International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V72I9P104
Volume 72 Issue 9, 56-67, September 2026
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Original Article

Neutrosophic Superhypersoft Topological Spaces

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Received: 16 July 2026 Revised: 21 August 2026 Accepted: 13 September 2026 Published: 30 September 2026

Abstract - *Neutrosophic Superhypersoft Topology is an innovative mathematical framework that integrates the principles of neutrosophic sets, Superhypersoft structures and topology to address the complexities and uncertainties inherent in real-world data. This advanced topology extends classical concepts by incorporating neutrosophic sets, where each element is characterized by degrees of truth, indeterminacy, and falsity. Superhypersoft operations, which act on the n-th power set (H), allow for multiple possible outcomes, accommodating the multifaceted nature of relationships between elements. The Superhypersoft axioms provide a generalized set of properties that these operations must satisfy, ensuring consistency and coherence within the structure. This article introduces and establishes several key theorems regarding the notion of neutrosophic Superhypersoft topology, neutrosophic Superhypersoft open (closed) sets, neutrosophic Superhypersoft interior, neutrosophic Superhypersoft closure, fuzzy hypersoft neutrosophic foundation, and neutrosophic Superhypersoft subspace topology.*

Keywords *- Neutrosophic Superhypersofttopology, Neutrosophic Superhypersoft interior, Neutrosophic Superhypersoft Closure.*

1. Introduction

Neutrosophic Superhypersoft Topology is a novel mathematical framework that extends classical topology by incorporating the principles of neutrosophy, which deals with the concept of indeterminacy and uncertainty. Developed by Florentin Smarandache, this topology introduces a new paradigm in the study of topological spaces, enabling researchers to explore complex relationships in various fields, including mathematics, computer science, and decision-making processes. Neutrosophic Superhypersoft Topology is built upon the n-th power set Pn(H) of a non-empty set H. Unlike traditional topological spaces, where the empty set is often excluded, neutrosophic Superhypersoft topology embraces the empty set as a legitimate member of the power set. This inclusion allows for a more nuanced representation of uncertainty, which is essential in many real-world applications where information may be incomplete or ambiguous. The fundamental concepts of this topology revolve around the definitions of neutrosophic open and closed sets, which serve as the building blocks for more complex structures. Neutrosophic Superhypersoft open sets are defined in a way that accommodates the inherent uncertainty of the elements they contain, while closed sets capture the complementary aspects of these uncertainties. This duality is crucial for understanding the interactions between different sets within the topology.

The objective and outcome of this study are to apply the idea of neutrosophic Superhypersoft to classical topology, neutrosophic Superhypersoft topology, neutrosophic Superhypersoft open (closed) sets, neutrosophic fuzzy interior, neutrosophic Superhypersoft closure, neutrosophic Superhypersoft base, and neutrosophic Superhypersoft subspace topology. We also establish some significant theorems pertaining to these spaces.

1.1. Literature Review

The idea of soft sets was presented by Molodtsov as an instrument to manage uncertainty [8]. The notion of fuzzy soft sets was created by Maji et al. [7] as an extension of the original concept; Ahmad and Kharal also made contributions to this area [6]. Chang has researched topological concepts within soft set theory, and many researchers expanded on this work by looking into fuzzy soft topological spaces [2][11][9]. Soft topological structures were also developed by Shabir and Naz [15]. Zadeh was responsible for laying the foundation of fuzzy set theory [13]. Smarandache's introduction of neutrosophic sets and neutrosophy [4][5] includes the uncertainty portion of many mathematical structures; their expansion to neutrosophic soft

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relations by Deli and Broumi [3], neutrosophic soft expert sets by Sahin et al. [14], and neutrosophic soft topological spaces by Bera and Mahapatra [16] provides the foundation for Neutrosophic SuperHypersoft Topological Spaces. Smarandache also proposed hypersoft sets [10], and many previously mentioned researchers have expanded to hypersoft processes through operations or foundational studies [1][18]. Yolcu and Ozturk have researched applications of these new concepts in the decision-making process [12], and Saqlain et al. [19] have formulated neutrosophic hypersoft sets with aggregation operators. Collectively, these contributions provide the groundwork for studying Neutrosophic SuperHypersoft Topological Spaces.

2. Premilinaries

Definition 2.1 The set (F, Λ) is called a soft set of the universe Δ, if and only if there exists a mapping F: Λ → $P(\Delta)$, from the indexing set Λ to the power set of Δ, therefore providing a parameterized collection of subsets of Δ.

