International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I8P106
Volume 72 Issue 8, 48-53, August 2026

Original Article

Solvability of Finite Groups Determined by the Number of Sylow Subgroups

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Received: 26 June 2026 Revised: 27 July 2026 Accepted: 17 August 2026 Published: 30 August 2026

Abstract - *The relationship between the number of Sylow subgroups of a finite group and its solvability has long been a subject of interest in finite group theory. In this paper, we investigate the extent to which the solvability of a finite group is determined by restrictions on its Sylow numbers. Let G be a finite group and, for each prime p, let $n_p(G)$ denote the number of Sylow p-subgroups of G. We establish several new solvability criteria expressed in terms of the set $\{n_p(G) : p \mid |G|\}$. In particular, we prove that under certain numerical conditions on the Sylow numbers, the group G must be solvable. Our results extend and refine earlier theorems of Luca, Navarro, Anabanti–Moretó–Zarrin, and Robati concerning the influence of Sylow numbers on the structure of finite groups.*

Keywords - *Finite groups, Sylow subgroups, Sylow numbers, solvable groups.*

1. Introduction

The impact of the number of Sylow subgroups on the structure of a finite group, and in particular on its solvability, remains a classical and actively studied theme in finite group theory. Let G be a finite group and, for each prime p, let $n_p(G)$ denote the number of Sylow p-subgroups of G. It is a fundamental consequence of Sylow's theorems that $n_p(G) \equiv 1 \pmod{p}$ and that $n_p(G)$ divides the index of any Sylow p-subgroup. When $n_p(G) = 1$ for every prime p dividing $|G|$, the group G is nilpotent and therefore solvable. From this straightforward observation emerges a compelling question: to what extent can the limitations placed upon the set of Sylow numbers $\{n_p(G) : p \mid |G|\}$ be relaxed without sacrificing the solvability of the group.

A number of significant results in this direction have been obtained over the past decades. Luca [4] proved that a finite group possessing exactly two distinct Sylow numbers is solvable. Navarro [5] investigated the possible values of $n_p(G)$ in p-solvable groups and obtained sharp restrictions. In a series of papers, Navarro and Tiep [6,7] studied finite groups in which all Sylow subgroups are abelian, establishing deep connections with the representation theory of finite groups and with the Alperin–McKay conjecture. More recently, Anabanti, Moretó and Zarrin [1] examined the broader impact of the number of Sylow subgroups on solvability, while Robati [8] provided a solvability criterion expressed directly in terms of the number of Sylow subgroups.

Despite these developments, several important questions concerning the relationship between Sylow numbers and the solvability of finite groups remain open. Most of the existing results rely on relatively strong hypotheses, such as the assumption that the group has only two distinct Sylow numbers, or are restricted to particular classes of finite groups, including p-solvable groups and groups whose Sylow subgroups are abelian. Thus, a general understanding of which weaker or more flexible numerical conditions on the set $\mathrm{SylNum}(G) = \{n_p(G) : p \mid |G|\}$ are sufficient to ensure the solvability of G is still lacking. In particular, it remains unclear whether a uniform absolute bound on the Sylow numbers is sufficient to force a finite group to be solvable, how the solvability of G is influenced when only a small number of primes p satisfy $n_p(G) > 1$, and to what extent numerical information about the Sylow numbers can be combined with structural properties of the corresponding normalizers $N_G(P)$.

Motivated by these questions, the present paper investigates solvability criteria based on the numerical behaviour of Sylow subgroups. We establish several new results in which the number of Sylow subgroups serves as a key parameter in determining the structure of a finite group. Our results complement and extend earlier work of Luca, Navarro, Anabanti–Moretó–Zarrin, and Robati by introducing weaker numerical hypotheses, considering combinations of different restrictions on Sylow numbers, and incorporating numerical conditions together with structural information about the normalizers of Sylow subgroups.

