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Original Article

Matrix Representation of Fuzzy Soft Graphs

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Abstract *- In this paper, a Fuzzy Soft Graph Matrix (FSGM) is formulated to represent fuzzy soft graphs in uncertain environments. This framework performs the fundamental operations such as union, intersection and complement. The effectiveness and applicability of the above operations in FSGMs are validated through suitable illustrations and a real-world application.*

Keywords *- Fuzzy Soft Set, Fuzzy Soft Graph, Fuzzy Soft Graph Matrix.*

1. Introduction

Molodtsov [1] developed the soft set theory as a generic mathematical framework that handles uncertainty without imposing limiting criteria to overcome these restrictions. Also, the Soft set theory has garnered interest since its inception, due to its adaptability and efficacy in simulating the unclear and complicated scenarios.

The Fuzzy soft sets enable parameterization and graded membership to be incorporated in a single framework, which was developed as a result of merging the soft set theory with fuzzy set theory. When compared to the traditional fuzzy or soft models by themselves, this combination provides a more comprehensive depiction of uncertainty. At the same time, fuzzy graph theory provides a useful tool for modeling systems where relationships between elements are not well defined. Also, the Fuzzy graphs provide a natural method of modeling imperfect connections in real-time and decision-oriented systems by giving vertices and edges membership values [2].

Expanding on these concepts, [3] presented the concept of fuzzy soft graphs and studied their basic operations, like intersection and union. Through their study, the basic structure of fuzzy soft graphs was developed, and its applicability to parameter-dependent systems was shown. The theoretical underpinnings of the soft graph models under uncertainty were later strengthened by Akram and Nawaz [4], who further developed this framework by defining the new operations and classifications. Although fuzzy soft graphs provides the strong modeling foundation, it has the difficulties in describing and manipulating them computationally, specifically when dealing with big systems or algorithmic implementations. By transforming the graph structures into numerical forms appropriate for algebraic analysis and computer processing, matrix representations are important to classical graph theory [5, 6]. Although, because fuzziness and parameter dependency coexist in fuzzy soft graphs, traditional adjacency matrices are not directly applicable to them.

Motivated by these concepts, this paper suggests the fuzzy soft graph matrix, that is matrix representation of the fuzzy soft graphs. This style makes the mathematical analysis easier and provides a methodical and condensed manner to store fuzzy soft graph information. Within the matrix framework, basic operations such as the union, intersection, complement, and Cartesian product are developed, and their key characteristics are examined. Also, the decision-making application demonstrates the practical utility of fuzzy soft graph matrices, emphasizing their efficiency in managing the imprecise and multi-parameter information.

2. Preliminaries

Definition 2.1 [3]

Let U, the universal set and A, the set of the parameters. P, power set of U. Then mapping $\mathcal{F}_A: A \to P$ is known as the fuzzy soft set over the pair (U, A), where $\mathcal{F}_A(a) = \mu^a_{\mathcal{F}_A}$, represents the collection of all fuzzy subsets of U, for each parameter $a \in A$.

This collection of all fuzzy soft sets defined on the pair (U, A) is represented by $FS(U, A)$.

Definition 2.2 [3]

Let V be a non-empty set of vertices and $\rho : V \to [0,1]$. Again, let E be a set of edges, $E = V \times V$ and $\mu : E \to [0,1]$ such that

$$\mu(x, y) \leq \rho(x) \wedge \rho(y) \ \forall\ (x, y) \in V \times V.$$

Then the pair $G = (\rho, \mu)$ is called a fuzzy graph over the set V. Here ρ and μ are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph $G = (\rho, \mu)$.

Definition 2.3 [3]

Let a simple graph $G^* = (V, E)$ where, $V = \{ v_1, v_2, v_3, \dots, v_n\}$ be the non-empty set of the vertices, E, the set of edges, if A represent the set of parameters and $A \subseteq E$.

