International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I7P104

Volume 72 Issue 7, 25-35, July 2026

Review Article

Quantifying Meaning: A Review of Semantic Information Analysis, Uncertainties, and Possibilistic Restrictions in Hybrid Systems

Jayesh Vijay Rao Karanjgaonkar

Department of Mathematics, Swami Atmanand Government English & Medium Model College, Raipur, Chhattisgarh, India.

Corresponding Author : drjayeshk1303@gmail.com

Received: 26 May 2026 Revised: 01 July 2026 Accepted: 17 July 2026 Published: 30 July 2026

Abstract *- Classical information theory, originating with Shannon's foundational work [1], treats information as a purely syntactic quantity divorced from meaning. Intelligent systems operating in real-world environments must nonetheless process semantic content carried in natural language — content permeated by vagueness, fuzziness, ambiguity, and multimodal uncertainty. This article presents a structured review of the conceptual and mathematical landscape surrounding semantic information theory, proceeding from Shannon's technical framework through Weaver's three-level communication model, Floridi's strongly semantic theory, Zadeh's restriction-centred view of natural language, and contemporary rate-distortion-based semantic communication theory, toward a possibilistic approach suited to modern human-machine hybrid systems. The review is organised along four axes: (i) a rigorous taxonomy of uncertainty types along the dimensions of nature, object, and severity; (ii) a hierarchy of Levels Of Abstraction (LOA) linking raw sensor data to high-level semantic content; (iii) a critical, comparative appraisal of classical and contemporary methodologies, including the Bar-Hillel–Carnap paradox, Floridi's veridical resolution, and recent synonymous-mapping semantic communication theory; and (iv) the Natural Possibility (NP) Framework, a data-driven possibilistic account that converts unimodal frequency data into natural possibility distributions via a modal conversion rule, and defines workable measures — Proximity Value (PV), Elemental Proximity Value (EPV), Mean Proximity Value (MPV), and a semantic measure SM — for precisiating vague human commands in machine reasoning. The article identifies a specific gap in the existing literature — the absence of an operational bridge linking uncertainty taxonomy, the LOA hierarchy, and computable possibilistic measures — and evaluates the extent to which the possibilistic account addresses it, alongside its limitations relative to contemporary coding-theoretic semantic communication frameworks. Open research directions in approximate reasoning, explainable AI, and 6G semantic communication are identified.*

Keywords *- Hybrid Systems, Levels Of Abstraction, Possibilistic Restrictions, Possibility Theory, Semantic Information Theory*.

1. Introduction

1.1. Background and Motivation

Shannon's 1948 paper established information theory as the science of reliable data transmission, measuring information in terms of statistical surprise — the logarithmic reduction in uncertainty when a message is received [1]. This framework is mathematically elegant but deliberately semantic-free: a string of random characters and a poem carry identical information if they share the same probability distribution over their constituent symbols.

Warren Weaver, in his commentary accompanying Shannon's work, classified communication problems into three levels [14]: the technical problem of how accurately symbols can be transmitted; the semantic problem of how precisely transmitted symbols convey the desired meaning; and the effectiveness (control) problem of how effectively the received meaning affects behaviour in the desired way. Shannon's theory addresses only the first level; the second and third remain long-standing open problems that become critical the moment human cognition is coupled with machine intelligence.

Human thought and everyday discourse are saturated with imprecision: words such as warm, likely, or soon are not crisp predicates but elastic constraints on an underlying variable [17]. Formalising such constraints mathematically is the agenda of fuzzy set theory, possibility theory, and the broader field of generalised uncertainty measures [3, 8].

The proliferation of human-machine hybrid systems — autonomous vehicles interpreting driver intent, clinical decision-support systems parsing physician notes, conversational agents responding to natural language commands — makes it increasingly important to close what this article terms the intelligence gap: machines operate on crisp numerical data, while humans communicate through highly abstract, vague natural language. Bridging this gap requires a theory of semantic information that is mathematically tractable and computationally implementable.

1.2. Research Gap and Contribution

Two broad strands of contemporary work bear on this problem, and neither, taken alone, closes the intelligence gap. The first is philosophical and linguistic: Floridi's veridicality-based semantic theory [6] and Zadeh's restriction-centred view of natural language [19] give conceptually rich accounts of meaning and vagueness but stop short of prescribing how a possibility distribution or an alethic value is to be constructed automatically from empirical data. The second is coding-theoretic: recent semantic communication theory formalises meaning through synonymous mappings and rate-distortion bounds tailored to task-oriented transmission (Niu & Zhang 2024 [11]; Tao, Niu & Wu 2025 [13]; Ye, Wu, Fan & Fan 2025 [15]), but this literature is oriented toward channel-level optimisation rather than toward interpreting vague natural-language commands issued by a human operator to a machine sub-system.