Definition 2.2: For a set Δ to be a universal set, the power set of Δ denoted by (Δ) exists. Suppose we have n distinct attributes denoted by $k_1, k_2, \ldots, k_n$and n sets corresponding to these distinct attributes denoted by $K_1, K_2, .., K_n$. with the following conditions: $K_i \cap K_j = \emptyset$ $(i \neq j)$ and $i, j\epsilon$ {1,2,3 … n}. Then the pair (F, $K_1 \times K_2 \times \ldots \times K_n$) is said to be HSS over Δ where F is a mapping from $K_1 \times K_2 \times \ldots \times K_n$ to $P(\Delta)$.

Definition 2.3 Let us define a Universal Set called Δ, and the Power Set of Δ is represented by the notation (Δ). For $n \geq 1$, there are n distinct attributes such as $k_1, k_2, \ldots, k_n$ an d$K_1, K_2, .., K_n$ are sets for corresponding attribute values, respectively, with conditions such as $K_i \cap K_j = \emptyset$ $(i \neq j)$ and $i, j\epsilon$ {1,2,3 … n}. Then the pair (F, Λ) is said to be an NHSS over Δ if there exists a relation. $K_1 \times K_2 \times \ldots \times K_n) = \Lambda$. F is a mapping from $K_1 \times K_2 \times \ldots \times K_n)$ to $P(\Delta)$ and F($K_1 \times K_2 \times \ldots \times K_n$)) = {<$u$,$u_A(u), v_A(u), w_A(u)$> : $u\in \Delta$ } where $\mathcal{u}$, $\mathcal{v}$, $\mathcal{w}$ are Membership values to represent truth, unclear and false, respectively, such that $\mathcal{u}$, $\mathcal{v}$, $\mathcal{w}$: $\mathbb{U} \rightarrow]0^-, 1^+[$ and $0^- \leq u_A(u), +v_A(u) + w_A(u) \leq 3^+$.

Definition 2.4 Let Δ be a universe of discourse, P($\mathbb{U}$) the power set of $\mathbb{U}$, and $k_1, k_2, \ldots \ldots, k_n$ (n ≥ 1) distinct attributes with disjoint corresponding sets $K_1, K_2, \ldots \ldots, K_n$. Let P($K_i$) denote the power set of $K_i$for i ∈ {1, 2, . . . , n}.
A Neutrosophic SuperHyperSoft Set (NSHSS) over $\mathbb{U}$ is defined as the pair
(S, P(K_1) × P(K_2) × · · · × P(K_n)) , where S : P(K_1) × P(K_2) × · · · × P(K_n)→ P($\mathbb{U}$), and
S = {$\alpha, \langle x, T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x)\rangle$: x ∈$\mathbb{U}$, α ∈ P($K_1$) × P($K_2$) × · · · × P($K_n$)}.
Here, $T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x) : \mathbb{U} \rightarrow [0, 1]$ represent the membership, indeterminacy, and non-membership degrees of x ∈$\mathbb{U}$ for each α, satisfying $0 \leq \left(T_{s(\alpha)}(x), I_{s(\alpha)}(x), F_{s(\alpha)}(x)\right) \leq 1$.

3. Neutrosophic Superhypersoft Set

Definition 3.1 Let Δ be the universe of discourse, $\mathcal{P}(\Delta)$ be the power set of Δ, *let* $b_1, b_2, \ldots \ldots, b_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attribute values are respectively the sets $B_1, B_2, \ldots \ldots, B_n$ with $B_i \cap B_j = \emptyset$ for $i \neq$i . Let $\mathcal{P}(B_1), \mathcal{P}(B_2), \ldots..\mathcal{P}(B_n)$ be the powers of $B_1, B_2, \ldots \ldots, B_n$ respectively. A neutrosophic superhypersoft set (NSHS) over Δ is the pair. $\left(\mathcal{S}, \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots..\times \mathcal{P}(B_n)\right)$ where $\mathcal{S}: \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots..\times \mathcal{P}(B_n) \rightarrow \mathcal{P}(\Delta)$ and $\mathcal{S}, (\mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots..\times \mathcal{P}(B_n)) = \{\alpha, \langle x, T_{\mathcal{S}(\alpha)}(x), I_{\mathcal{S}(\alpha)}(x), F_{\mathcal{S}(\alpha)}(x)\rangle: x \in \Delta, \alpha \in (\mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots..\times \mathcal{P}(B_n))\}$.

