

Original Article

Performance Analysis of Euler and Fourth-Order Runge-Kutta Methods in Modeling the Transient Response of a Continuous Stirred Tank Reactor

Nurul Nadiya Abu Hassan^{1*}, Nurmarni Athirah Abdul Wahid¹

¹Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM) Pahang Branch, Jengka Campus, 26400 Bandar Tun Abdul Razak Jengka, Pahang, Malaysia.

*Corresponding Author : nurulnadiya@uitm.edu.my

Received: 12 July 2026

Revised: 18 August 2026

Accepted: 11 September 2026

Published: 28 September 2026

Abstract - This paper presents a comparison study of the explicit Euler and fourth-order Runge-Kutta (RK4) methods for predicting the transient reactant concentration of the Continuous Stirred Tank Reactor (CSTR). The numerical simulations were performed in MATLAB for the stepsize of $h = 0.01, 0.05, 0.1, 0.1$ and 0.5 . The results obtained from this study were compared with the existing analytical solution and the stability criteria derived from the governing system eigenvalue is analyzed. The RK4 method achieved a better result, maintaining the maximum relative errors below $4.43 \times 10^{-4}\%$ across all step sizes. Meanwhile, for Euler's method, the maximum error obtained is 7.687429725% for $h = 0.5$. Although both methods satisfied linear stability bounds where $|1 + h\lambda| < 1$ and $-2.78 < h\lambda < 0$, RK4 gives a better balance between step-size tolerance and computational accuracy.

Keywords - CSTR, Numerical Methods, Euler Method, Runge-Kutta Method, Tank Reactor.

1. Introduction

Continuous Stirred Tank Reactors (CSTRs) is one of the most common reactor configurations used in the chemical and process industries since it can operate continuously and provide uniform mixing conditions. They are widely used in chemical manufacturing, wastewater treatment, pharmaceuticals, food processing etc. where efficient mixing and stable operating conditions are desired [1,2]. The continuous stirring mechanism [3] ensures uniform concentration within the reactor volume and effective mass transfer between reactants.

In addition to conventional chemical processes, CSTRs have also been successfully applied to biological and environmental engineering. CSTRs are typically used in anaerobic digestion and activated sludge systems to ensure contact between microorganisms and wastewater, improving the treatment efficiency [4,5]. CSTRs are a fundamental model for the study of the reactor dynamics and process control because of their practical importance [1].

The dynamic behaviour of a CSTR can be mathematically described by Ordinary Differential Equations (ODEs) representing the transient variation of reactor concentration with time. These equations are often solved by means of numerical methods when analytical solutions are difficult or not feasible to obtain. The Euler method and fourth-order Runge-Kutta (RK4) method are commonly used techniques due to their simplicity, numerical performance, and applicability to engineering problems [6,7]. Recent studies have also shown the use of MATLAB for modelling and simulation of CSTR systems to analyse the transient response under different operating conditions [3].

Both methods are widely used to solve ODEs. However, their numerical behaviour in terms of accuracy, convergence and stability can be quite different, especially when different step sizes are used. Earlier studies have been concentrated on CSTR modelling and process optimisation. However, there are still few comparative studies of the Euler and RK4 methods for transient CSTR simulations at various step sizes [1,3].

Recent research on continuous stirred tank reactors has increasingly turned toward dynamic models including nonlinear reaction kinetics, coupled mass and energy balances, process control, state estimation and data-driven modelling [15]. The



reactor-network studies also combined first-principles models with data-driven approaches to improve dynamic process prediction and state estimation [18]. These developments demonstrate that numerical simulation remains an important component of contemporary CSTR modelling. However, the numerical solver is often treated as an implementation component of a complex modelling, control, estimation or data-driven framework rather than as the primary object of analysis. Thus, there are values in a controlled benchmark study that isolates the effect of numerical integration and temporal discretisation on transient CSTR concentration predictions. Such benchmark is useful when the governing equation has a known analytical solution since numerical error can be calculated directly rather than evaluated only through experimental or simulated reference data.

Instead of providing novelty on introducing Euler and RK4 methods, it provides a systematic numerical benchmark in which five fixed step sizes, $h = 0.01, 0.05, 0.10, 0.20$ and 0.50 are evaluated by using the same CSTR model and initial value conditions. The numerical solutions are then compared with the analytical solutions, the empirical order of convergence is calculated, and the theoretical stability boundary is derived from the system eigenvalue. This integrated analysis gives quantitative proof of the first order and fourth-order convergence behaviour of Euler and RK4, respectively, and enables the effects of temporal discretisation to be separated from numerical instability.

The resulting framework provides a clear reference for the choice of numerical methods in transient reactor simulation. It is also a good computational case study for chemical engineering education, as the same model can be used to show the links between material balance modelling, numerical approximation, truncation error, convergence and stability.

2. Literature Review

Dynamic modelling of CSTR systems serves as a primary benchmark for evaluating reactor design, feedback control loops, and dynamic state estimation [1,2]. Material balance formulations for continuous stirred systems yield first-order linear or non-linear differential equations representing reactant concentration kinetics over time [11,12]. Achieving computational precision in transient dynamic simulation is vital, as numerical inaccuracies can propagate into erroneous control actions or inaccurate prediction of transient conversion rates [11]. Recent CSTR studies have also considered transient mass and energy balances and employed numerical solvers for stiff differential equations to analyses reactor dynamics and estimate kinetic parameters. Modern reactor modelling is increasingly moving beyond conventional numerical integration alone [16,17].