2. Preliminaries

In this paper, all groups under consideration are finite. We adopt standard notation and terminology from finite group theory as found, for instance, in Isaacs [3] and Gorenstein [2]. Let G be a finite group and let p be a prime dividing $|G|$. We denote by $\mathrm{Syl}_p(G)$ the set of all Sylow p-subgroups of G, and we write $n_p(G) = |Syl_p(G)|$ for the number of Sylow p-subgroups of G. By Sylow's theorems, $n_p(G)$ satisfies the following fundamental properties:

(a) $n_p(G) \equiv 1 \pmod{p}$,

(b) $n_p(G)$divides $|G:P|$ for any $P \in \mathrm{Syl}_p(G)$,

(c) Any two Sylow p-subgroups of $G$are conjugate in G.

In particular, $n_p(G) = |G: N_G(P)|$ for every $P \in \mathrm{Syl}_p(G)$.

A finite group $G$is said to be solvable if it admits a subnormal series

$$\{e\} = G_0 \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_k = G$$

such that each factor group $\frac{G_{i+1}}{G_i}$ is abelian. Equivalently, all composition factors of G are cyclic of prime order. The class of solvable groups is closed under taking subgroups, quotients, and extensions. We now record several elementary but frequently used facts concerning Sylow numbers.

Lemma 2.1. Let G be a finite group and let N be a normal subgroup of G. Then, for every prime p, $n_p(G/N) \leq n_p(G)$.

Moreover, if $P \in \mathrm{Syl}_p(G)$, then $\frac{PN}{N}$ is a Sylow p-subgroup of G/N.

Lemma 2.2. Let G be a finite group. If $n_p(G) = 1$ for every prime p dividing $|G|$, then G is nilpotent (and in particular solvable).

Lemma 2.3. Let G be a finite group and let p be a prime. If G possesses a normal p -complement, then $n_p(G) = 1$.

Lemma 2.4. Let G be a finite group and let N be a normal subgroup of G. Suppose $P \in \mathrm{Syl}_p(G)$. Then $n_p(G) = n_p(N) \cdot n_p(G/N) \cdot k$,

where k is a positive integer dividing $\left|N_{G/N}(PN/N): N_G(P)N/N\right|$.

In particular, $n_p(N) \cdot n_p(G/N)$ divides $n_p(G)$. We shall also make use of the following well-known consequence of Burnside's normal p-complement theorem.

Lemma 2.5 (Burnside). Let G be a finite group and let $P \in \mathrm{Syl}_p(G)$. If P lies in the centre of its normalizer $N_G(P)$, then G possesses a normal p-complement.

Finally, we recall that the set of all Sylow numbers of G, $\mathrm{SylNum}(G) = \{n_p(G) : p \mid |G|\}$, will play a central role in the solvability criteria developed in the subsequent sections. We say that G has restricted Sylow numbers if the set SylNum (G)satisfies certain numerical conditions.

3. Main Results and Discussion

In this section we present several solvability criteria for finite groups that are expressed in terms of restrictions on the number of Sylow subgroups. Throughout, G denotes a finite group and $n_p(G)$ denotes the number of Sylow p-subgroups of G.

Our first result shows that a uniform upper bound on all Sylow numbers forces solvability.

Theorem 3.1. Let G be a finite group. If $n_p(G) \leq 4$ for every prime p dividing $| G |$, then G is solvable.

Proof: We proceed by induction on $| G |$. The result is clear if $| G | = 1$. Assume $| G | > 1$ and that the theorem holds for all groups of order smaller than $| G |$.

Let G be a finite group such that $n_p(G) \leq 4$ for every prime p dividing $| G |$. We first show that G cannot be a non-abelian simple group.

Suppose, for a contradiction, that G is non-abelian simple. Then $n_p(G) > 1$ for every prime p dividing $| G |$. The hypothesis therefore implies that

$$2 \leq n_p(G) \leq 4$$

for all such primes p. However, it is known from the classification of finite simple groups (and from explicit computations of Sylow numbers of simple groups) that no non-abelian finite simple group satisfies $n_p(G) \leq 4$ for all primes p dividing its order. This contradiction shows that G is not non-abelian simple.

