coloring, every vertex of P_n is colored with colors c_1 and c_2 alternatively, and every vertex of P_m is colored with colors c_3 and c_4 alternatively. Since every end vertex of the i-th edge of P_n is joined to every vertex in the i-th copy of P_m.

Thus, $P_n \diamond P_m$ admits Johan Coloring.

The Johan Coloring of the edge corona product of a path graph with a path graph is the (minimum degree of the corona product of $P_n \diamond P_m$) + 1.

$$\therefore J(P_n \diamond P_m) = \delta(P_n \diamond P_m) + 1.$$

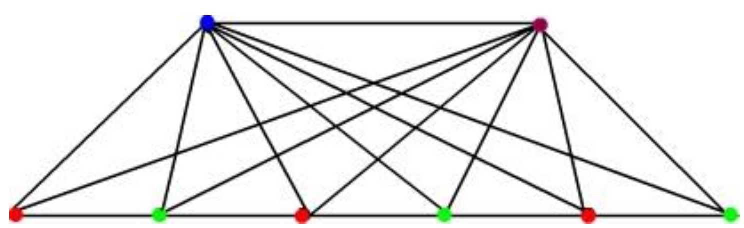

Fig. 1 Johan Coloring of $P_6 \diamond P_2$

3.2. Theorem

The edge corona product of the path graph with the cycle graph admits Johan Coloring.

Proof:

Let us consider a path graph P_n, n≥2, and a cycle graph C_m, where m is even.

Let the vertices and edges be

$V(P_n) = \{u_i: 1 \leq i \leq n\}$

$V(C_m) = \{v_i: 1 \leq i \leq m\}$

$E(P_n) = \{u_i u_{i+1}: 1 \leq i \leq n-1\}$

$E(C_m) = \{v_i v_{i+1}: 1 \leq i \leq m-1\} \cup \{v_1, v_m\}$

The edge corona product of P_n and C_m, denoted as $P_n \diamond C_m$, is obtained by taking one copy of Pn and |E(Pn)| copies of Cm and joining each end vertex of the i-th edge of Pn to every vertex in the i-th copy of Cm.

By the definition of Johan Coloring, every vertex of Pn is colored with colors c_1 and c_2. alternatively, and every vertex of C_m is colored with colors c_1, c_2, c_3. Since every end vertex of the i-th edge of P_n is joined to every vertex in the i-th copy of C_m.

Thus, $P_n \diamond C_m$ admits a Johan Coloring.

The Johan Coloring of the edge corona product of the path graph with the cycle graph is the minimum degree of the corona product $P_n \diamond C_m$

$$\therefore J(P_n \diamond C_m) = \delta(P_n \diamond C_m)$$

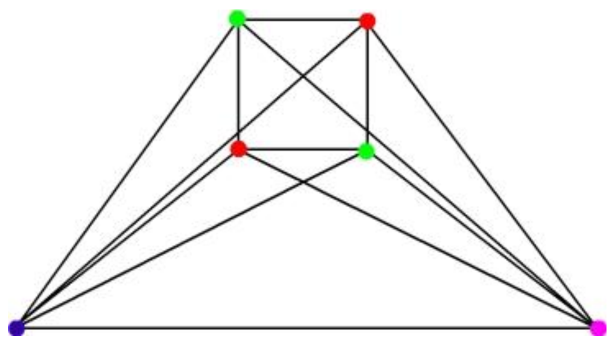

Fig. 2 Johan Coloring of $P_2 \diamond C_4$

3.3. Theorem

The edge corona product of a path graph with a complete graph admits Johan Coloring.

Proof:

Let us consider a path graph P_n, n≥2, and a complete graph K_m, m≥2.

Let the vertices and edges be

$V(P_n) = \{u_i: 1 \leq i \leq n\}$

$V(K_m) = m$

$E(P_n) = \{u_i u_{i+1}: 1 \leq i \leq n-1\}$

$E(K_m) = \dfrac{m(m-1)}{2}$

The edge corona product of P_n and K_m, denoted as $P_n \diamond K_m$, is obtained by taking one copy of P_n and $|E(P_n)|$ copies of K_m and joining each end vertex of the i-th edge of P_n to every vertex in the i-th copy of K_m .

By the definition of Johan Coloring, every vertex of Pn is colored with colors c_1 and c_2. alternatively, and every vertex of K_m is colored with colors $c_1, c_2, c_3, \ldots c_m$. Since every end vertex of the i-th edge of P_n is joined to every vertex in the i-th copy of K_m.

Thus, $P_n \diamond K_m$ admits Johan Coloring.

The Johan Coloring of the edge corona product of the path graph with the complete graph is (number of vertices of K_m) +2.

$$\therefore J(P_n \diamond K_m) = m+2.$$

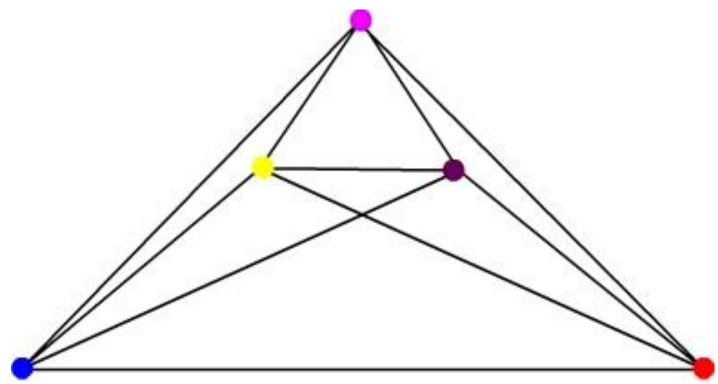

Fig. 3 Johan Coloring of $P_2 \diamond K_3$

3.4. Theorem

Edge Corona Product of path graph with star graph admits Johan Coloring

Proof:

Let us consider a path graph P_n, n≥2, and a star graph S_m, m≥2.