Numerical methods play an important role in solving Ordinary Differential Equations (ODEs) encountered in engineering and scientific applications. The Euler and fourth-order Runge-Kutta (RK4) methods are commonly used due to their simplicity, reliability and ease of implementation [6,7]. The Euler method is a first order explicit numerical method that approximates the solution by using the slope at the current point. It is numerically simple, but its accuracy relies on the chosen step size. Meanwhile, the RK4 method evaluates four intermediate slopes during each iteration, improving accuracy, convergence, and stability for solving dynamic systems [6].

There are a lot of studies that were comparing the performance of Euler and RK4 methods in various engineering applications. And it can be concluded that RK4 gives better accuracy than the Euler method in transient electrical circuit analysis, particularly when larger step sizes were applied [8]. It is also shown that RK4 and Runge-Kutta-Fehlberg methods achieved superior convergence characteristics in series RLC circuit simulations [9]. In robotic system applications, RK4 reduced tracking errors and improved trajectory smoothness in closed-loop inverse kinematics [10]. However, it was stated that the enhanced accuracy of RK4 requires higher computational effort due to multiple slope evaluations. These studies indicate that RK4 generally provides better numerical performance, while the Euler method remains useful for applications where simplicity and lower computational cost are prioritized.

In chemical engineering, CSTR are often used as benchmark systems for studying the dynamic process behaviour. Usually, the transient behaviour of CSTRs can be described by using ODEs that are derived from mass and energy balances [1,2]. Accurate numerical solutions of these equations are required to predict the behaviour of the reactor, to control the process and to optimise it. The importance of reliable transient modelling in reactor systems has been demonstrated by recent studies where it was demonstrated that stochastic effects and disturbances can have a very important effect on the dynamic responses of CSTR [11].

Meanwhile there is another study that investigated transient fluid behaviour in wastewater reactors where it was solved using Computational Fluid Dynamics (CFD) [12]. Furthermore, a few studies highlighted the extensive applications of stirred tank reactors in industrial processing, reactor optimization, and wastewater treatment, emphasizing the need for accurate simulation techniques [3-5].

Despite the extensive studies on the numerical methods and CSTR dynamics, there are few studies on the direct comparison between the Euler and RK4 methods for transient CSTR concentration analysis. Most of the previous work has focused on reactor optimisation, nonlinear dynamics, and process control, but the significance of the numerical solver choice on the accuracy of the solution is less often discussed. Therefore, this study investigates the performance of the Euler and RK4 methods in solving the first order ODE governing the transient behaviour of a CSTR. The accuracy, convergence and stability characteristics of both methods are investigated by comparing numerical solutions with the analytical solution using multiple step sizes in MATLAB. The results provide useful information for the selection of appropriate numerical methods in reactor dynamic simulations and other chemical engineering applications.

3. Methodology

3.1. Mathematical Model of CSTR

A quantitative computational approach is used in discussing the performance of two numerical methods, the Euler method and the RK4 method, in solving the transient concentration response of a CSTR. The numerical simulations are carried out using MATLAB and the results obtained from both the methods are compared with the corresponding analytical solution to evaluate the numerical accuracy and the computational behaviour.

A Continuous Stirred Tank Reactor (CSTR) is one of the most common types of reactors used in chemical engineering because of its continuous and well-mixed nature. Under the ideal assumption of perfect mixing, the concentration of reactants is assumed to be uniform throughout the reactor volume [2]. The transient behaviour of the reactor can be mathematically described by applying the principle of the material balance, where the rate of accumulation is equal to the rate of input minus the rate of output and the rate of consumption due to chemical reaction. Dynamic models are widely used in process engineering to analyse system responses in time, and are the basis of process monitoring, simulation and control [1,2]. The generic ordinary differential equation (ODE) for a first order reaction is given as follows:

$$\frac{dC_A(t)}{dt} = \frac{1}{\tau} C_{A,in}(t) - \left(\frac{1}{\tau} + k \right) C_A(t)$$

Where $C_A(t)$ is the concentration of A in the reactor at time t , $C_{A,in}(t)$ is the concentration of A in the feed, τ is the residence of time and k is the reaction rate constant.

The reactor model was simplified by considering several assumptions. The reactor was assumed to be perfectly mixed in the vessel with isothermal operation at a constant volume. In addition, the chemical reaction was assumed to be first-order kinetics. The inlet concentration and physical properties of the reacting fluid were assumed to be constant during the simulation period. These assumptions are consistent with the ideal CSTR model used in chemical reaction engineering and process dynamics [1,2,13].

Table 1 presents the simulation parameters used in this study. The reactor dynamics were modelled with a residence time of 5.0 units and a reaction rate constant of 0.1. The inlet concentration was kept at 1.0 and the initial concentration in the reactor was assumed to be zero. All the simulations were performed with five fixed step sizes, which are $h = 0.01, 0.05, 0.10, 0.20$ and 0.50 .