A fuzzy soft graph is the ordered pair.

$$G_{(A,V)} = (G^*, \rho, \mu)$$

i) $\rho_a : A \to P(V)$ is a fuzzy soft vertex function

ii) $\mu_a : A \to P(V \times V)$ is a fuzzy soft edge function such that,

$\mu_a(v_i, v_j) \leq \min\{\rho_a(v_i),\ \rho_a(v_j)\}$ holds for all $a \in A$ and for all $i, j = 1,2, \dots, n$.

Here, $P(V)$ denotes the fuzzy power set of V and $P(V \times V)$denotes the fuzzy power set of E.

Example 2.4

The simple graph G^* with the set of vertices $V = \{v_1, v_2, v_3, v_4\}$ and a set of edges $E = \{e_1, e_2, e_3, e_4, e_5\}$ as shown in Figure.

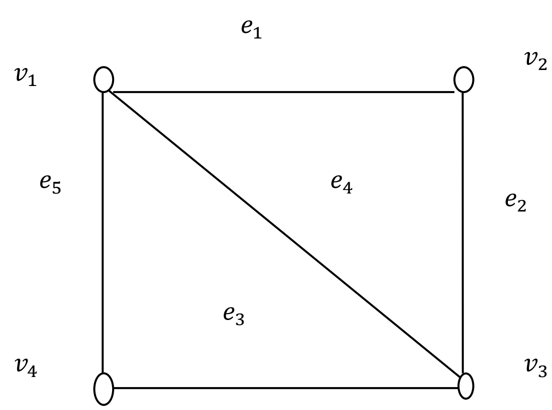
Consider parameter set $A = \{a_1, a_2, a_3\}$ and the fuzzy soft vertex function $\rho: A \to P(V)$ is defined as follows:

$$\rho(a_1) = \{v_1|0.6, v_2|\ 0.8, v_3|0, v_4|\ 0.5\ \}$$
$$\rho(a_2) = \{v_1|0.4, v_2|\ 0, v_3|0.7, v_4|\ 0.5\ \}$$

The fuzzy soft edge function μ is defined as follows:

$$\mu(a_1) = \{e_1|0.2, e_2|\ 0, e_3|0, e_4|\ 0, e_5|0.4\}$$
$$\mu(a_2) = \{e_1|0, e_2|\ 0, e_3|0.5, e_4|\ 0.6, e_5|0.4\}$$

Therefore, $G_{(A,V)}\ (a_1)$ is represented as $(\rho(a_1), \mu(a_1))$, and $G_{(A,V)}(a_2)$ is represented as $(\rho(a_2), \mu(a_2))$, both of which are FSG of G^* as shown in Figure 1.



G^*

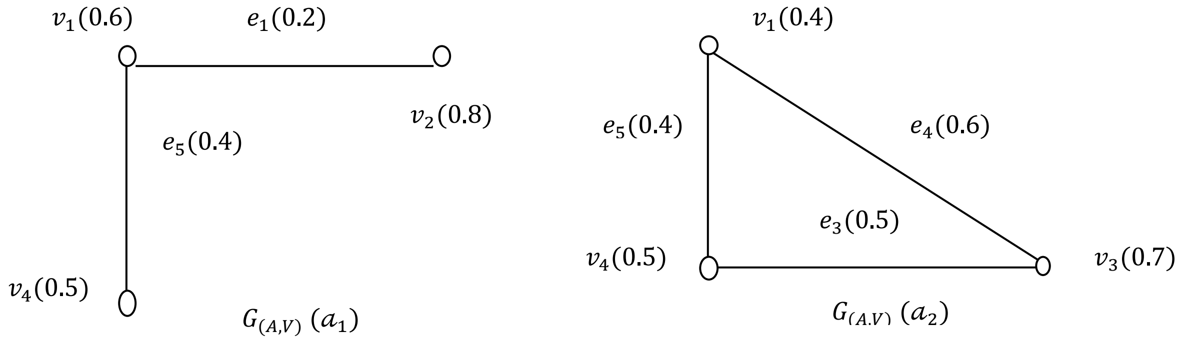


Fig. 1 Fuzzy soft graphs corresponding to the given parameters

Definition 2.5 [3]