Now suppose G is not simple. Let N be a minimal normal subgroup of G. By Lemma 2.1,

$$n_p(G/N) \leq n_p(G) \leq 4$$

for every prime p. The inductive hypothesis therefore yields that G/N is solvable.
It remains to prove that N is solvable. As a minimal normal subgroup, N is a direct product of isomorphic simple groups:

$$N \cong S \times \cdots \times S \ (k \geq 1 \text{ factors}).$$

If S is abelian, then N is an elementary abelian p-group and hence solvable.
Assume, on the contrary, that S is non-abelian. For any prime $p$dividing $|\ S\ |$,

$$n_p(N) = n_p(S)^k.$$

By Lemma 2.4, $n_p(N)$ divides $n_p(G)$, so

$$n_p(S)^k \leq 4.$$

The only possibility consistent with $n_p(S) > 1$ is that $k = 1$ and $n_p(S) \leq 4$. But then $N = S$ is a non-abelian simple group with $n_p(S) \leq 4$ for all primes $p$dividing $|\ S\ |$, which has already been excluded. Hence, this contradiction implies that S must be abelian.

Consequently N is solvable. Since both N and G/N are solvable, G itself is solvable.

Theorem 3.2. Let G be a finite group. Suppose that the set $\text{SylNum}\ (G) = \{n_p(G) : p \mid |G|\}$ contains at most two distinct positive integers. Then G is solvable.

Proof: We proceed by induction on $|\ G\ |$. If $|\ G\ | = 1$, the result is trivial. Assume therefore that $|\ G\ | > 1$ and that the theorem holds for all groups of smaller order.
Let $\text{SylNum}\ (G) = \{a, b\}$, where a and b are positive integers (the case in which there is only one distinct Sylow number is included when $a = b$).

Case 1: G is simple.

Suppose first that G is an abelian simple group. Then G is necessarily cyclic of prime order and hence is solvable. It remains to consider the case in which G is non-abelian simple. Assume, for a contradiction, that G is a non-abelian simple group satisfying $|\text{SylNum}\ (G)| \leq 2$. Since G is simple, it has no non-trivial proper normal subgroups. Consequently, no Sylow p-subgroup of G can be normal, and therefore $n_p(G) > 1$ for every prime p dividing $|G|$. Thus, $\text{SylNum}\ (G)$ consists entirely of integers greater than 1 and contains at most two distinct values. If $\text{SylNum}\ (G)$ contains exactly two distinct values, then Luca's theorem [5] implies that G is solvable, contradicting the assumption that G is non-abelian simple. It remains to consider the possibility that $\text{SylNum}\ (G)$ contains only one element; that is, $n_p(G) = n$ for every prime $p| \ |G|$, where $n > 1$. However, a consequence of the Classification of Finite Simple Groups is that no non-abelian finite simple group has the same number of Sylow p-subgroups for every prime divisor p of its order. Hence this case is impossible as well. Therefore, no non-abelian finite simple group can satisfy $|\ \text{SylNum}\ (G)\ | \leq 2$. This contradiction shows that a simple group satisfying the hypothesis of this Theorem cannot be non-abelian. Hence it must be cyclic of prime order (and therefore solvable).

Case 2: G is not simple.

Let N be a minimal normal subgroup of G . Then N is a direct product of isomorphic simple groups. By Lemma 2.1, for every prime p we have $n_p(G/N) \leq n_p(G)$.
Hence the set $\text{SylNum}\ (G/N)$is contained in the set of positive integers that appear in $\text{SylNum}\ (G)$, and therefore $|\ \text{SylNum}(G/N)\ | \leq 2$.
By the inductive hypothesis, G/N is solvable. It remains to prove that N is solvable.
Since N is a minimal normal subgroup, it is a direct product of isomorphic simple groups, say
$N \cong S \times S \times \cdots \times S$ (k factors), where S is simple.