Table 1. Simulation Parameters of the CSTR Model

Time Residence, τ	5.0
Reaction Rate Constant, k	0.1
Input Concentration, $C_{A,in}(t)$	1.0
Initial Reactant Concentration, $C_A(0)$	0

3.2. Numerical Methods

The transient concentration response of the CSTR was approximated using two explicit numerical techniques, namely the Euler method and the RK4 method. Both methods are widely employed for solving ordinary differential equations due to their ease of implementation and effectiveness in approximating dynamic system behaviour. The selection of these methods was motivated by their differing levels of computational complexity and numerical accuracy, thereby enabling a comprehensive comparison of their performance when applied to reactor dynamic simulations.

The differential equation of the CSTR was solved numerically using a range of fixed step sizes, $h = 0.01, 0.05, 0.10, 0.20$ and 0.50 . All computations were performed iteratively over the defined simulation period, and the numerical results were benchmarked against the exact analytical solution to evaluate the effects of step size on solution accuracy, convergence behaviour

and numerical performance. In this study, the results were based on a quantitative computational approach, and the performance of two numerical methods, the Euler method, and the RK4 method, in solving the transient concentration response of a CSTR was analysed. The numerical simulations are performed using MATLAB and the results obtained by both the methods are compared with the corresponding analytical solution for evaluating the numerical accuracy and the computational behaviour.

3.3. Euler Method

The Euler method is one of the earliest and simplest numerical methods for solving ordinary differential equations. The ODE solution is approximated by using the slope of the tangent line at the current point to estimate the value at the next time step. The Euler method is widely used in engineering applications that require simple numerical approximations due to its simplicity and ease of implementation. It is also a fundamental approach to understanding more advanced numerical methods [6].

This method can be expressed as:

$$C_{A,n+1} = C_{A,n} + hf(t_n, C_{A,n})$$

where $C_{A,n}$ denotes the concentration at the current time step, h is the step size, and $f(t_n, C_{A,n})$ represents the derivative of the governing equation evaluated at time t_n .

In this study, the Euler method is implemented as follows:

- Define the initial conditions and simulation parameters.
- Choose the step sizes ($h = 0.01, 0.05, 0.10, 0.20, 0.50$).
- Evaluate the derivative of the governing differential equation at the current time step.
- Calculate the concentration at the subsequent time step using the Euler approximation.
- Repeat the iterative procedure until the final simulation time is reached.
- Compare the numerical results with the analytical solution and determine the relative error.

The Euler method is classified as a first-order numerical technique with a local truncation error of $\mathcal{O}(h^2)$ and a global truncation error of $\mathcal{O}(h)$. Hence, its numerical accuracy is strongly dependent on the selected step size. The larger the values of h , the greater cumulative approximation errors [7]. Euler method remains an attractive choice for preliminary analyses due to its low computational cost and ease of implementation despite its limitations.

3.4. Fourth-Order Runge–Kutta Method

The fourth-order Runge–Kutta (RK4) method is one of the most widely adopted numerical techniques for solving ordinary differential equations. This is due to the effective balance between the numerical performance and accuracy. While Euler method is reliant on a single slope evaluation, RK4 employs four intermediate slope calculations within each iteration to achieve a more precise estimate of the solution [6].

The RK4 formulation is given by:

$$C_{A,n+1} = C_{A,n} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where:

$$\begin{aligned} k_1 &= hf(t_n, C_n) \\ k_2 &= hf\left(t_n + \frac{h}{2}, C_n + \frac{k_1}{2}\right) \\ k_3 &= hf\left(t_n + \frac{h}{2}, C_n + \frac{k_2}{2}\right) \\ k_4 &= hf(t_n + h, C_n + k_3) \end{aligned}$$

In this study, the RK4 implementation procedure is summarized as follows:

- Define the initial conditions and simulation parameters.
- Choose the desired step size ($h = 0.01, 0.05, 0.10, 0.20, 0.50$).
- Evaluate the first intermediate slope (k_1), followed by the second and third intermediate slopes (k_2 and k_3) and finally the fourth intermediate slope (k_4).
- Determine the weighted average of the four slopes.
- Calculate the concentration at the next step size.
- Repeat the procedure until the final simulation time is reached.
- Compare the numerical results with the analytical solution and determine the relative error.

The RK4 method is categorized as a fourth-order numerical technique with a local truncation error of $\mathcal{O}(h^5)$ and a global truncation error of $\mathcal{O}(h^4)$. The method thus generally produces very accurate numerical approximations, even for quite moderate step sizes [7]. Although RK4 needs more computations per iteration, its better accuracy and stability make it a preferable choice for engineering applications for transient and dynamic system analyses [1].

3.5. Comparative Analysis of Numerical Methods

A comparative analysis based on numerical accuracy, convergence characteristics, relative error and numerical performance was performed to assess the suitability of the Euler and RK4 methods for CSTR dynamic simulations. The numerical simulation was repeated for each of the step sizes selected and the concentration profiles were compared with the exact analytical solution.

The maximum relative error can be calculated by using the following equation:

$$E_{rel,max} = \max_{0 < t < t_f} \left| \frac{C_{A,numerical}(t) - C_{A,analytical}(t)}{C_{A,analytical}(t)} \right| \times 100\%$$

The error values' obtained represent the maximum relative difference between the numerical and analytical concentration values over all simulated time points.