Consider, $G_{(A,V)}\ (a_1) = (G^*, \rho_{a_1}, \mu_{a_1})$ and $G_{(A,V)}\ (a_2) = \left(G^*, \rho_{a_2}, \mu_{a_2}\right)$ be two fuzzy soft graphs defined on G^*. Then the union of two FSG $G_{(A,V)}\ (a_1)$ and $G_{(A,V)}(a_2)$ is defined by the,

$$G_{(A,V)}(a_1)\ \cup\ G_{(A,V)}(a_2) = G_{(A,V)}\ (a) = (G^*, \rho_a, \mu_a)$$

where, $\rho_a(v_i)\ = \max\{\rho_{a_1}, \rho_{a_2}\}$ for all $v_i \in V$ and for all $a \in A$, also $\mu_a\left(v_i,\ v_j\right) = \min\{\mu_{a_1}\left(v_i,\ v_j\right) \vee \mu_{a_2}\left(v_i,\ v_j\right)\}\ if\ v_i$, $v_j \in V$ and for all $a \in A$.

Definition 2.6 [3]

Consider, $G_{(A,V)}\ (a_1) = (G^*, \rho_{a_1}, \mu_{a_1})$ and $G_{(A,V)}\ (a_2) = \left(G^*, \rho_{a_2}, \mu_{a_2}\right)$ be two fuzzy soft graphs defined on G^*. Then the intersection of two FSG $G_{(A,V)}\ (a_1)$ and $G_{(A,V)}(a_2)$is defined by,

$$G_{(A,V)}(a_1)\ \cap\ G_{(A,V)}(a_2) = G_{(A,V)}\ (a) = (G^*, \rho_a, \mu_a)$$

where, $\rho_a(v_i)\ = \min\{\rho_{a_1}, \rho_{a_2}\}$ for all $v_i \in V$ and for all $a \in A$, also $\mu_a\left(v_i,\ v_j\right) = \max\left\{\mu_{a_1}\left(v_i,\ v_j\right) \vee \mu_{a_2}\left(v_i,\ v_j\right)\right\}\ if\ v_i,\ v_j \in V$ and for all $a \in A$.

Definition 2.7 [3]

The complement of an FSG, $G_{(A,V)}(a) = (G^*, \rho_a, \mu_a)$ is defined as $G^c_{(A,V)}(a) = (G^*, \rho^c_a, \mu^c_a)$, where $\rho^c_a(v_i) = 1 - \rho_a(v)\ \ \forall\ v_i \in V$ and $a\ \in A$, and $\mu^c_a\left(v_i,\ v_j\right) =\ 1 - \mu_a\left(v_i,\ v_j\right)\ if\ \ v_i,\ v_j \in V$ and $a \in A$.

Definition 2.8 [3]

Adjacency matrix of the fuzzy graph $G^* = (\ V, E)$ is,

$$A_{FG} = \begin{cases} \mu(u, v), & if\ i, j\ in\ the\ neighbourhood \\ 0, & otherwise \end{cases}$$

Example 2.9

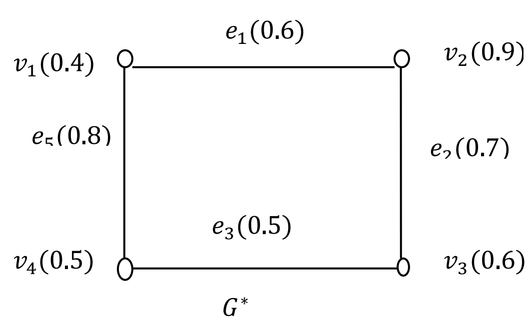


Fig. 2 Adjacency matrix representation of a fuzzy graph

Fuzzy adjacent matrix of the abovementioned fuzzy graph is,

$$A_{FG} = \begin{bmatrix} 0 & 0.6 & 0 & 0.8 \\ 0.6 & 0 & 0.7 & 0 \\ 0 & 0.7 & 0 & 0.5 \\ 0.8 & 0 & 0.5 & 0 \end{bmatrix}$$