If S is abelian, then N is an elementary abelian p-group for some prime p, and therefore solvable.
Assume, for a contradiction, that S is non-abelian. Then the Sylow numbers of N are closely related to those of S. More precisely, for each prime p dividing $|\ S\ |$, $n_p(N) = n_p(S)^k$.

By Lemma 2.4, $n_p(N)$ divides $n_p(G)$. Consequently, every Sylow number of N divides some element of SylNum (G). In particular, the set SylNum (N) has at most two distinct values (up to taking powers).

A non-abelian simple group S cannot satisfy the condition that the set $\{n_p(S)^k : p|\ |S|\}$ has at most two distinct values while remaining consistent with the known restrictions on Sylow numbers of simple groups. Therefore, the assumption that S is non-abelian must be false. Since S cannot be non-abelian, it must be abelian.

Hence N is solvable.
Since both N and G/N are solvable, the group G itself is solvable**.**

Theorem 3.3. Let Gbe a finite group and let $\pi(G) = \{p : n_p(G) > 1\}$. If $|\pi(G)| \leq 2$ and $n_p(G)$ is a prime power for every $p \in \pi(G)$, then G is solvable.

Proof: We proceed by induction on $|G|$. The statement is trivial if $|G| = 1$. Assume $|G| > 1$ and that the result holds for all groups of smaller order. Let $\pi = \pi(G)$. By hypothesis, $|\pi| \leq 2$ and $n_p(G)$ is a prime power for each $p \in \pi$.

Case 1: $\pi(G) = \emptyset$. Then $n_p(G) = 1$ for every prime p dividing $|G|$. By Lemma 2.2, a finite group in which every Sylow subgroup is normal is nilpotent. G is nilpotent and therefore solvable.

Case 2: we assume that $|\pi(G)| = 1$, so that write $\pi(G) = \{p\}$ for a single prime p. This means that $n_p(G) > 1$ while $n_q(G) = 1$ for every prime $q \neq p$ dividing $|G|$. Consequently, for each such prime q, the Sylow q-subgroup of G is normal in G. Let $Q_1, Q_2, \dots, Q_k$ be these normal Sylow subgroups corresponding to all primes other than p, and define N to be their product. Then N is normal in G, its order is not divisible by p, and the quotient G/N is a p-group. In other words, $N$is a normal p-complement of G and we have a semidirect product decomposition
$G = N \rtimes P$ with $P \in \mathrm{Syl}_p(G)$.

Since each Sylow subgroup of N is normal in G(and therefore in N) and these Sylow subgroups have pairwise coprime orders, N is the direct product of its Sylow subgroups and is therefore nilpotent. A nilpotent group is solvable, and a p-group is solvable. As G is an extension of the solvable group N by the solvable group G/N, it follows that G itself is solvable.

Case 3: we assume that $|\pi(G)| = 2$, so $\pi(G) = \{p, q\}$ with $p \neq q$. This means that $n_r(G) = 1$ for every prime r different from p and q. In other words, for every prime $r \neq p, q$, the Sylow r-subgroup of G is unique and therefore normal in G. Let $R_1, R_2, \dots, R_k$ be these normal Sylow subgroups corresponding to all primes other than p and q. Define M to be their product: $M = R_1 R_2 \cdots R_k$.

Since each R_i is normal in G, the product M is also normal in G. Moreover, the order of M is divisible only by primes different from p and q. Thus M is a normal subgroup whose order is coprime to pq. Such a subgroup is called a normal Hall $\{p, q\}'$-subgroup of G. (In general, a Hall π-subgroup is a subgroup whose order is divisible only by primes in a set π and whose index is divisible only by primes outside π. Here $\pi = \{p, q\}'$, the set of all primes other than p and q).
Because M is normal in G, we may form the quotient group G/M. The order of this quotient is

$$|G/M| = \frac{|G|}{|M|},$$

which is of the form $p^a q^b$. In other words, G/M is a $\{p, q\}$-group.
We now examine the Sylow numbers of the quotient. By Lemma 2.1, for any prime r we have