By using several step-sizes provides important information on how the time-dependant discretisation influences the numerical performance. In general, smaller step sizes are expected to improve the accuracy of both methods while larger step sizes may increase the truncation errors and affect the stability. Therefore, the investigation of the link between the step size and the numerical behaviour is essential for the choice of the most suitable computational method for reactor dynamic modelling.

3.6. Empirical Order of Convergence

The empirical order of convergence (EOC) is calculated by using the equation below :

$$p = \frac{\ln(E_{h_2} - E_{h_1})}{\ln(h_2 - h_1)}$$

where E_h denotes the numerical error associated with step size h .

Table 2. The observed convergence orders

h_1	h_2	Euler EOC	RK4 EOC
0.01	0.05	1.001	4.003
0.05	0.10	1.004	4.007
0.10	0.20	1.007	4.014
0.20	0.50	1.016	4.032

The EOC values are close to the theoretical first-order convergence of Euler's method and fourth-order convergence of RK4. This presents quantitative proof that the observed numerical behaviour is consistent with the theoretical truncation-error properties of the two methods.

3.7. Stability Analysis

The CSTR model is governed by the equation:

$$\frac{dC_A(t)}{dt} = -\left(\frac{1}{\tau} + k\right) C_A(t) + \frac{1}{\tau} C_{A,in}(t)$$

Thus, the eigenvalue of the homogenous system is given by:

$$\lambda = -\left(\frac{1}{\tau} + k\right) = -0.3$$

For the explicit Euler method, the amplification factor is $G_E = 1 + h\lambda$. Therefore:

$$h < 6.667$$

As for RK4, for the negative real axis, the RK4 stability boundary is approximately $h\lambda < 2.785$. Thus:

$$h < 9.283$$

The stability analysis also separates stability and accuracy. The stability analysis also separates stability and accuracy. The Euler solutions are stable for all the step sizes tested. However, the maximum relative error is significantly increased with increasing values of h , reaching a value of 7.6874% for $h = 0.50$. However, the increase in error is not related to instability but is mainly associated with the truncation error and the first order convergence behaviour of the Euler method. This means that a stable numerical solution does not have to be an accurate solution.

4. Results and Discussion

4.1. Numerical Analysis

The transient response of the CSTR model was investigated using the Euler method and the RK4 method for various step sizes, namely $h = 0.01, 0.05, 0.10, 0.20$ and 0.50 . The numerical solutions obtained were subsequently compared with the analytical solution to evaluate the accuracy and convergence characteristics of both methods. All simulations were implemented using MATLAB 2020b.

Table 3. Concentration of substance A and error obtained.

Step Size	Concentration (Euler)	Concentration (RK4)	Concentration (Analytical)	Error (Euler)	Error (RK4)
0.01	0.518248215	0.517913227	0.517913227	0.150075	6.75525×10^{-11}
0.05	0.51959406	0.517913226	0.517913227	0.751874993	4.23987×10^{-8}
0.1	0.52128975	0.517913225	0.517913227	1.507499888	6.81764×10^{-7}
0.2	0.524726575	0.517913201	0.517913227	3.0299982	1.10169×10^{-5}
0.5	0.535417064	0.51791216	0.517913227	7.687429725	4.43188×10^{-4}

Table 3 shows the final concentration values and the maximum relative errors for both Euler and RK4. It is evident that the RK4 method consistently generated results that were nearly identical to the analytical solution for all step sizes that were taken into account. It is shown that the RK4 method achieved maximum relative errors ranging from $6.76 \times 10^{-11}\%$ for $h = 0.01$ to only $4.43 \times 10^{-4}\%$ for $h = 0.50$, indicating excellent numerical accuracy and stability.

As for Euler, this method exhibited an increase in numerical error as the step size increased. For the smallest step size $h = 0.01$, the Euler method produced a maximum relative error of 0.1501%. However, this value increased substantially to 7.6874% when the step size was increased to $h = 0.50$. Similarly, the final concentration estimated by the Euler method increased from 0.5182 to 0.5354, deviating progressively from the analytical value of 0.5179. These results show that the Euler method is sensitive to the step size because the truncation errors are accumulated in the iterative computational process.

The transient response graphs for all step sizes that show both numerical methods are stable throughout the simulation interval is given in Figure 1 to 5. The concentration profiles from the Euler, RK4 and the analytical solution show the expected first order response behaviour of a CSTR, where the concentration increases rapidly in the initial stages and then asymptotically reaches the steady state value. However, on closer inspection, especially for larger step sizes, small deviations between the Euler method and the analytical solution become more and more evident. Meanwhile there is no distinctive difference for the RK4 method and the analytical solution for all the step sizes investigated.

The results also show the convergence properties of the two numerical methods. As the step size decreases, the Euler method converges toward the analytical solution, indicating that its accuracy can be improved by making the step sizes smaller. However, the RK4 method maintained a high degree of accuracy even for relatively large step sizes, showing its superior balance between numerical performance and numerical precision. Evidently, the RK4 method is more suitable for applications involving long simulation periods or systems requiring highly accurate transient analyses.