3. Fuzzy Soft Graph Matrix

Definition 3.1

Let, $G_{(A,V)} = (G^*, \rho, \mu)$, is a fuzzy soft graph, where $G^* = (V, E)$ is the simple fuzzy graph and A is a non-empty parameter set for each parameter $a \in A$, an adjacency matrix is defined as:

$$M_{FSG[G_{A,V}\,(a_i)]} = \begin{cases} \mu(u,v), & \textit{if there is an edge between the vertices} \\ 0, & \textit{otherwise} \end{cases}$$

The matrix $M_{FSG[G_{A,V}\,(a_i)]}$ denoted as Fuzzy Soft Graph Matrix for each parameter $a_i \in A$.

Note

The matrix M_{FSG} is designated as the Fuzzy Soft Graph Matrix.

Example 3.2

Consider the graph $G^*$with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ Let $A = \{a_1, a_2, a_3\}$ be a set of parameters associated with the graph. The fuzzy soft vertex and edge membership functions are defined for each parameter in A.

$$\rho(a_1) = \{v_1|0.4, v_2|\ 0.2, v_3|0, v_4|\ 0.8\}$$
$$\rho(a_2) = \{v_1|0.5,\ v_2|\ 0, v_3|0.6, v_4|\ 0.7\}$$
$$\rho(a_3) = \{v_1|0, v_2|\ 0.5,\ v_3|0.7,\ v_4|\ 0.8\}$$

$$\mu(a_1) = \{e_1|0.1, e_2|\ 0, e_3|0, e_4|\ 0.2, e_5|0.4\}$$
$$\mu(a_2) = \{e_1|0, e_2|\ 0, e_3|0.5, e_4|\ 0, e_5|0.4\}$$
$$\mu(a_3) = \{e_1|0, e_2|\ 0.4, e_3|0.6, e_4|\ 0.3, e_5|0\}$$

Consequently, $G_{(A,V)}\ (a_1)$ is represented as $(\rho(a_1), \mu(a_1))$, $G_{(A,V)}\ (a_2)$ is expressed as $(\rho(a_2), \mu(a_2))$, and $G_{(A,V)}(a_2)$is denoted as $(\rho(a_3), \mu(a_3))$, all of which are FSG of G^*.