Definition 3.2 Let Δ be the universal set. $(\mathcal{S}_1, \Gamma_1)$, $(\mathcal{S}_2, \Gamma_2)$ be two neutrosophic superhypersoft sets over the universe Δ. The union $(\mathcal{S}_1, \Gamma_1) \cup (\mathcal{S}_2, \Gamma_2)$ is defined as

$$T\left((\mathcal{S}_1, \Gamma_1) \cup (\mathcal{S}_2, \Gamma_2)\right) = \begin{cases} T_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ T_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ max\{T_{\mathcal{S}_1(\alpha)}(x),\ T_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x\epsilon\Gamma_1 \cup \Gamma_2 \end{cases}$$

$$I\left((\mathcal{S}_1, \Gamma_1) \cup (\mathcal{S}_2, \Gamma_2)\right) = \begin{cases} I_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ I_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ max\{I_{\mathcal{S}_1(\alpha)}(x),\ I_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x\epsilon\Gamma_1 \cup \Gamma_2 \end{cases}$$

$$F\left((\mathcal{S}_1, \Gamma_1) \cup (\mathcal{S}_2, \Gamma_2)\right) = \begin{cases} F_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ F_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ min\{F_{\mathcal{S}_1(\alpha)}(x),\ F_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x\epsilon\Gamma_1 \cup \Gamma_2 \end{cases}$$

Definition 3.3 Let Δ be the universal set over which the two neutrosophic Superhypersoft sets. $(\mathcal{S}_1, \Gamma_1)$, $(\mathcal{S}_2, \Gamma_2)$are located. The definition of the intersection $(\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)$is

$$T\big((\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)\big) = \begin{cases} T_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ T_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ min\{T_{\mathcal{S}_1(\alpha)}(x),\ T_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x \epsilon \Gamma_1 \cap \Gamma_2 \end{cases}$$

$$I\big((\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)\big) = \begin{cases} I_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ I_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ min\{I_{\mathcal{S}_1(\alpha)}(x),\ I_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x \epsilon \Gamma_1 \cap \Gamma_2 \end{cases}$$

$$F\big((\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)\big) = \begin{cases} F_{\mathcal{S}_1(\alpha)}(x) if\ x \in \Gamma_1 - \Gamma_2 \\ F_{\mathcal{S}_2(\alpha)}(x) if\ x \in \Gamma_2 - \Gamma_1 \\ max\{F_{\mathcal{S}_1(\alpha)}(x),\ F_{\mathcal{S}_2(\alpha)}(x)\}\ if\ x \epsilon \Gamma_1 \cap \Gamma_2 \end{cases}.$$

Definition 3.4 Let Δ be the universal set. $(\mathcal{S}_1, \Gamma_1)$, $(\mathcal{S}_2, \Gamma_2)$ be two neutrosophic superhypersoft sets over the universe Δ. The 'AND' operator$(\mathcal{S}_1, \Gamma_1) \wedge (\mathcal{S}_2, \Gamma_2) = \mathcal{S}(\Gamma_1 \times \Gamma_2)$ is defined as

$T\big((\mathcal{S}_1, \Gamma_1) \wedge (\mathcal{S}_2, \Gamma_2)\big) = \min\{T_{\mathcal{S}_1(\alpha)}(x), T_{\mathcal{S}_2(\alpha)}(x)\}$

$I\big((\mathcal{S}_1, \Gamma_1) \wedge (\mathcal{S}_2, \Gamma_2)\big) = \min\{I_{\mathcal{S}_1(\alpha)}(x), I_{\mathcal{S}_2(\alpha)}(x)\}$

$F\big((\mathcal{S}_1, \Gamma_1) \wedge (\mathcal{S}_2, \Gamma_2)\big) = \max\{F_{\mathcal{S}_1(\alpha)}(x), F_{\mathcal{S}_2(\alpha)}(x)\}$.