What is comparatively underdeveloped is an operational bridge that (a) situates these accounts within a single taxonomy of uncertainty types, (b) locates them within a hierarchy of levels of abstraction (LOA) running from raw sensor signal to pragmatic effect, and (c) supplies a lightweight, data-driven procedure for converting empirical frequency data into a possibility distribution without requiring hand-crafted membership functions or externally assigned truth values. This is the gap the present review addresses. Its contribution is twofold: first, a comparative synthesis that places classical eliminative, veridical, restriction-centred, and contemporary coding-theoretic accounts of semantic information on a common footing (Section 4); and second, a detailed exposition of the Natural Possibility (NP) Framework as one candidate operational bridge, together with an explicit assessment of where it improves on, and where it falls short of, the alternatives reviewed (Sections 5–6).

1.3. Review Methodology

Sources were identified through structured searches of Google Scholar, Scopus, IEEE Xplore, arXiv, and the MDPI journal index, using combinations of the terms "semantic information theory", "possibility theory", "uncertainty taxonomy", "levels of abstraction", "semantic communication", and "possibilistic restriction". Foundational sources were drawn from the period 1948–2021, and contemporary sources were prioritised from the 2023–2026 window to ensure the review reflects the current state of the field. A source was included if it directly addressed semantic-level (as opposed to purely syntactic) information, formal models of uncertainty or vagueness, or possibilistic/fuzzy restriction; sources confined to Shannon-capacity or channel-coding results without a semantic treatment were excluded. Table 1 summarises the principal sources reviewed, their focus, and their relation to the present synthesis.

Table 1. Summary of principal sources reviewed and their relation to this article

Source	Year	Focus	Relation to this Review
Shannon [1]	1948	Syntactic information, channel capacity	Baseline against which semantic accounts are contrasted
Bar-Hillel & Carnap [9]	1953	Eliminative semantic content	Classical methodology motivates the veridicality critique
Zadeh [16–19]	1965–2009	Fuzzy sets, possibility, restriction	Conceptual foundation for possibilistic restriction
Dubois & Prade [3]	1988	Possibility theory formalism	Formal grounding for possibility distributions
Floridi [6]	2011	Strongly semantic, veridical information	Resolves the Bar-Hillel–Carnap paradox; benchmark for operationalisability
Bradley & Drechsler [2]	2014	Taxonomy of uncertainty	Basis of the three-dimensional taxonomy in Section 2
Niu & Zhang [11]	2024	Synonymous-mapping semantic communication theory	Contemporary coding-theoretic comparator (Section 4.4)

Tao, Niu & Wu [13]	2025	Semantic information theory and applications	A recent survey of the semantic-communication research front
Ye et al. [15]	2025	Semantic communication in the Internet of Vehicles	Illustrates the applied scope of coding-theoretic semantic methods
Förster et al. [7]	2025	Taxonomy for uncertainty-aware explainable AI	Motivates the XAI open-research direction in Section 6.4

1.4. Paper Organisation

Section 2 surveys the taxonomy and typology of uncertainty. Section 3 develops the hierarchy of levels of abstraction. Section 4 critically and comparatively examines historical and contemporary methodologies. Section 5 presents the NP Framework in formal detail, together with extended application scenarios. Section 6 discusses impact, comparative standing, limitations, and open problems. Section 7 concludes.

2. Taxonomy and Typology of Uncertainties

The word uncertainty is used informally to cover a heterogeneous family of phenomena. In natural language, the terms ambiguity, vagueness, fuzziness, possibility, and probability are often treated as synonyms, yet they correspond to mathematically and ontologically distinct structures [10, 12]. A rigorous framework for semantic information must distinguish these forms precisely.

2.1. Three Dimensions of Uncertainty

Following Bradley & Drechsler [2], uncertainty can be classified along three independent dimensions.

2.1.1. Dimension 1 — Nature of Uncertainty

Empirical (ontic) uncertainty arises from incomplete knowledge of a factual state of the world; normative/ethical uncertainty concerns how to evaluate or rank possible states when values are unclear; modal uncertainty concerns what is possible, necessary, or contingent, independent of any particular factual state.

2.1.2. Dimension 2 — Object of Uncertainty

Factual uncertainty concerns propositions about the actual world (“Is the temperature above 30°C?”); counterfactual/option uncertainty concerns propositions about hypothetical or future states.

2.1.3. Dimension 3 — Severity of Uncertainty

Severity is a meta-level assessment of how resistant a form of uncertainty is to reduction through information acquisition. Bradley & Drechsler [2] note that collapsing distinct types of uncertainty into a single category of “empirical state uncertainty” increases overall severity, because the modelling tool no longer captures the full structure of the problem.