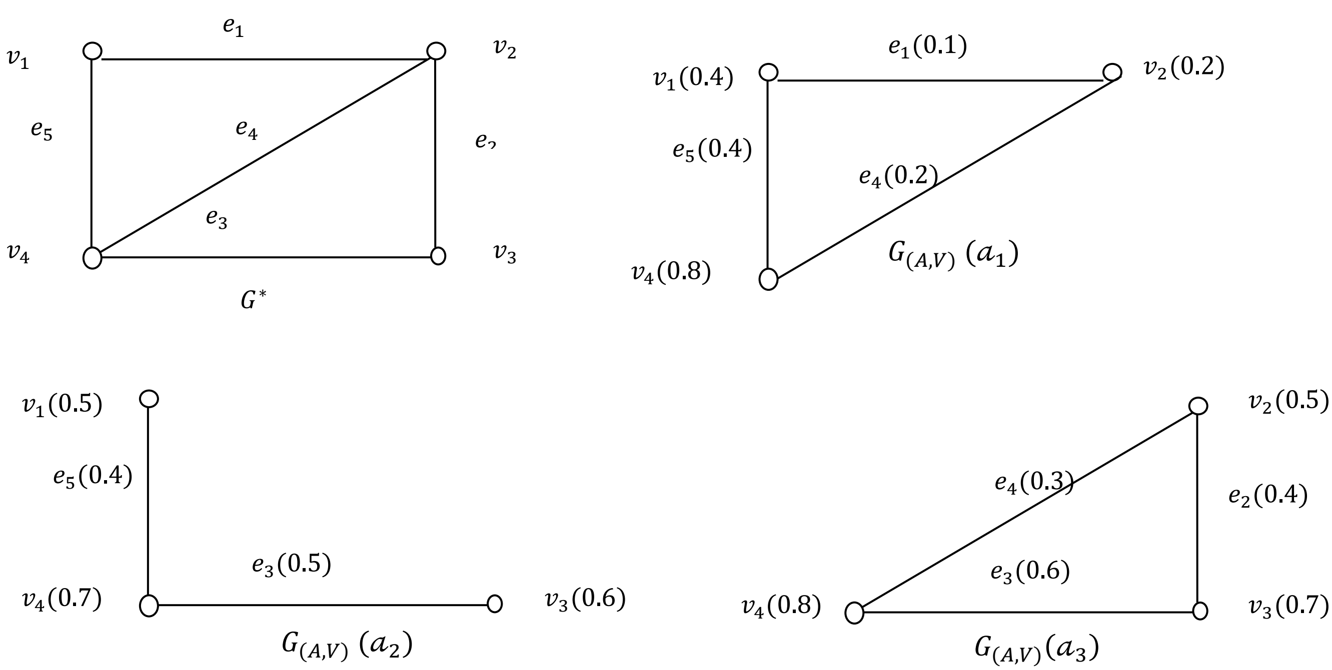


Fig. 3 Fuzzy soft graphs for the given parameter set

$$M_{FSG[G_{A,V}(a_1)]} = \begin{bmatrix} 0 & 0.1 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0 & 0 \end{bmatrix}$$

$$M_{FSG[G_{A,V}(a_2)]} = \begin{bmatrix} 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.4 & 0 & 0.5 & 0 \end{bmatrix}$$

$$M_{FSG[G_{A,V}(a_3)]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0.4 & 0 & 0.6 \\ 0 & 0.3 & 0.6 & 0 \end{bmatrix}$$

4. Operations on Fuzzy Soft Graph Matrix

Definition 4.1

Let $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = m_{ij}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = n_{ij}$ be two M_{FSG} associated with parameters $a_1, a_2 \in A$ respectively. The union of the two M_{FSG} is defined as,

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cup M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = l_{ij}$$

Where, $l_{ij} = \max\{m_{ij}, n_{ij}\}\ \forall\, i, j$.

Example 4.2

Using the fuzzy soft graph matrices obtained in Example 3.2, the union operation is performed by taking the maximum of the corresponding entries in the two matrices.

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = \begin{bmatrix} 0 & 0.1 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0 & 0 \end{bmatrix}$$

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = \begin{bmatrix} 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.4 & 0 & 0.5 & 0 \end{bmatrix}$$

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cup M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = \begin{bmatrix} 0 & 0.1 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0.5 \\ 0.4 & 0.2 & 0.5 & 0 \end{bmatrix}$$

Definition 4.3

Let $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = m_{ij}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = n_{ij}$ be two M_{FSG} associated with parameters $a_1, a_2 \in A$ respectively. The intersection of the two M_{FSG} is:

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cap M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = l_{ij}$$

Where, $l_{ij} = \min\{m_{ij}, n_{ij}\}\ \forall\, i, j$.

Example 4.4

The intersection of the matrices corresponding to Example 3.2 is obtained by selecting the minimum value of each pair of corresponding entries.

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = \begin{bmatrix} 0 & 0.1 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0 & 0 \end{bmatrix}$$

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = \begin{bmatrix} 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.4 & 0 & 0.5 & 0 \end{bmatrix}$$

The resulting matrix represents the intersection of associated fuzzy soft graphs.

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cap M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = \begin{bmatrix} 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \end{bmatrix}$$

Definition 4.5

Let $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = m_{ij}$be the fuzzy soft graph matrix associated with the parameter $a \in A$. Then $M^c_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]}$ Complement is then defined as

$$M^c_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = 1 - m_{ij} \ \forall\, i, j.$$

Definition 4.6

Let $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = m_{ij}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = n_{kl}$be two M_{FSG} associated with parameters $a_1, a_2 \in A$ respectively. The Cartesian product of the two M_{FSG} is defined as,

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \times M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = l_{(i,k)(j,l)}$$

Where, $l_{(i,k)(j,l)} = \min\{m_{ij}, n_{kl}\} \ \forall\, i, j$.