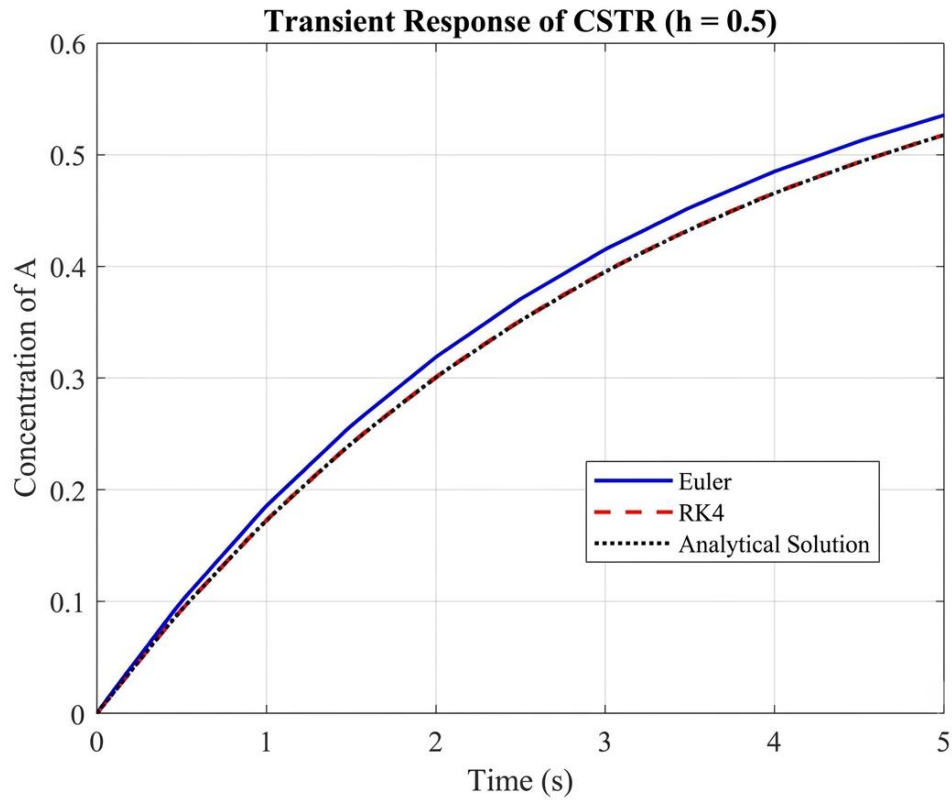


Fig. 1 Transient Response of CSTR for $h=0.5$

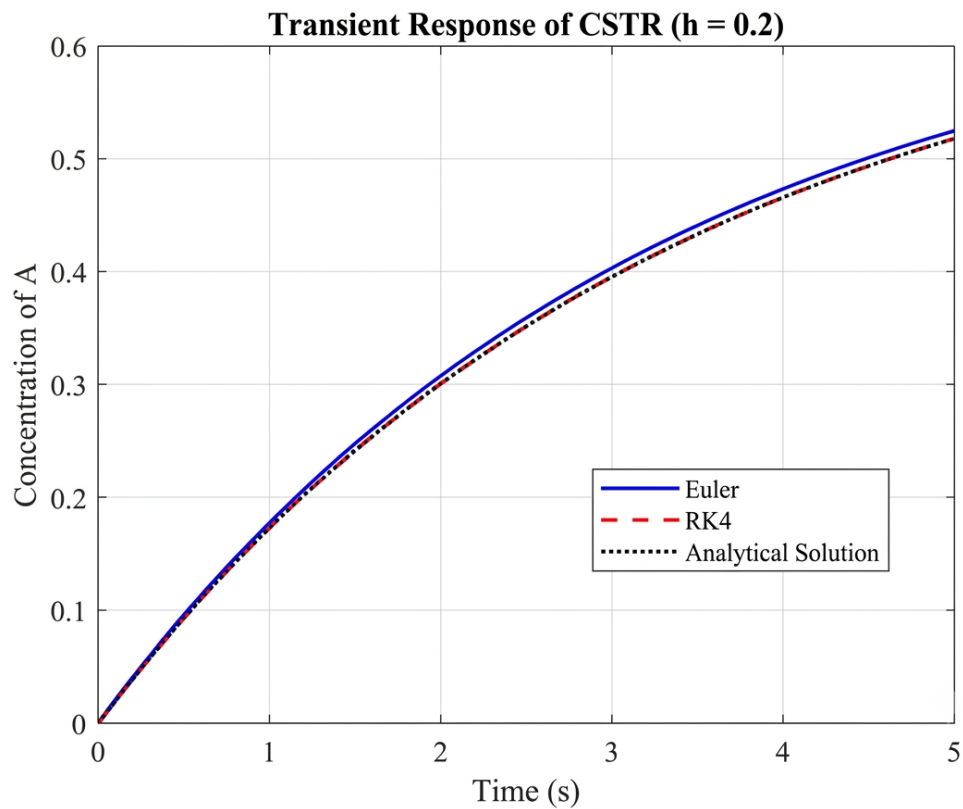


Fig. 2 Transient Response of CSTR for $h=0.2$