2.2. Key Distinctions in Semantic Uncertainty

Definition 2.1 (Vagueness). A predicate P is vague if there exist borderline cases x for which neither $P(x)$ nor $\neg P(x)$ is definitively true [4]. Vagueness is modelled by graded membership (fuzzy sets) rather than classical two-valued membership.

Definition 2.2 (Fuzziness). Fuzziness measures the lack of sharp boundaries in a concept or class. A fuzzy set A in universe X is characterised by a membership function $\mu A: X \to [0,1]$, where $\mu A(x) = 1$ denotes full membership and $\mu A(x) = 0$ denotes non-membership [16].

Definition 2.3 (Possibility distribution). A possibility distribution on X is a function $\pi: X \to [0,1]$ with $\sup_{x\in X} \pi(x) = 1$, representing the degree to which each value of x is compatible with available knowledge [3].

Definition 2.4 (Probability distribution). A probability distribution $p: X \to [0,1]$ satisfies $\Sigma_{x\in X}\, p(x) = 1$ (discrete case) and models frequency of occurrence in repeated trials [8].

Remark 2.1. The fundamental distinction between possibility and probability is that possibility captures plausibility (what is consistent with available constraints), while probability captures likelihood (relative frequency or calibrated degree of belief). Every probability distribution induces a possibility distribution via $\pi(x) \geq p(x)$, but the converse does not hold in general [3].

Vagueness and uncertainty also interact via the non-transitivity of indiscriminability: if a is indiscriminable from b, and b from c, it does not follow that a is indiscriminable from c [5]. This phenomenon underlies the sorites paradox and motivates graded rather than sharp semantic categories.

2.3. Tabular Classification

Table 2 summarises the principal uncertainty types encountered in semantic information analysis.

Table 2. Classification of uncertainty types relevant to semantic information

Type	Mathematical Model	Reducible by Data?	Typical Source
Stochastic uncertainty	Probability measure P	Yes	Sensor noise, sampling
Fuzziness/vagueness	Fuzzy set μA	Partially	Natural language predicates
Possibilistic uncertainty	Possibility distribution π	Partially	Linguistic constraints
Ambiguity (lexical)	Multiple crisp meanings	Yes (context)	Polysemy, homonymy
Ignorance (total)	Vacuous possibility $\pi \equiv 1$	No	Unknown unknowns
Epistemic uncertainty	Interval / set-valued	Yes (more data)	Incomplete knowledge
Modal uncertainty	Modal logic operators	No (in principle)	Counterfactuals

3. Levels of Abstraction in Information Processing

The transformation from raw sensor data to high-level human meaning does not occur in a single step; it progresses through a hierarchy of Levels Of Abstraction (LOA), a concept developed within the philosophy of information by Floridi [6].

3.1. The LOA Hierarchy

Definition 3.1 (Level of Abstraction, Floridi 2011). A level of abstraction L is a finite, non-empty set of observables {o1, o2, …, on}, each typed (given a domain and range), together with a set of relations and constraints among them. Different LOAs yield different informational models of the same system.

In a human-machine hybrid system, the LOA hierarchy can be articulated as follows.

LOA 1 (Physical / signal level): raw transducer outputs — voltages, byte streams, pixel values. Uncertainty is predominantly stochastic (sensor noise), modelled by probability distributions.

LOA 2 (Data level): structured, calibrated measurements. Uncertainty reduces but retains an epistemic component (calibration error, quantisation).

LOA 3 (Information level): patterns extracted from data (features, statistics, classifications). Uncertainty becomes increasingly epistemic and context-dependent.

LOA 4 (Semantic level): meaning assigned by an interpreter, expressed in natural language. Uncertainty is predominantly possibilistic and fuzzy: the sentence "the room is warm" does not specify an exact temperature.

LOA 5 (Pragmatic / effectiveness level): the impact of semantic content on action. Uncertainty concerns the alignment of intent with outcome.

A key observation is that as one ascends the LOA hierarchy, uncertainty becomes less amenable to reduction by data acquisition alone and increasingly requires contextual and possibilistic modelling.

3.2. Semantic Noise

At the semantic level, inaccurate or false information introduces semantic noise — analogous to channel noise at the physical level but qualitatively different in character. Forms of semantic noise include misinformation (unintentionally false), disinformation (deliberately false), and under-specification (truthful but insufficiently precise).

Unlike physical noise, semantic noise cannot be reduced by increasing signal power or adding redundancy; it requires truth-verification mechanisms and contextual disambiguation [6].