Definition 3.5 Let Δ be the universal set. $(\mathcal{S}_1, \Gamma_1)$, $(\mathcal{S}_2, \Gamma_2)$ be two neutrosophic superhypersoft sets over the universe Δ. The 'OR' operator$(\mathcal{S}_1, \Gamma_1) \vee (\mathcal{S}_2, \Gamma_2) = \mathcal{S}(\Gamma_1 \times \Gamma_2)$ is defined as

$T\big((\mathcal{S}_1, \Gamma_1) \vee (\mathcal{S}_2, \Gamma_2)\big) = \max\{T_{\mathcal{S}_1(\alpha)}(x), T_{\mathcal{S}_2(\alpha)}(x)\}$

$I\big((\mathcal{S}_1, \Gamma_1) \vee (\mathcal{S}_2, \Gamma_2)\big) = \max\{I_{\mathcal{S}_1(\alpha)}(x), I_{\mathcal{S}_2(\alpha)}(x)\}$

$F\big((\mathcal{S}_1, \Gamma_1) \vee (\mathcal{S}_2, \Gamma_2)\big) = \min\{F_{\mathcal{S}_1(\alpha)}(x), F_{\mathcal{S}_2(\alpha)}(x)\}$.

Example 3.1 Let $\Delta = (x_1, x_2, x_3)$ be an initial universe and $b_1, b_2, \ldots \ldots, b_n$ be sets of attributes. Attributes are given as

$$\mathrm{B}_1 = \{\{\mathrm{a}_1\}, \{\mathrm{a}_2\}, \{\mathrm{a}_3\}\}, \mathrm{B}_2 = \{\{\mathrm{c}_1\}, \{\mathrm{c}_2\}\}, \mathrm{B}_3 = \{\{\mathrm{d}_1\}, \{\mathrm{d}_2\}\},$$
$$\mathrm{C}_1 = \{\{\mathrm{a}_1\}\}, \mathrm{C}_2 = \{\{\mathrm{c}_1\}, \{\mathrm{c}_2\}\}, \mathrm{C}_3 = \{\{\mathrm{d}_1\}, \{\mathrm{d}_2\}\},$$

$D_1 = \{\{\mathrm{a}_1\}, \{\mathrm{a}_2\}\}, \mathrm{D}_2 = \{\{\mathrm{c}_1\}, \{\mathrm{c}_2\}\}, \mathrm{D}_3 = \{\{\mathrm{d}_2\}\}$

Are subsets of B_i for each $i = 1,2,3,..$ then NSHS sets $(\mathcal{S}_1,\ \Gamma_1)$ and $(\mathcal{S}_2,\ \Gamma_2)$ over the universe Δ as follows

$$(\mathcal{S}_1, \Gamma_1) = \left\{ \begin{array}{l} \langle(\{a_1\}, \{c_1\}, \{d_1\}) \left\{ \frac{x_1}{\{0.8\}, \{0.5\}, \{0.2\}}, \frac{x_2}{\{0.6\}, \{0.5\}, \{0.4\}}, \frac{x_3}{\{0.3\}, \{0.6\}, \{0.2\}} \right\} \rangle \\ \langle(\{a_1\}, \{c_1\}, \{d_2\}) \left\{ \frac{x_1}{\{0.8\}, \{0.2\}, \{0.5\}}, \frac{x_2}{\{0.5\}, \{0.7\}, \{0.1\}}, \frac{x_3}{\{0.5\}, \{0.5\}, \{0.4\}} \right\} \rangle \\ \langle(\{a_1\}, \{c_2\}, \{d_1\}) \left\{ \frac{x_1}{\{0.6\}, \{0.4\}, \{0.5\}}, \frac{x_2}{\{0.5\}, \{0.7\}, \{0.4\}}, \frac{x_3}{\{0.7\}, \{0.3\}, \{0.2\}} \right\} \rangle \\ \langle(\{a_1\}, \{c_2\}, \{d_2\}) \left\{ \frac{x_1}{\{0.6\}, \{0.2\}, \{0.8\}}, \frac{x_2}{\{0.3\}, \{0.4\}, \{0.5\}}, \frac{x_3}{\{0.1\}, \{0.3\}, \{0.7\}} \right\} \rangle \end{array} \right\}$$

$$(\mathcal{S}_2,\Gamma_2)=\begin{Bmatrix}\langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.9\},\{0.2\},\{0.1\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.1\},\{0.7\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.4\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.6\},\{0.8\}}\right\}\rangle\\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.3\},\{0.7\}}\right\}\rangle\\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.6\},\{0.1\},\{0.7\}}\right\}\rangle\end{Bmatrix}$$