Theorem 4.7

The fuzzy soft graph matrix corresponding to an undirected fuzzy soft graph is symmetric.

Proof

Let, $G_{(A,V)} = (G^*, \rho, \mu)$ be the fuzzy soft graph, where $G^* = (V, E)$the simple fuzzy graph, and A is the associated non-empty parameter set.

For each parameter $a \in A$, the fuzzy soft graph matrix

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = m_{ij}$$

Is defined such that, $m_{ij} = \mu_{\mathfrak{a}}(u, v)$

Where, $\mu_{\mathfrak{a}}$denotes the fuzzy soft edge membership function corresponding to parameter $\mathfrak{a}$.

Since,G^* be the undirected simple fuzzy graph.

$$\Rightarrow \mu_{\mathfrak{a}}(v_i, v_j) = \mu(v_j, v_i) \ \forall\, i, j.$$

Therefore,$m_{ij} = m_{ji} \forall\, i, j$, which implies that M_{FSG} is symmetric.

Theorem 4.8

The union of two fuzzy soft graph matrices is the fuzzy soft graph matrix.

Proof

Let, $G_{(A,V)} = (G^*, \rho, \mu)$is a fuzzy soft graph, where $G^* = (V, E)$ is a simple fuzzy graph, and A is the associated non-empty parameter set.

For each parameter $a_1, a_2 \in A$, the fuzzy soft graph matrix $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = m_{ij}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = n_{ij}$.

Then the Union of two M_{FSG} is,

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cup M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]}, \forall\, i, j.$$

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = l_{ij}$$

Where, $l_{ij} = \max\{m_{ij}, n_{ij}\}$.

Since both $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]}$ are fuzzy soft graph matrices, their entries satisfy $0 \leq m_{ij}, n_{ij} \leq 1$for all i, j.
Hence, $0 \leq l_{ij} \leq 1$, which ensures that $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]}$ is a valid matrix.
Furthermore, for each parameter, the original fuzzy soft graph consistency condition

$$\mu_a(v_i, v_j) \leq \max\{\rho_a(v_i), \rho_a(v_j)\}$$

Taking the maximum of two such admissible membership values preserves this inequality, since the maximum of two values that individually satisfy the condition cannot exceed the corresponding vertex membership bounds.

Thus, the matrix satisfies all requirements of a fuzzy soft graph matrix with respect to the same underlying graph G*. Therefore, the union of two fuzzy soft graph matrices is itself a fuzzy soft graph matrix.

Theorem 4.9

The intersection of two fuzzy soft graph matrices is also a fuzzy soft graph matrix.

Proof

Let, $G_{(A,V)} = (G^*, \rho, \mu)$ be a fuzzy soft graph, where $G^* = (V, E)$ is a simple fuzzy graph, and A is the associated non-empty parameter set.

For each parameter $a_1, a_2 \in A$, the fuzzy soft graph matrix$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} = m_{ij}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = n_{ij}$.

Then the Intersection of the two M_{FSG} is,

$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]} \cap M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]} = M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]}, \forall\, i, j.$$
$$M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]} = l_{ij}$$

Where, $l_{ij} = \min\{m_{ij}, n_{ij}\}$.

Since both $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_1)]}$ and $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a_2)]}$ are fuzzy soft graph matrices, their entries satisfy $0 \leq m_{ij}, n_{ij} \leq 1$for all i, j.

Hence, $0 \leq l_{ij} \leq 1$, which ensures that $M_{FSG[G_{\mathbb{A},\mathbb{V}}(a)]}$ is a valid matrix.

Moreover, for each parameter $a \in A$, the corresponding fuzzy soft graph matrix entries satisfy the edge – vertex consistency condition.

$$l_{ij} \leq \min\{\rho_a(v_i), \rho_a(v_j)\}$$

Therefore, the intersection matrix satisfies all the defining conditions of a fuzzy soft graph matrix with respect to the underlying graph G*.