Let the vertices and edges be

$V(P_n) = \{u_i : 1 \leq i \leq n\}$

$V(S_m) = m + 1$

$E(P_n) = \{u_i u_{i+1} : 1 \leq i \leq n - 1\}$

$E(S_m) = m$

The edge corona product of P_n and S_m, denoted as Pn ⋄ Sm, is obtained by taking one copy of Pn and $|E(P_n)|$ copies of S_m and joining each end vertex of the i-th edge of Pn to every vertex in the i-th copy of S_m.

By the definition of Johan Coloring, every vertex of P_n is colored with colors c_1 and c_2 alternately, and every vertex of S_m is colored with colors c_1, c_2, c_3,... Since every end vertex of the i-th edge of P_n is joined to every vertex in the i-th copy of S_m.

Thus, $P_n \diamond S_m$ admits Johan Coloring.

The Johan Coloring of the edge corona product of the path graph with the star graph is the (minimum degree of edge corona product of $P_n \diamond S_m$)+1

$$\therefore J(P_n \diamond S_m) = \delta(P_n \diamond S_m) + 1.$$

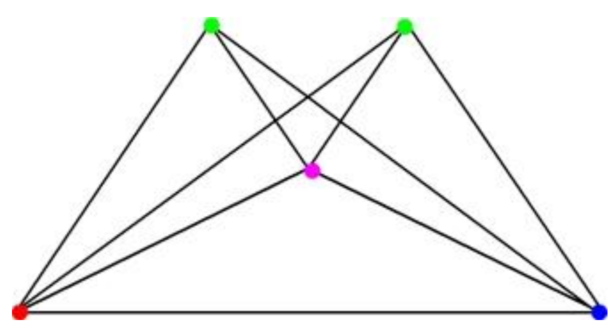

Fig. 4 Johan Coloring of $P_2 \diamond S_3$

3.5. Theorem

Edge Corona Product of path graph with wheel graph admits Johan Coloring.

Proof:

Let us consider a path graph P_n, n≥2, and a wheel graph W_m, where m is odd.

Let the vertices and edges be

$V(P_n) = \{u_i : 1 \leq i \leq n\}$

$E(P_n) = \{u_i u_{i+1}: 1 \le i \le n-1\}$

The Wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle.

$V(W_m) = \{v_i: 1 \le i \le m\} \cup \{v_0\}$, where v0 is the universal vertex and
$E(W_m) = \{v_{i+1}: 1 \le i \le m-1\} \cup \{v_1, v_m\}$

The edge corona product of P_n and Wm, denoted as $P_n \diamond W_m$, is obtained by taking one copy of P_n and $|E(P_n)|$ copies of W_m and joining each end vertex of the i-th edge of P_n to every vertex in the i-th copy of W_m.

By the definition of Johan Coloring, every vertex of Pn is colored with colors c_1 and c_2 alternately, and every vertex of Wm is colored with colors $c_{1,}$ c_2, c_3,.. Since every end vertex of the i-th edge of P_n is joined to every vertex in the i-th copy of W_m.

Thus, $P_n \diamond W_m$ admits Johan Coloring.

The Johan Coloring of the edge corona product of the path graph with the wheel graph is the (minimum degree of the edge corona product of $P_n \diamond W_m$).

$\therefore J(P_n \diamond W_m) = \Delta(P_n \diamond W_m)$

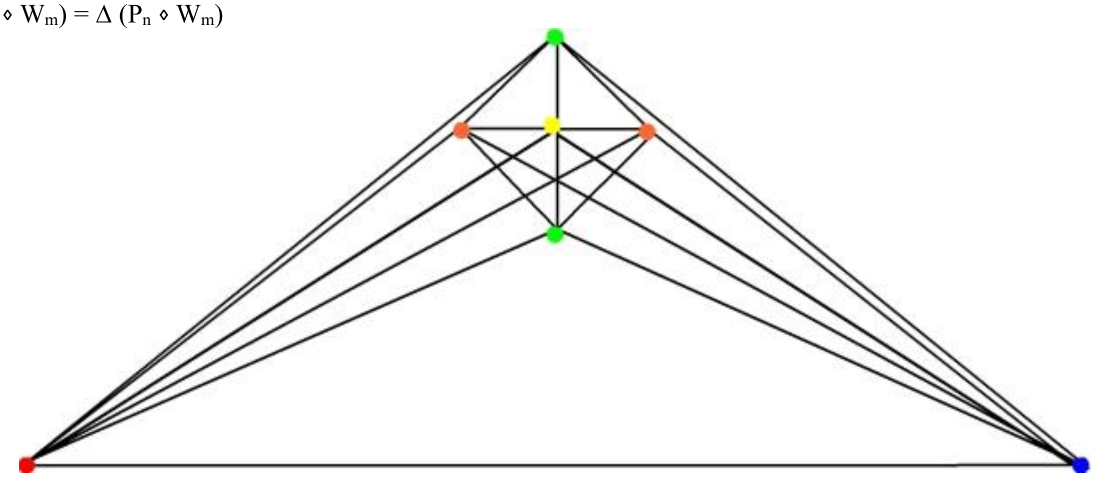

Fig. 5 Johan Coloring of $P_2 \diamond W_5$

4. Conclusion

In this paper, we have calculated the Johan chromatic number for the edge corona products of a path graph with path, complete, cycle, star, and wheel graphs. Future research will focus on determining the Johan chromatic number for edge corona products of other graph families.

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