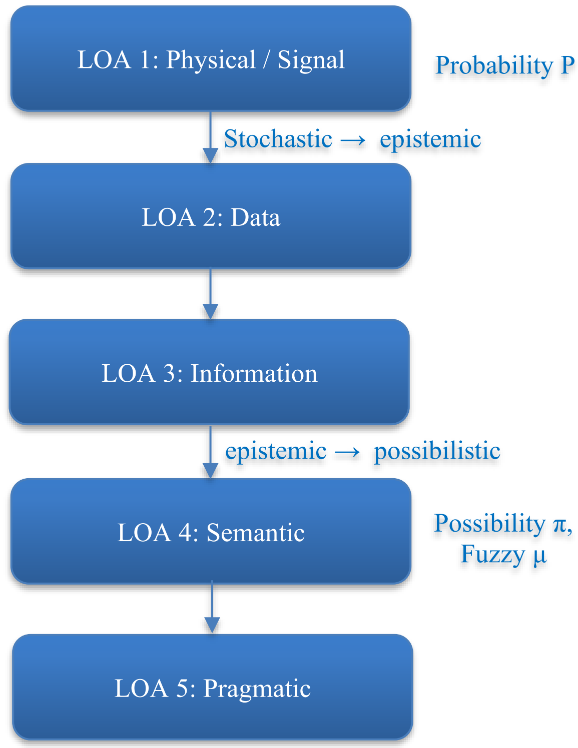

Fig. 1 The LOA hierarchy in a human-machine hybrid system. As abstraction increases, the dominant uncertainty model shifts from probabilistic to possibilistic

4. Methodologies of Semantic Information Analysis

Several distinct programmes have attempted to put semantic information on a mathematical footing. This section reviews four such programmes, spanning classical eliminative theory to contemporary coding-theoretic semantic communication, and identifies their respective limitations.

4.1. Classical Semantic Theory: Bar-Hillel and Carnap

Bar-Hillel & Carnap [9] proposed measuring the semantic content of a proposition p in a formal language by the proportion of state descriptions it rules out. In a language with n atomic propositions, there are 2^n state descriptions; if p is consistent with k of them, its semantic content is given by Eq. (1).

$$Cont(p) = 1 - \frac{k}{2^n}, \quad Inf(p) = -\log_2\left(\frac{k}{2^n}\right) \tag{1}$$

This treats informativeness as eliminative content: the more possible worlds a statement rules out, the more informative it is.

Example 4.1 (Bar-Hillel–Carnap Paradox). Let $p = q \wedge \neg q$ for any proposition q. Then p is a contradiction and is consistent with $k = 0$ state descriptions. From Eq. (1), $Cont(p) = 1$ and $Inf(p) = +\infty$: a self-contradictory statement receives maximum semantic information under this measure, which is paradoxical.

The paradox arises because eliminative content does not respect truth: a contradiction rules out everything, not because it is maximally informative but because it is false.

4.2. Strongly Semantic Theory: Floridi

Floridi [6] resolves the Bar-Hillel–Carnap Paradox by insisting that genuine semantic information must be well-formed, meaningful, and truthful (the "veridicality thesis"). On this view, a contradiction fails the truthfulness condition and is therefore not information at all.

Floridi's informativeness is measured relative to an alethic value $\alpha \in [0,1]$ encoding proximity to actual truth, with the informativeness curve taking the parabolic form of Equation (2), which is maximised at $\alpha = 0.5$: a statement that is neither trivially true nor trivially false carries the most information.

$$\Phi(\alpha) = \alpha(1 - \alpha) \quad (2)$$

Limitation: Floridi's framework is philosophically powerful but difficult to operationalise in engineering systems. The alethic value α must be externally assigned; the theory does not specify a mechanism for extracting α automatically from sensor data or corpus statistics.

4.3. Restriction-Centred Theory: Zadeh

Zadeh [19] proposed that information is restriction: a natural language statement such as "the traffic is heavy" is not a crisp proposition but an elastic, imprecise constraint on the set of possible traffic states. Zadeh formalised this as a possibilistic restriction R(X) on a variable X, represented by a possibility distribution πX. Computing with words (CWW) then becomes computation with possibility distributions, enabling machines to reason approximately under vague human instructions [18].

Limitation: Zadeh's framework does not prescribe how to construct the possibility distribution πX automatically from real data — a gap addressed directly by the NP Framework discussed in Section 5.

4.4. Contemporary Semantic Communication Theory

A distinct, coding-theoretic strand has developed rapidly since 2022 under the banner of semantic communication. Niu & Zhang [11] formalise a mathematical theory of semantic information built on synonymous mappings between physical and semantic symbol sets, deriving semantic source-, channel-, and rate-distortion coding theorems that extend Shannon's classical bounds. Tao, Niu & Wu [13] survey the resulting research front, spanning task-oriented transmission, semantic entropy, and semantic rate-distortion theory, while Ye, Wu, Fan & Fan [14] document applied deployments in the Internet of Vehicles, covering semantic extraction, system architecture, and resource allocation for traffic perception and intelligent driving.