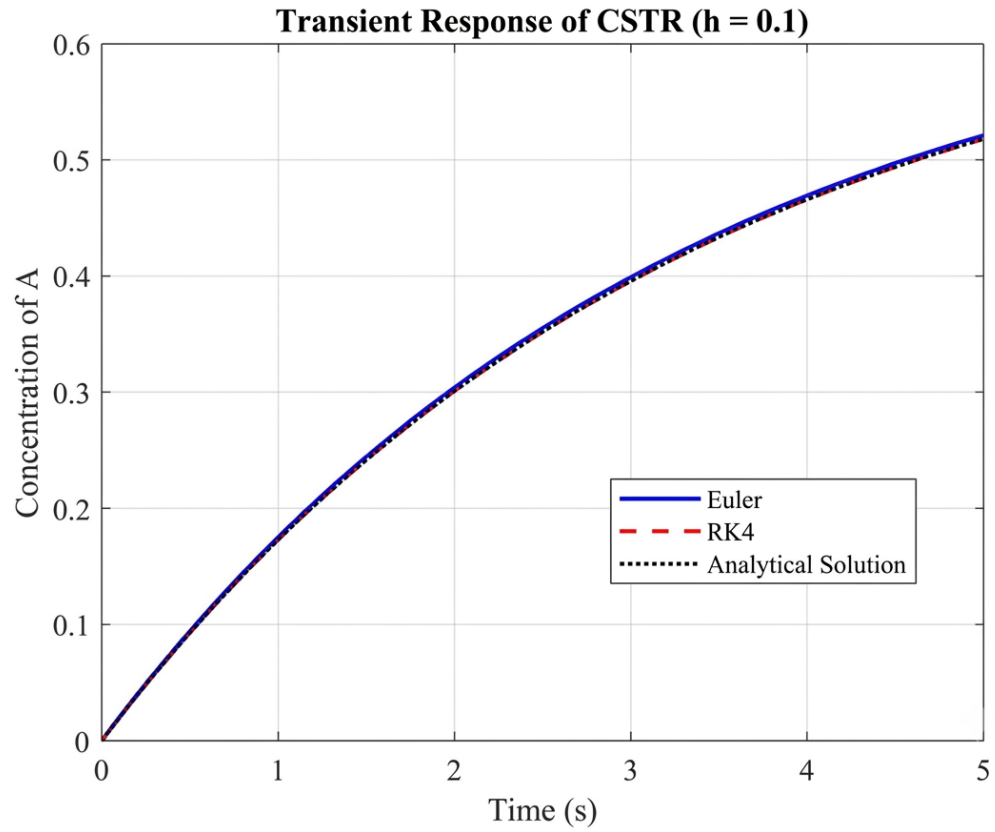


Fig. 3 Transient Response of CSTR for $h=0.1$

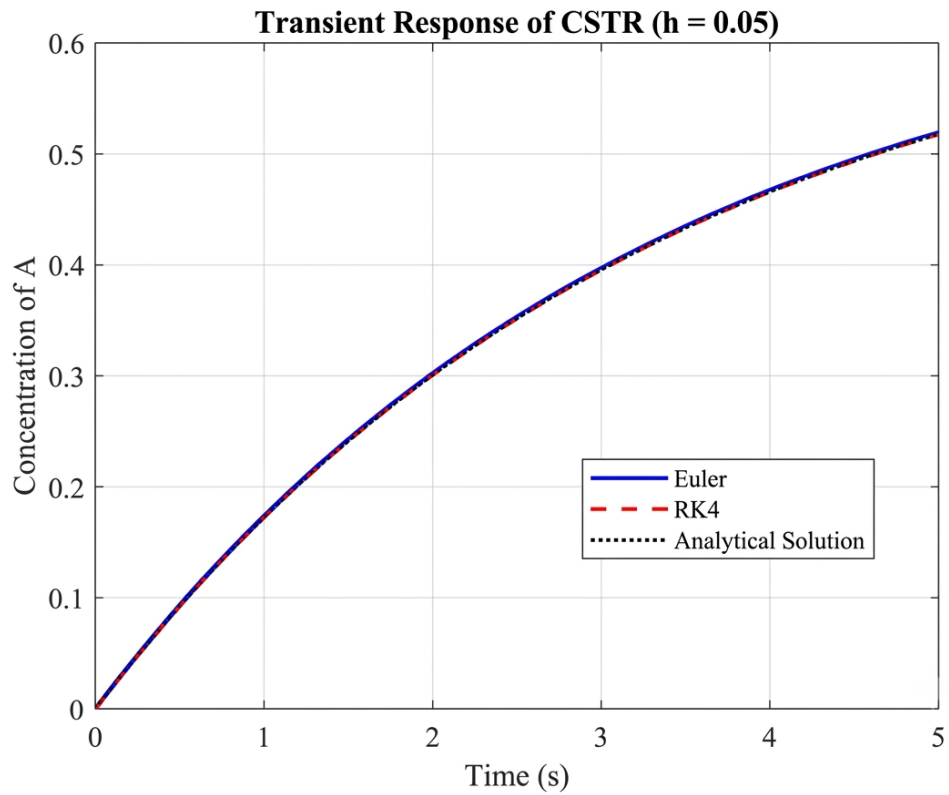
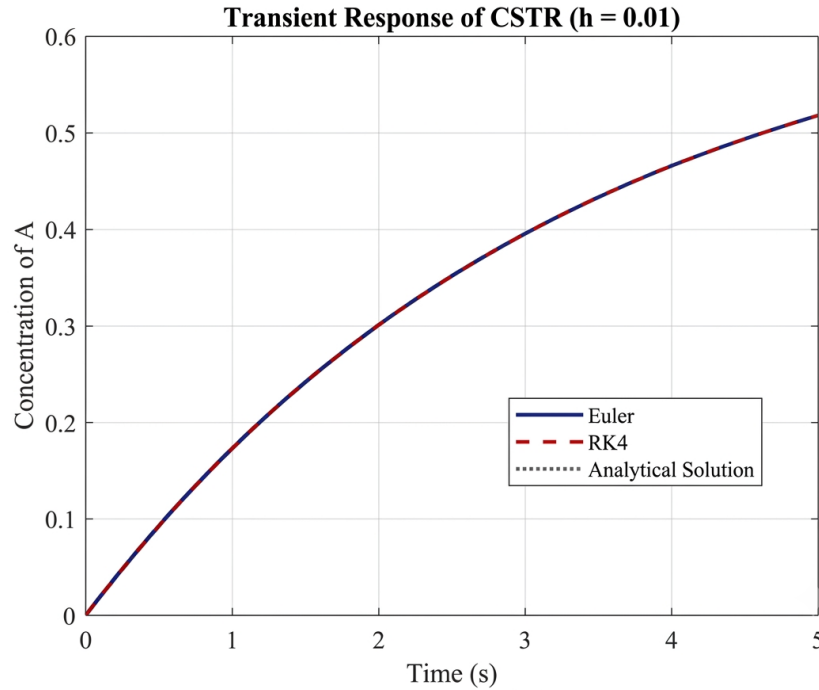