Hence, the intersection of two fuzzy soft graph matrices is also a fuzzy soft graph matrix

5. Application

A vehicle selection problem is considered to demonstrate the practical usefulness of the proposed fuzzy soft graph matrix framework.

Let, $V = \{v_1, v_2, v_3, v_4, v_5\}$ be the set of cars of the same model manufactured in different countries, where each v_i represent the distinct alternative.

The major objective is to analyze these alternatives on the basis of the multiple qualitative parameters and to determine whether any car satisfies all chosen criteria simultaneously.

Let the parameter set be defined as $A = . \{a_1, a_2, a_3, a_4\}$ where, a_1 represents safety, a_2 represents affordability, a_3 represent maintenance and a_4 represents performance.

These parameters are chosen because they are commonly used, mutually independent, and essential factors in automobile evaluation. Since their assessment depends largely on human perception and expert judgment, fuzzy modeling is appropriate.

For each parameter $a_k \in A$, fuzzy membership values

$$\rho_{a_k}(v_i) \in [0,1]$$

are assigned based on expert opinion and normalized qualitative assessment. A higher value indicates better satisfaction of the corresponding parameter.

The fuzzy approximate function $\rho : A \rightarrow P(V)$ defined by

$$\rho(a_1) = \{v_1|0.8, v_3|\ 0.2, v_4|0.5\}$$
$$\rho(a_2) = \{\ v_2|1, v_3|\ 0.3, v_5|0.7\}$$
$$\rho(a_3) = \{v_1|0.4, v_2|0.5, v_4|\ 0.3, v_5|0.5\}$$
$$\rho(a_4) = \{v_1|0.2, v_2|0.4, v_3|0.6, v_4|0.4\}$$

The fuzzy approximate function $\mu : A \rightarrow P(E)$ defined by

$$\mu(a_1) = \{\ e_3|\ 0.2, e_6|0.4, e_7|0.2\}$$
$$\mu(a_2) = \{\ e_2|\ 0.3, e_8|0.6, e_9|0.3\}$$
$$\mu(a_3) = \{e_1|0.4, e_4|\ 0.3, e_5|0.4, e_6|\ 0.3, e_8|0.5, e_{10}|0.2\}$$
$$\mu(a_4) = \{e_1|0.2, e_2|\ 0.3, e_3|0.4, e_6|\ 0.2, e_7|0.1, e_{10}|0.4\}$$

Thus, $G_{A,V}(a_1) = (\rho(a_1), \mu(a_1))$, $G_{A,V}(a_2) = (\rho(a_2), \mu(a_2))$, $G_{A,V}(a_3) = (\rho(a_3), \mu(a_3))$ *and* $G_{A,V}(a_4) = (\rho(a_4), \mu(a_4))$ are fuzzy soft graphs of G^* with an incidence function of $\phi = \{(v_i, v_j) | v_i, v_j$ have similarity between them}