Limitation: this body of work is oriented toward channel-level fidelity and transmission efficiency for machine-to-machine or machine-to-human data flows; it does not directly address the construction of possibility distributions from vague, human-issued natural-language commands, which is the specific problem addressed by the NP Framework in Section 5.

4.5. Comparative Summary of Methodologies

Table 3 places the four methodologies reviewed above on a common footing across four dimensions: whether the construction of the semantic measure is data-driven, whether it is directly operationalisable in an engineering pipeline, its characteristic output, and its principal domain of application.

Table 3. Comparative summary of semantic information methodologies

Framework	Data-Driven?	Characteristic Output	Principal Domain
Bar-Hillel–Carnap [9]	No (logical)	Eliminative content measure	Formal semantics
Floridi (veridical) [6]	No (externally assigned α)	Alethic informativeness Φ(α)	Philosophy of information
Zadeh (restriction) [19]	Partially (expert-elicited)	Possibility distribution πX	Computing with words
Synonymous-mapping SIT [11, 13]	Yes (learned mappings)	Semantic entropy / rate-distortion bound	Semantic communication, 6G
NP Framework (Section 5)	Yes (frequency data)	Natural possibility distribution πM, PV/EPV/MPV/SM	Human-machine command interpretation

5. The Natural Possibility (NP) Framework

The NP Framework, developed in the author's doctoral research, provides an end-to-end, data-driven methodology for converting unimodal empirical frequency distributions into natural possibility distributions, and for computing semantic proximity measures that support approximate reasoning in hybrid systems. Later subsections refer to this source simply as "the NP Framework" rather than repeating the citation at each definition.

5.1. The Intelligence Gap

A human-machine hybrid system faces a fundamental intelligence gap: the machine sub-system processes crisp, numerical sensor data (LOA 1–3), while the human sub-system communicates via natural language operating at LOA 4. Existing approaches either require manual construction of membership functions by domain experts or lose semantic content by mapping natural language to crisp rules. The NP Framework addresses this by inserting a modal conversion layer between the data level and the semantic level, as illustrated in Figure 2.

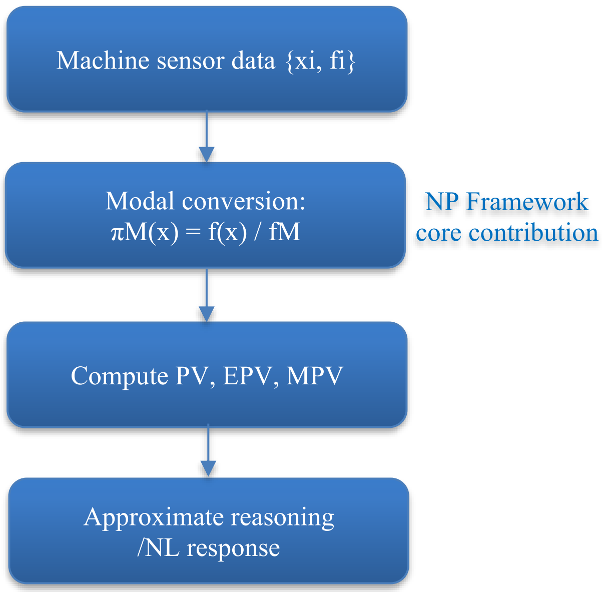

Fig. 2 Architecture of the NP Framework within a hybrid human-machine system. The highlighted block (modal conversion layer) is the core contribution, automatically bridging LOA 2 data and LOA 4 semantic content

5.2. Modal Conversion: From Frequency to Possibility

Definition 5.1 (Unimodal frequency distribution). Let X = {x1, x2, …, xn} be a finite domain observed via sensor readings, and let f: X → $\mathbb{N}_0$ be the observed frequency function satisfying Σf(xi) = N (total observations). The distribution is unimodal if there exists a unique modal value fM = maxx∈X f(x) achieved at a unique x* ∈ X.

Definition 5.2 (Natural possibility distribution). Given a unimodal frequency distribution (X, f, fM), the natural possibility distribution πM: X → [0,1] is defined by Eq. (3).

$$\pi_M(x_i) = \frac{f(xi)}{fM}, \quad i = 1, 2, \ldots, n \tag{3}$$

Remark 5.1. By construction, maxx πM(x) = 1 (the normalisation condition for possibility distributions is automatically satisfied) and πM(xi) ≤ 1 for all i. The distribution assigns full possibility to the modal value — the most frequently observed state — and decreasing possibility to states observed less frequently, encoding the intuition that the most typical outcome is the most plausible outcome.

Proposition 5.1 (Consistency with Zadeh's framework). The natural possibility distribution πM in Definition 5.2 is consistent with Zadeh's possibilistic restriction framework [19]: it represents the elastic constraint "X takes its most typical value with full possibility, and other values with proportionally reduced possibility."