The union, intersection, AND, and OR of these sets are as follows:

$$(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)=\begin{Bmatrix}\langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.9\},\{0.2\},\{0.1\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.5\},\{0.4\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.6\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.4\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.6\},\{0.7\}}\right\}\rangle\\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.3\},\{0.7\}}\right\}\rangle\\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.6\},\{0.1\},\{0.7\}}\right\}\rangle\end{Bmatrix}$$

$$(\mathcal{S}_1,\Gamma_1)\cap(\mathcal{S}_2,\Gamma_2)=\begin{Bmatrix}\langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.1\},\{0.7\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.3\},\{0.8\}}\right\}\rangle\end{Bmatrix}$$

Let us assume $(\{a_1\},\{c_1\},\{d_1\})=f_1$,$(\{a_1\},\{c_1\},\{d_2\})=f_2$, $(\{a_1\},\{c_2\},\{d_1\})=f_3$, $(\{a_1\},\{c_2\},\{d_2\})=f_4$, and $(\{a_1\},\{c_1\},\{d_2\})=g_1$,$(\{a_1\},\{c_2\},\{d_2\})=g_2$, $(\{a_3\},\{c_1\},\{d_2\})=g_3$,$(\{a_3\},\{c_2\},\{d_2\})=g_4$.

$$(\mathcal{S}_1,\Gamma_1)\wedge(\mathcal{S}_2,\Gamma_2)=\begin{Bmatrix}\left(f_1\times g_1,\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.2\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.1\},\{0.7\}}\right\}\right)\\ \left(f_1\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.6\},\{0.4\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.6\},\{0.8\}}\right\}\right)\\ \left(f_1\times g_3,\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_1\times g_4,\left\{\frac{x_1}{\{0.4\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.3\},\{0.1\},\{0.5\}}\right\}\right)\\ \left(f_2\times g_1,\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.1\},\{0.7\}}\right\}\right)\\ \left(f_2\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.4\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.5\},\{0.8\}}\right\}\right)\\ \left(f_2\times g_3,\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.5\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.5\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_2\times g_4,\left\{\frac{x_1}{\{0.4\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.5\},\{0.1\},\{0.5\}}\right\}\right)\\ \left(f_3\times g_1,\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.1\},\{0.7\}}\right\}\right)\\ \left(f_3\times g_2,\left\{\frac{x_1}{\{0.6\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.4\},\{0.4\}},\frac{x_3}{\{0.4\},\{0.3\},\{0.8\}}\right\}\right)\\ \left(f_3\times g_3,\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.5\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_3\times g_4,\left\{\frac{x_1}{\{0.4\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.6\},\{0.1\},\{0.5\}}\right\}\right)\\ \left(f_4\times g_1,\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.2\},\{0.2\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.1\},\{0.7\}}\right\}\right)\\ \left(f_4\times g_2,\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.3\},\{0.8\}}\right\}\right)\\ \left(f_4\times g_3,\left\{\frac{x_1}{\{0.5\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_4\times g_4,\left\{\frac{x_1}{\{0.4\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.1\},\{0.1\},\{0.7\}}\right\}\right)\end{Bmatrix}$$

$$(\mathcal{S}_1,\Gamma_1)\wedge(\mathcal{S}_2,\Gamma_2)=\left\{\begin{array}{l}\left(f_1\times g_1,\left\{\frac{x_1}{\{0.9\},\{0.5\},\{0.1\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.6\},\{0.2\}}\right\}\right)\\ \left(f_1\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.5\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.6\},\{0.2\}}\right\}\right)\\ \left(f_1\times g_3,\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.6\},\{0.2\}}\right\}\right)\\ \left(f_1\times g_4,\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.6\},\{0.6\},\{0.2\}}\right\}\right)\\ \left(f_2\times g_1,\left\{\frac{x_1}{\{0.9\},\{0.2\},\{0.1\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.5\},\{0.4\}}\right\}\right)\\ \left(f_2\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.6\},\{0.4\}}\right\}\right)\\ \left(f_2\times g_3,\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.6\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.8\},\{0.5\},\{0.4\}}\right\}\right)\\ \left(f_2\times g_4,\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.6\},\{0.5\},\{0.4\}}\right\}\right)\\ \left(f_3\times g_1,\left\{\frac{x_1}{\{0.9\},\{0.4\},\{0.1\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\right)\\ \left(f_3\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.7\},\{0.2\}},\frac{x_3}{\{0.7\},\{0.6\},\{0.2\}}\right\}\right)\\ \left(f_3\times g_3,\left\{\frac{x_1}{\{0.6\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.6\},\{0.7\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.3\},\{0.2\}}\right\}\right)\\ \left(f_3\times g_4,\left\{\frac{x_1}{\{0.6\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.7\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\right)\\ \left(f_4\times g_1,\left\{\frac{x_1}{\{0.9\},\{0.2\},\{0.1\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_4\times g_2,\left\{\frac{x_1}{\{0.8\},\{0.4\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.4\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.6\},\{0.7\}}\right\}\right)\\ \left(f_4\times g_3,\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.6\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.3\},\{0.7\}}\right\}\right)\\ \left(f_4\times g_4,\left\{\frac{x_1}{\{0.6\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.6\},\{0.3\},\{0.5\}}\right\}\right)\end{array}\right\}$$