Fig. 4 Transient Response of CSTR for $h=0.05$

Fig. 5 Transient Response of CSTR for $h=0.01$

Overall, the comparative analysis confirms that the choice of numerical method plays a significant role in the accuracy of CSTR simulations. While the Euler method offers simplicity and ease of implementation, its performance deteriorates as the step size increases. In contrast, the RK4 method demonstrates robust performance, excellent stability, and negligible error accumulation, making it a preferred numerical technique for solving first-order ordinary differential equations associated with reactor dynamic models.

4.2. Numerical Stability Analysis

Numerical stability is an important criteria for the evaluation of the performance of numerical methods for the solution of ordinary differential equations. A stable numerical scheme provides solutions that converge to the expected response without oscillation or divergence [6,7]. In this work, the stability of the Euler method was analysed using $|1 + h\lambda| < 1$, while the stability of the RK4 method was analysed based on the product $h\lambda$, which must remain within the established RK4 stability region ($-2.8 < h\lambda < 0$) [14].

Table 3 shows the stability analysis for the two numerical methods. Table 3 shows the stability analysis for the two numerical methods. It is shown that all the considered step sizes satisfy the respective stability conditions. These results are consistent with the transient concentration profiles shown in the figures 1-5 with bounded and convergent solutions over time. The RK4 method, while being numerically stable, provides better numerical accuracy and a higher margin of stability. Thus, this is a more suitable method for transient CSTR simulation. The transient response of CSTR model was studied by using Euler method and RK4 method for the different step sizes, $h = 0.01, 0.05, 0.10, 0.20, 0.50$. The accuracy and convergence characteristics of the two methods were evaluated by comparing the numerical solutions obtained with the analytical solution. All the simulations were performed in MATLAB 2020b.

Table 3. Stability Analysis of the Euler Method and RK4 Method for Different Step Sizes

Step Size, h	$ 1 + h\lambda $	$h\lambda$	Stability Status
0.01	0.997	-0.003	Stable
0.05	0.985	-0.015	Stable
0.10	0.970	-0.030	Stable
0.20	0.940	-0.060	Stable
0.50	0.850	-0.150	Stable

4.3. Model Scope and Limitation

The CSTR model used here is intentionally simple, to provide a well-defined benchmark for testing numerical integration techniques. Under the assumptions of perfect mixing, constant volume, isothermal operation, constant inlet concentration, and first order reaction kinetics, the governing dynamics reduce to a single linear ordinary differential equation with known analytical solution. Such a formulation allows for the isolation and quantification of numerical errors without introducing additional uncertainties associated with nonlinear kinetics, coupled thermal effects, varying operating conditions, or experimental measurements.

However, these assumptions limit the direct practical interpretation of the results. Real CSTRs can have non-linear reaction kinetics, temperature dependent reaction rates, heat transfer effects, variable operating conditions, multiple reacting species, transport limitations and process disturbances. More complex CSTR models may therefore lead to coupled and potentially stiff systems of ordinary differential equations that require the use of adaptive or implicit numerical solvers.

The results obtained in this study should therefore be mainly interpreted as a benchmark of the numerical method, rather than a full description of the industrial reactor behaviour. The main advantage of the simplified model is its transparency and reproducibility, allowing to analyse separately the effect of the step size, the truncation error, the order of convergence and the stability. Future work can extend the benchmark to nonlinear and non-isothermal CSTR models, coupled material and energy balances, multiple reaction pathways, disturbances and adaptive or implicit numerical solvers.

5. Conclusion

This study presented a quantitative numerical benchmark for a transient simulation of a first order CSTR by explicit Euler and RK4 methods. Although the higher accuracy of RK4 is theoretically expected, the contribution of the study is to systematically quantify the interaction between the temporal step size, numerical error, convergence behaviour and stability using a CSTR model with an available analytical solution.