5.3. Proximity Value and Elemental Proximity Value

The core semantic measures of the NP Framework precisiate how close an observed data value is to the modal (prototypical) value.

Definition 5.3 (Proximity Value). Let (X, f, fM) be a unimodal frequency distribution with modal value fM at x*. For any xi ∈ X, the Proximity Value is given by Eq. (4).

$$PV(x_i) = \pi_M(x_i) = \frac{f(xi)}{fM} \tag{4}$$

Definition 5.4 (Elemental Proximity Value). The Elemental Proximity Value quantifies the contribution of each xi to aggregate semantic proximity, weighted by its relative frequency, as in Eq. (5).

$$EPV(x_i) = PV(x_i) \cdot \frac{f(xi)}{N} = \frac{f(xi)^2}{fM \cdot N} \tag{5}$$

Definition 5.5 (Mean Proximity Value). The Mean Proximity Value aggregates EPVs over the entire domain, as in Eq. (6).

$$MPV = \Sigma i\ EPV(xi) = \frac{1}{fM \cdot N} \Sigma i\ f(xi)^2 \tag{6}$$

Remark 5.2. MPV has the character of a normalised second moment of the frequency distribution, penalised by the modal frequency. It is related to the Herfindahl–Hirschman index in economics and to the ℓ²-norm of the frequency vector. High MPV indicates that observations are concentrated near the modal value (sharp possibility distribution, high specificity); low MPV indicates a flat, dispersed distribution.

5.4. Specificity and Semantic Measure

Definition 5.6 (Specificity). The Specificity of the natural possibility distribution πM is Sp(πM) = 1 − MPV, measuring the sharpness (low spread) of the possibility distribution. High specificity corresponds to a highly informative, precise linguistic constraint.

Definition 5.7 (Semantic Measure). The Semantic Measure combines specificity and possibilistic content into a scalar index, given by Eq. (7).

$$SM = Sp(\pi M) \cdot \pi M(x*) = (1 - MPV) \cdot 1 = 1 - MPV \tag{7}$$

Remark 5.3. SM ∈ [0,1]. When SM → 1, the distribution is sharp and nearly all observations coincide with the modal value — the natural language constraint "precisiated" to near-crisp form. When SM → 0, the distribution is maximally diffuse (total ignorance).

5.5. Illustrative Example

Example 5.1 (Room temperature monitoring). A temperature sensor records the following frequency distribution over N = 50 readings in units of °C (Table 4).

Table 4. Frequency-to-possibility conversion for the room-temperature example

xi (°C)	f(xi)	πM(xi) = f(xi)/25
22	2	0.08
23	5	0.20
24	12	0.48
25	25	1.00
26	4	0.16
27	2	0.08

Here fM = 25 at x* = 25°C. A human operator issues the command "the room is warm"; the system maps this to the natural possibility distribution above, treating 25°C as the prototypical referent of "warm" in this context.

$$MPV = \frac{1}{25\times 50}(4 + 25 + 144 + 625 + 16 + 4) = \frac{818}{1250} \approx 0.654 \tag{8}$$

$$SM = 1 - 0.654 = 0.346 \tag{9}$$

The moderate SM reflects genuine spread around the modal temperature, consistent with natural variation in a room. The machine sub-system can reason about "warm" using the computed πM rather than requiring a hand-crafted membership function.

5.6. Extended Application Scenarios

Two brief scenarios beyond the temperature example illustrate the scope of the modal conversion procedure. In autonomous-vehicle teleoperation, a supervisory operator may issue the vague instruction "slow down a little"; historical logs of operator-issued deceleration values for that phrase form a frequency distribution over discrete speed decrements, from which πM and SM can be computed to select the deceleration value the vehicle controller should treat as most plausible, while retaining a graded set of acceptable alternatives. In clinical alert triage, nurse-recorded frequency counts of the linguistic label "mildly elevated" applied to a given vital-sign reading (for example, heart rate) can similarly be converted into a possibility distribution over numeric ranges, allowing a decision-support system to align its numeric thresholds with the clinical staff's actual usage of the term rather than with a fixed, externally imposed cut-off. In both cases, the procedure is identical to that of Section 5.2–V-D; only the domain of X and the source of the frequency counts change.

6. Discussion: Impact, Limitations, and Open Problems

6.1. Bridging the Intelligence Gap

The principal contribution of the NP Framework is operational: it provides a data-driven pipeline from raw sensor observations to semantic possibility distributions, without requiring expert elicitation of fuzzy membership functions. This has immediate relevance for hybrid systems in which a robotic system must interpret vague natural language commands ("move a little to the left"), a clinical decision-support system must align sensor-derived patient measurements with clinical linguistic categories ("mildly elevated", "borderline"), or a smart home system must translate occupant comfort preferences into actuator setpoints.