Definition 3.6 The complementof neutrosophic Superhypersoft $(\mathcal{S},\Gamma)$ over the universal set Δ is denoted by $(\mathcal{S},\Gamma)^c$ and defined as

$$T^c{}_{\mathcal{S}(\alpha)}(x)=\ F_{\mathcal{S}(\alpha)}(x)$$
$$I^c{}_{\mathcal{S}(\alpha)}(x)=1-\ I_{\mathcal{S}(\alpha)}(x)$$

$F^c{}_{\mathcal{S}(\alpha)}(x)=\ T_{\mathcal{S}(\alpha)}(x)$.

Example 3.2. We consider the attributes in Example 3.1

$$(\mathcal{S}_1,\Gamma_1)=\left\{\begin{array}{l}\langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.5\},\{0.4\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.6\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.3\},\{0.7\}}\right\}\rangle\end{array}\right\}$$

$$(\mathcal{S}_1,\Gamma_1)^c=\left\{\begin{array}{l}\langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.2\},\{0.5\},\{0.8\}},\frac{x_2}{\{0.4\},\{0.5\},\{0.6\}},\frac{x_3}{\{0.7\},\{0.4\},\{0.8\}}\right\}\rangle\\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.8\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.3\},\{0.9\}},\frac{x_3}{\{0.5\},\{0.5\},\{0.6\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.4\},\{0.6\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.3\},\{0.6\}},\frac{x_3}{\{0.3\},\{0.7\},\{0.8\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.8\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.6\},\{0.5\}},\frac{x_3}{\{0.9\},\{0.7\},\{0.3\}}\right\}\rangle\end{array}\right\}$$

Definition 3.7: In the universe Δ, a neutrosophic Superhypersoftset ($\mathcal{S}$,Γ) is considered null if, for every x ϵ Δ and α ϵ Γ, $T_{\mathcal{S}(\alpha)}(x)=0,\ I_{\mathcal{S}(\alpha)}(x)=0,\ F_{\mathcal{S}(\alpha)}(x)=1$. To represent it, use $0_{(\Delta_{NSHS},\ \wedge)}$.

Definition 3.8: A set of fuzzy hypersoft sets ($\mathcal{S}$,Γ) that is neutrosophic over a universe Δ is considered absolute if, for any x that is ϵ Δ and α that is ϵ Γ, $T_{\mathcal{S}(\alpha)}(x)=1,\ I_{\mathcal{S}(\alpha)}(x)=1,\ F_{\mathcal{S}(\alpha)}(x)=0$. $1_{(\Delta_{NSHS},\ \wedge)}$ is used to denote it.

4. Neutrosophic Superhypersoft Topological Spaces

Definition 4.1 Let NSHS(Δ, Λ) be the set of all neutrosophic superhypersoft sets of (Δ, Λ) over the universe Δ and $\bar{\tau}$ be a subfamily of NSHS(Δ, Λ). Then $\bar{\tau}$ is called a neutrosophic Superhypersofttopology on Δ if the following conditions are satisfied

$0_{(\Delta_{NSHS},\ \wedge)}$ and $1_{(\Delta_{NSHS},\ \wedge)}$ belongs to $\bar{\tau}$

The union of any number of neutrosophic Superhypersoftsets in $\bar{\tau}$ belongs to $\bar{\tau}$

The intersection of any two neutrosophic Superhypersoftsets in $\bar{\tau}$ belongs to $\bar{\tau}$

Then $(\Delta,\ \bar{\tau},\Lambda)$ is called a neutrosophic Superhypersoft topological space over Δ. Every member of $\bar{\tau}$ is called a neutrosophic Superhypersoft open set in Δ.