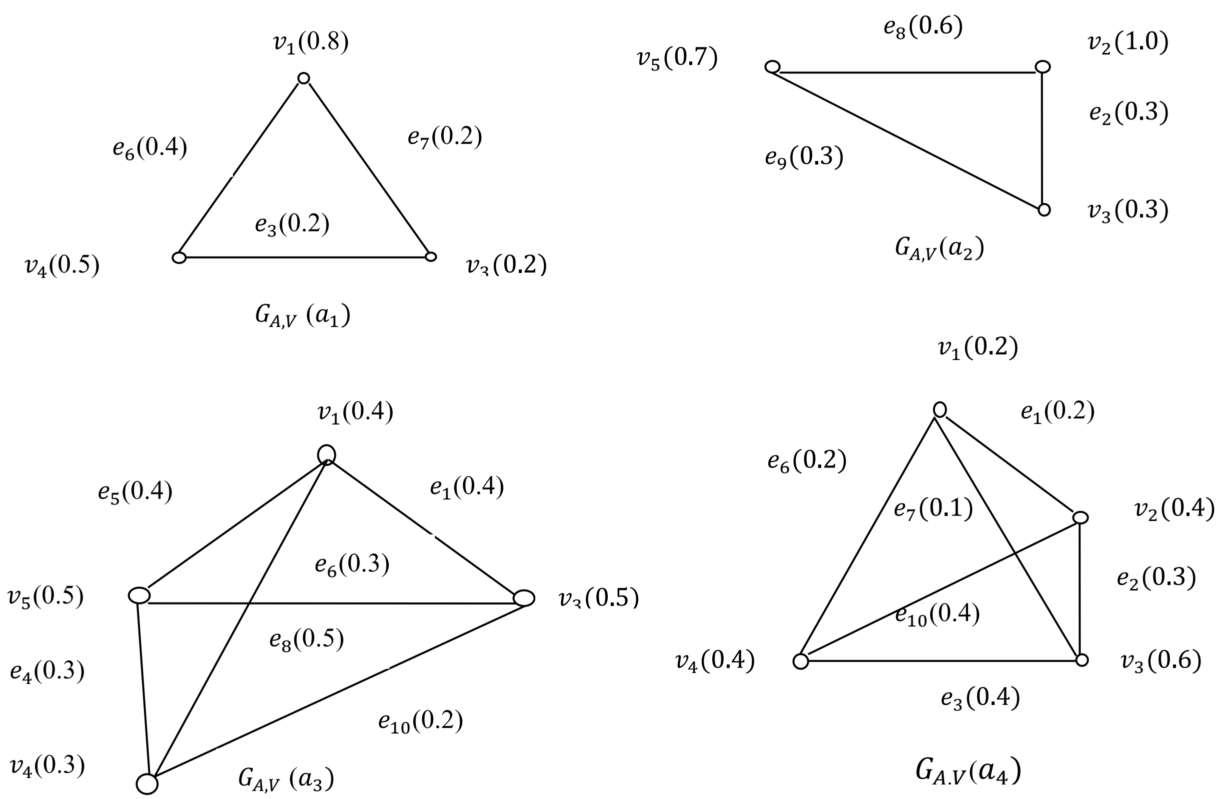


Fig. 4 Fuzzy soft graph representation of the car selection alternatives

The M_{FSG} of the above FSG are,

$$M_{FSG[G_{A,V}(a_1)]} = \begin{pmatrix} 0 & 0 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_{FSG[G_{A,V}(a_2)]} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0.6 \\ 0 & 0.3 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0.3 & 0 & 0 \end{pmatrix}$$

$$M_{FSG[G_{A,V}(a_3)]} = \begin{pmatrix} 0 & 0.4 & 0 & 0.3 & 0.4 \\ 0.4 & 0 & 0 & 0.2 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0 & 0 & 0.3 \\ 0.4 & 0.5 & 0 & 0.3 & 0 \end{pmatrix}$$

$$M_{FSG[G_{A,V}(a_4)]} = \begin{pmatrix} 0 & 0.2 & 0.1 & 0.2 & 0 \\ 0.2 & 0 & 0.3 & 0.4 & 0 \\ 0.1 & 0.3 & 0 & 0.4 & 0 \\ 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

To identify a car that satisfies all the selected parameters simultaneously, the intersection operation on the corresponding fuzzy soft graph matrices is performed. This operation captured common structural relationships among alternatives in all considered criteria. Finally, the resulting intersection matrix represent combined influences of the safety, affordability, maintenance, and performance.

$$\left(M_{FSG[G_{A,V}(a_1)]} \cap M_{FSG[G_{A,V}(a_2)]}\right) \cap M_{FSG[G_{A,V}(a_4)]} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In this instance, there are only 0 entries in the intersection matrix. This shows that, under specified fuzzy thresholds, no accessible car possibilities satisfy all requirements simultaneously.

6. Conclusion

In this study, Fuzzy soft graph matrices are presented as an organized method to express fuzzy soft graphs and examine their basic operations. Then the multi-criteria decision-making application is utilized to show the practical applicability of the suggested framework and to show how subjective characteristics are methodically represented and examined. This framework may be expanded in future studies by involving the weighted models, ranking techniques, and dynamic decision systems.

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