By grounding possibility distributions in empirical frequency data, the framework also provides a calibration that is absent from purely expert-driven approaches: the modal value corresponds to the empirically most frequent observation, not an arbitrary expert judgment.

6.2. Comparative Standing Relative to Contemporary Approaches

Relative to the synonymous-mapping semantic communication theory of Niu & Zhang [11] and Tao, Niu & Wu [13], the possibilistic account traded here favours interpretability and low data requirements over asymptotic optimality. The synonymous-mapping approach derives coding theorems with provable rate-distortion bounds, which the possibilistic account does not attempt to match; its contribution is instead a closed-form, easily computed distribution (Equation 3) that requires only a histogram of past observations rather than a learned synonymous mapping trained on large paired corpora. In settings where such training data are unavailable — for example, a newly deployed clinical instrument or a bespoke robotic command vocabulary with only a modest operational history — the modal-conversion procedure is expected to be applicable where a trained semantic encoder would not yet be. Conversely, in high-throughput channel-limited applications such as 6G semantic transmission, the coding-theoretic approach offers rate-distortion guarantees that the possibilistic account does not provide; the two approaches are complementary rather than competing, and the possibilistic account should be read as addressing the natural-language-command interpretation problem rather than the channel-fidelity problem.

Relative to Floridi's veridical theory, the possibilistic account replaces an externally assigned alethic value α with an empirically computed modal frequency ratio, which removes the need for a human judge to assign a truth-proximity score to each candidate meaning; the trade-off is that the possibilistic account is silent on the truthfulness of the underlying data, a question veridical theory treats as central. The NP Framework should therefore be read as complementary to, rather than a replacement for, existing truth-conditional and coding-theoretic accounts of semantic information [3, 8].

6.3. Limitations

(a) Unimodality assumption: the modal conversion rule in Definition 5.2 requires the frequency distribution to be unimodal. Multimodal distributions — common in heterogeneous sensor data — require extension, for example, a mixture-of-possibility-distributions.

(b) Stationarity: the framework assumes that the frequency distribution is stationary over the observation window. In dynamic environments, adaptive or sliding-window versions are needed.

(c) High-dimensional extensions: the current formulation addresses univariate X. Extension to multivariate and structured data (text, images) requires joint possibility distributions and copula-based methods.

(d) Validation breadth: empirical validation of the framework across diverse application domains, beyond the illustrative examples in Section 5, would strengthen the case for its generality; this is identified as a priority for future work rather than a claim resolved by the present review.

6.4. Open Research Directions

6.4.1. R1. Multimodal NP Framework

Extend the modal conversion to mixture models, enabling handling of bimodal or polymodal frequency data.

6.4.2. R2. Adaptive Possibility Updating

Develop a Bayesian-style updating rule for πM as new observations arrive, analogous to Dempster–Shafer belief revision.

6.4.3. R3. NP Framework for 6G Semantic Communication

6G networks are expected to transmit semantic content rather than raw bits [10]. Integrating the modal-conversion layer with the rate-distortion bounds of synonymous-mapping semantic communication theory [11] is a natural next step.

6.4.4. R4. Explainable AI (XAI) via Possibilistic Explanations

Recent taxonomies of uncertainty-aware XAI identify data, model, method, and human uncertainty as distinct sources requiring distinct treatment (Förster et al. 2025); the NP Framework's possibility distributions offer one principled way to construct explanations addressing the data-uncertainty component of that taxonomy.

6.4.5. R5. Chronobiological and Health Informatics Applications

Vague circadian-rhythm data ("sleep quality", "alertness") are natural candidates for NP-based semantic encoding, linking wearable sensor data to clinical language.

6.4.6. R6. Axiomatic Characterisation

A uniqueness theorem establishing that πM in Eq. (3) is the unique possibility distribution satisfying a specified set of axioms (modality, proportionality, normalisation) would strengthen the theoretical foundations.

7. Conclusion

This review has traced the conceptual and mathematical development of semantic information theory from Shannon's purely syntactic framework, through Weaver's three-level model, the Bar-Hillel–Carnap classical approach and its paradox, Floridi's veridical resolution, Zadeh's restriction-centred paradigm, contemporary synonymous-mapping semantic communication theory, and the possibilistic NP Framework.

Three findings follow from the comparative synthesis in Sections 4 and 6. First, no single existing methodology closes the intelligence gap between crisp machine perception and vague human natural language on its own: philosophical accounts (Floridi, Zadeh) under-specify the construction procedure, while coding-theoretic accounts (synonymous-mapping semantic communication) are oriented toward channel fidelity rather than command interpretation. Second, the NP Framework's data-driven modal conversion from empirical frequency distributions to natural possibility distributions, together with its suite of semantic measures (PV, EPV, MPV, SM), offers one operationally lightweight bridge, at the cost of the unimodality and stationarity assumptions detailed in Section 6.3. Third, the remaining research gap is empirical rather than conceptual: validation of the modal-conversion procedure across heterogeneous, multimodal, real-world data streams, and its integration with rate-distortion-based semantic communication theory, are identified as the most immediate next steps. The intersection of information theory, fuzzy and possibilistic systems, and natural language processing is expected to remain a juncture of growing importance as intelligent hybrid systems become pervasive, and the comparative account offered here is intended as a starting point for that continuing research.