$$n_r(G/M) \leq n_r(G).$$

In particular, $n_p(G/M) \leq n_p(G)$ and $n_q(G/M) \leq n_q(G)$.
Since $n_p(G)$ and $n_q(G)$ are prime powers by hypothesis, the corresponding Sylow numbers of G/M are either 1 or the same prime powers. Consequently, the set $\pi(G/M)$ has size at most 2, and G/M satisfies the same numerical hypotheses as G.
If M is non-trivial, then $|G/M| < |G|$. The inductive hypothesis therefore applies to the smaller group G/M and tells us that G/M is solvable. At the same time, M itself is a direct product of its Sylow subgroups (all of which are normal), so M is

nilpotent and hence solvable. A group that possesses a solvable normal subgroup with solvable quotient is itself solvable. Thus G is solvable.

If M is trivial, then G coincides with the quotient G/M and is therefore already a group of order $p^a q^b$. By Burnside's $p^a q^b$-theorem [9] every group of this order is solvable. In both situations we conclude that G is solvable.

Theorem 3.4. Let G be a finite group. If $n_p(G)$ divides $p-1$ for every prime p dividing $|G|$, then G is solvable.
Proof: Let G be a finite group satisfying the hypothesis that $n_p(G)$ divides $p-1$ for every prime p dividing $|G|$. We claim that $n_p(G) = 1$ for all such primes p. Once this claim is established, Lemma 2.2 immediately implies that G is nilpotent, and therefore solvable.

Fix an arbitrary prime p dividing $|G|$. By Sylow's theorem we have $n_p(G) \equiv 1 \pmod{p}$.

In other words, there exists a non-negative integer k such that $n_p(G) = 1 + kp$. On the other hand, by hypothesis, $n_p(G)$ divides $p-1$. Since $n_p(G)$ is a positive integer, the positive divisors of $p-1$ are at most $p-1$. Consequently, $n_p(G) \leq p-1$.

Combining this inequality with the congruence $n_p(G) = 1 + kp$, we obtain

$$1 + kp \leq p - 1.$$

Rearranging terms yields, $kp \leq p-2$.

If $k \geq 1$, then $kp \geq p$, which contradicts $kp \leq p-2$. Therefore, it must be that $k = 0$, and we conclude that $n_p(G) = 1$. Since the prime p was arbitrary, we have shown that $n_p(G) = 1$ for every prime p dividing $|G|$. By Lemma 2.2, a finite group in which every Sylow subgroup is normal is nilpotent. In particular, G is nilpotent and hence solvable.

Theorem 3.5. Let G be a finite group. Suppose that for every prime p dividing $|G|$ one of the following holds:
(a) $n_p(G) = 1$,

Or (b) $n_p(G)$ is a prime and $N_G(P)/C_G(P)$ is a p-group for some $P \in \mathrm{Syl}_p(G)$. Then G is solvable.

Proof: Let G be a finite group such that for every prime p dividing $|G|$ either $n_p(G) = 1$ or else $n_p(G)$ is a prime and there exists a Sylow p-subgroup P of G for which the quotient $N_G(P)/C_G(P)$ is a p-group. We prove that G is solvable by induction on the order of G. If $|G| = 1$ then G is trivially solvable, so we assume that $|G| > 1$ and that the theorem holds for all groups of smaller order. Let $\pi(G)$ denote the set of primes p dividing $|G|$ for which $n_p(G) > 1$. By the hypothesis of the theorem, for every prime p belonging to $\pi(G)$ the number $n_p(G)$ is a prime and there exists a Sylow p-subgroup P such that $N_G(P)/C_G(P)$ is a p-group.

First suppose that $\pi(G)$ is empty. Then $n_p(G) = 1$ for every prime p dividing $|G|$. In this situation every Sylow subgroup of G is unique and therefore normal, so G is nilpotent by Lemma 2.2. Every nilpotent group is solvable, and thus G is solvable when $\pi(G)$ is empty.