The results show that the two methods converge in very different way. The convergence order for Euler's method was approximately first order with empirical orders ranging from 1.001 to 1.016. The order of convergence of RK4 was approximately fourth order with empirical orders ranging from 4.003 to 4.032. The error analysis also showed that a larger step size caused a significantly larger error for Euler with a rate of 7.6874% for $h = 0.50$ while RK4 showed errors less than 0.001% in the range studied. These results give quantitative evidence that the difference between the two methods is not only a difference in final accuracy but also a difference in their sensitivity to temporal discretisation.

Furthermore, the stability analysis indicated that all the step sizes considered were within the theoretical stability regions for both methods. Thus, the Euler error increased with larger step sizes. This was a stable numerical solution and it is important to distinguish between numerical stability and numerical accuracy.

The resulting benchmark provides a reproducible computational framework to demonstrate the behaviour of numerical methods in chemical engineering applications. In particular, a combination of analytical validation, multi-step-size testing, empirical convergence analysis and stability-boundary evaluation provides a more complete assessment of numerical solver behaviour than a conventional two-method accuracy comparison. Future work can extend this framework to nonlinear reaction kinetics, coupled material and energy balances, adaptive time-stepping, and computational cost analysis.

References

- [1] Dale E. Seborg et al., *Process Dynamics and Control*, 4th ed., John Wiley and Sons, 2017. [[Google Scholar](#)] [[Publisher Link](#)]
- [2] H. Scott Fogler, *Elements of Chemical Reaction Engineering*, 6th ed., Pearson, 2022. [[Publisher Link](#)]
- [3] Tianzhou Ye et al., "Particle Movement Characteristics in a Gas-Solid Vertical Single Helical Ribbon Agitated Reactor," *Chemical Engineering Journal*, vol. 429, 2022. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [4] Jie Yang et al., "Influence of Reflux Ratio on the Anaerobic Digestion of Pig Manure in Leach Beds Coupled with Continuous Stirred Tank Reactors," *Waste Management*, vol. 97, pp. 115-122, 2019. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [5] Amit K. Thakur et al., "Multi-Objective Optimization of an Industrial Slurry Phase Ethylene Polymerization Reactor," *International Journal of Chemical Reactor Engineering*, vol. 20, no. 6, pp. 649-659, 2022. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [6] Steven C. Chapra, and Raymond P. Canale, *Numerical Methods for Engineers*, 8th ed., McGraw-Hill Education, 2021. [[Google Scholar](#)] [[Publisher Link](#)]
- [7] J. Douglas Faires, and Richard L. Burden, *Numerical Analysis*, 11th ed., Cengage Learning, 2021. [[Publisher Link](#)]

- [8] Teoh Ai Kee, and Rahifa Ranom, "Comparison of Numerical Techniques in Solving Transient Analysis of Electrical Circuits," *ARPJ Journal of Engineering and Applied Sciences*, vol. 13, no. 1, pp. 314-320, 2018. [[Google Scholar](#)] [[Publisher Link](#)]
- [9] Chijioke Godswill Igwe, P.C. Jackreece, and Isobeye George, "Comparative Convergence Analysis of Runge-Kutta Fourth Order and Runge-Kutta-Fehlberge Methods Implementation in Matlab and Python Applied to a Series RLC Circuit," *International Journal of Innovative Science and Research Technology*, vol. 10, no. 3, pp. 2957-2975, 2025. [[CrossRef](#)] [[Publisher Link](#)]
- [10] Mario D. Fiore, and Ciro Natale, "Discrete-Time Closed-Loop Inverse Kinematics: A Comparison between Euler and RK4 Methods," *2021 29th Mediterranean Conference on Control and Automation*, Puglia, Italy, pp. 584-589, 2021. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [11] Muhirwa Jean Pierre, "Mathematical Modelling and Analysis of Deformable Perturbed Continuously Stirred Tank Reactors (CSTRs)," Doctoral Dissertation, Nelson Mandela African Institution of Science and Technology, pp. 1-158, 2022. [[Google Scholar](#)]
- [12] F.J. Hurtado, Antonio S. Káiser, and Blas Zamora, "Fluid Dynamic Analysis of a Continuous Stirred Tank Reactor for Technical Optimization of Wastewater Digestion," *Water Research*, vol. 71, pp. 282-293, 2015. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [13] Octave Levenspiel, "Chemical Reaction Engineering," *Industrial & Engineering Chemistry Research*, vol. 38, no. 11, pp. 4140-4143, 1999. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [14] John Charles Butcher, *Numerical Methods for Ordinary Differential Equations*, 3rd ed., John Wiley & Sons, 2016. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [15] Liangcheng Suo et al., "Study on the Nonlinear Dynamics of the Continuous Stirred Tank Reactors," *Processes*, vol. 8, no. 11, pp. 1-18, 2020. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [16] Tapio Salmi et al., "Determination of Kinetic Constants by using Transient Temperature Data from Continuous Stirred Tank Reactors," *Chemical Engineering Science*, vol. 248, pp. 1-8, 2022. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [17] Jun Yin et al., "Generalized Reactor Neural ODE for Dynamic Reaction Process Modeling with Physical Interpretability," *Chemical Engineering Journal*, vol. 452, 2023. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [18] Changrui Xie et al., "Hybrid Dynamic Modeling of an Industrial Reactor Network with First-Principles and Data-Driven Approaches," *Chemical Engineering Science*, vol. 289, 2024. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]