Acknowledgment

The author thanks Prof. Purushottam Jha (Pt. Ravishankar Shukla University, Raipur) for guidance during doctoral work that forms part of the foundation of this review.

References

[1] Claude Elwood Shannon, "A Mathematical Theory of Communication," *The Bell System Technical Journal*, vol. 27, no. 3, pp. 379-423, 1948. [CrossRef] [Google Scholar] [Publisher Link]

[2] Richard Bradley, and Mareile Drechsler, "Types of Uncertainty," *Knowledge*, vol. 79, no. 6, pp. 1225-1248, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[3] Didier Dubois, and Henri Prade, *Possibility Theory*, Granular, Fuzzy, and Soft Computing, New York, NY: Springer US, pp. 859-876, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[4] Dorothy Edgington, "Validity, Uncertainty and Vagueness," *Analysis*, vol. 52, no. 4, pp. 193-204, 1992. [CrossRef] [Google Scholar] [Publisher Link]

[5] Paul Égré, and Denis Bonnay, "Vagueness, Uncertainty and Degrees of Clarity," *Synthese*, vol. 174, no. 1, pp. 47-78, 2009. [CrossRef] [Google Scholar] [Publisher Link]

[6] Luciano Floridi, *The Philosophy of Information*, Oxford: Oxford University Press, 2011. [CrossRef] [Publisher Link]

[7] Maximilian Förster et al., "A Taxonomy for Uncertainty-Aware Explainable AI," *Proceedings European Conference on Information Systems*, pp. 1-8, 2025. [Google Scholar] [Publisher Link]

[8] George J. Klir, and Richard M. Smith, "On Measuring Uncertainty and Uncertainty-based Information: Recent Developments," *Annals of Mathematics and Artificial Intelligence*, vol. 32, no. 1, pp. 5-33, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[9] Yehoshua Bar-Hillel, and Rudolf Carnap, "Semantic Information," *The British Journal for the Philosophy of Science*, vol. 4, no. 14, pp. 147-157, 1953. [CrossRef] [Google Scholar] [Publisher Link]

[10] Victor Raskin, and Julia M. Taylor, "Fuzziness, Uncertainty, Vagueness, Possibility, and Probability in Natural Language," *2014 IEEE Conference on Norbert Wiener in the 21st Century*, Boston, MA, USA, pp. 1-6, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[11] Kai Niu, and Ping Zhang, "A Mathematical Theory of Semantic Communication," *arXiv preprint*, pp. 1-6, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[12] Om Prakash Singh, and Manoj E. Patil, "Analysis of Ambiguity, Vagueness, Fuzziness, Uncertainty, Possibility and Probability in the Natural Language Semantics with Fuzzy Logic," *International Research Journal on Advanced Engineering Hub*, vol. 2, no. 5, pp. 1478-1483, 2024. [CrossRef] [Publisher Link]

[13] Meixia Tao, Kai Niu, and Youlong Wu, "Semantic Information Theory and Applications," *Entropy*, vol. 27, no. 11, pp. 1-4, 2025. [CrossRef] [Google Scholar] [Publisher Link]

[14] Warren Weaver, "Recent Contributions to the Mathematical Theory of Communication," *A Review of General Semantics*, vol. 74, no. 1-2, pp. 136-157, 2017. [Google Scholar] [Publisher Link]

[15] Sha Ye et al., "A Survey on Semantic Communications in Internet of Vehicles," *Entropy*, vol. 27, no. 4, pp. 1-49, 2025. [CrossRef] [Google Scholar] [Publisher Link]

[16] Lotfi A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965. [CrossRef] [Google Scholar] [Publisher Link]

[17] Lotfi A. Zadeh, "Fuzzy Sets as a Basis for a Theory of Possibility," *Fuzzy Sets and Systems*, vol. 1, no. 1, pp. 3-28, 1978. [CrossRef] [Google Scholar] [Publisher Link]

[18] Lotfi A. Zadeh, "From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 46, no. 1, pp. 105-119, 1999. [CrossRef] [Google Scholar] [Publisher Link]

[19] Lotfi A. Zadeh, "Toward Extended Fuzzy Logic – A First Step," *Fuzzy Sets and Systems*, vol. 160, no. 21, pp. 3175-3181, 2009. [CrossRef] [Google Scholar] [Publisher Link]