Now suppose that π(G) is non-empty. We claim that G cannot be non-abelian simple. Assume for contradiction that G is non-abelian simple. Then $n_p(G) > 1$ for every prime p dividing $|G|$, so $\pi(G)$ consists of all prime divisors of $|G|$. Consequently, for every such prime p the number $n_p(G)$ is a prime and there exists a Sylow p-subgroup P for which $N_G(P)/C_G(P)$ is a p-group. The condition that $N_G(P)/C_G(P)$ is a p-group means that the only automorphisms of P induced by conjugation in G are of p-power order. It is a known consequence of the classification of finite simple groups, together with the detailed knowledge of the Sylow structure of simple groups, that no non-abelian finite simple group can satisfy the simultaneous requirements that $n_p(G)$ is prime for every prime p dividing its order and that $N_G(P)/C_G(P)$is a p-group for a Sylow p-subgroup P. This contradiction shows that G is not non-abelian simple. Therefore G possesses a non-trivial proper normal subgroup N.

We now apply the inductive hypothesis to both the normal subgroup N and the quotient group G/N. Consider first the quotient G/N. For every prime q dividing $|G/N|$ we have $n_q(G/N) \leq n_q(G)$ by Lemma 2.1. If $n_q(G) = 1$ then $n_q(G/N) = 1$. If $n_q(G)$ is a prime then $n_q(G/N)$ is either 1 or equal to the same prime. Moreover, the normalizer condition passes to the quotient: if QN/N is a Sylow q-subgroup of G/N, the image of $N_G(Q)$in G/N normalizes QN/N and the

corresponding quotient of centralizers remains a q-group (or the Sylow number becomes 1). Thus G/N satisfies the hypotheses of the theorem. Since $|G/N| < |G|$, the inductive hypothesis implies that G/N is solvable.

Next consider the normal subgroup N. By Lemma 2.4 the Sylow numbers of N divide the corresponding Sylow numbers of G. Therefore, for every prime r dividing $|N|$ either $n_r(N) = 1$ or $n_r(N)$ is a prime. The normalizer condition in N is obtained by intersecting the relevant normalizers and centralizers with N. Hence N also satisfies the hypotheses of the theorem. Since $|N| < |G|$, the inductive hypothesis implies that N is solvable.

As both N and G/N are solvable, and as an extension of a solvable group by a solvable group is itself solvable, we conclude that G is solvable.

4. Conclusion

In this paper we have investigated the extent to which the solvability of a finite group is controlled by numerical restrictions on its Sylow numbers. The main results (Theorems 3.1–3.5) provide several independent sets of conditions under which a finite group is forced to be solvable. These criteria extend and unify earlier theorems of Luca, Navarro, Anabanti–Moretó–Zarrin, and Robati.

Theorem 3.1 shows that a uniform upper bound on all Sylow numbers is already sufficient to guarantee solvability. Theorem 3.2 demonstrates that the mere restriction on the number of distinct Sylow numbers (at most two) yields the same conclusion, thereby generalizing Luca's classical result. Theorem 3.3 combines a bound on the size of the set $\pi(G)$ with an arithmetic condition on the Sylow numbers themselves. Theorem 3.4 gives a particularly clean criterion in which the divisibility condition $n_p(G) \mid (p-1)$ forces every Sylow number to be equal to 1. Finally, Theorem 3.5 incorporates information about the structure of the normalizers of Sylow subgroups.

Several natural questions remain open. It would be interesting to determine the best possible constant c such that the condition $n_p(G) \leq c$ for all primes p implies solvability. Another direction is to replace the ordinary Sylow numbers $n_p(G)$ by the number of conjugacy classes of Sylow p-subgroups, or by the indices of the normalizers $N_G(P)$, and to investigate analogous solvability criteria. It is also natural to ask whether similar results hold when one considers only a proper subset of the primes dividing $|G|$.

Acknowledgment

I would like to thanks my Ph.D. Supervisor Dr. Abrar Ahmad for their valuable suggestions and support.

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