Definition 4.2

Let NSHS(Δ, Λ) be the family of all neutrosophic hypersoft sets over the universe set Δ. Then

(1) If $\bar{\tau}=\{0_{(\Delta_{NSHS},\ \wedge)}1_{(\Delta_{NSHS},\ \wedge)}\}$, then $\bar{\tau}$ is said to be a neutrosophic superhypersoft indiscrete topology and $(\Delta,\ \bar{\tau},\Lambda)$ is said to be a neutrosophic Superhypersoftindiscrete topological space over Δ.

(2) If $\bar{\tau}=NSHS(\Delta,\Lambda)$ then $\bar{\tau}$ is said to be a neutrosophic superhypersoft discrete topology and $(\Delta,\ \bar{\tau},\Lambda)$ is said to be a neutrosophic superhypersoft discrete topological space over Δ.

Proposition 4.1 Let $(\Delta,\ \bar{\tau_1},\ \Lambda)$ and $(\Delta,\ \bar{\tau_2},\ \Lambda)$ be two neutrosophic Superhypersofttopological spaces over Δ. Then $(\Delta,\bar{\tau_1}\cap\ \bar{\tau_2},\Lambda)$ is a neutrosophic Superhypersofttopological space over the universe Δ.

Proof:

1. Since $0_{(\Delta_{NSHS},\ \wedge)},1_{(\Delta_{NSHS},\ \wedge)}\in\ \bar{\tau_1}$ and $0_{(\Delta_{NSHS},\ \wedge)},1_{(\Delta_{NSHS},\ \wedge)}\in\ \bar{\tau_2}$, then $0_{(\Delta_{NSHS},\ \wedge)},1_{(\Delta_{NSHS},\ \wedge)}\in\ \bar{\tau_1}\cap\bar{\tau_2}$.
2. Suppose that$\{(\{(\mathcal{S}_i,\Gamma_i):i\in I\}$ be a family of neutrosophic Superhypersoft sets in $\bar{\tau_1}\cap\ \bar{\tau_2}$. Then $(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}$ and $(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_2}$ for all $\in I$, so $\cup_{i=I}\ (\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}$ and $\cup_{i=I}\ (\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_2}$. Thus $\cup_{i=I}\ (\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}\cap\bar{\tau_2}$.
3. Suppose that $\{(\mathcal{S}_i,\Gamma_i):i=\overline{1,k}\}$ be a family of a finite number of neutrosophic hypersoft sets in $\bar{\tau_1}\cap\ \bar{\tau_2}$. Then Then $(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}$ and $(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_2}$ for all $i=\overline{1,k}$, so $\bigcap_{i=1}^{k}(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}$ and $\bigcap_{i=1}^{k}(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_2}$. Thus $\bigcap_{i=1}^{k}(\mathcal{S}_i,\Gamma_i)\in\ \bar{\tau_1}\cap\bar{\tau_2}$.

Remark 4.1 The unionof two neutrosophic fuzzy topologies over Δ may not be a neutrosophic Superhypersofttopology on Δ.

Example 4.1: We take example 3.1 into consideration

$$\overline{\tau_1}=\{0_{(\Delta_{NSHS},\ \wedge)},1_{(\Delta_{NSHS},\ \wedge)},(\mathcal{S}_1,\Gamma_1),(\mathcal{S}_2,\Gamma_2)\}$$
$$\overline{\tau_2}=\{0_{(\Delta_{NSHS},\ \wedge)},1_{(\Delta_{NSHS},\ \wedge)},(\mathcal{S}_3,\Gamma_3),(\mathcal{S}_4,\Gamma_4)\}$$

Be two neutrosophic superhypersoft topologies over Δ. Here, the neutrosophic superhypersoft sets $(\mathcal{S}_1,\Gamma_1),(\mathcal{S}_2,\Gamma_2),(\mathcal{S}_3,\Gamma_3)$ and $(\mathcal{S}_4,\Gamma_4)$ over Δ are defined as follows:

$$(\mathcal{S}_1,\Gamma_1)=\left\{\begin{array}{l}\langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.5\},\{0.4\}},\frac{x_3}{\{0.3\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.5\},\{0.4\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.6\},\{0.4\},\{0.5\}},\frac{x_2}{\{0.5\},\{0.7\},\{0.4\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.6\},\{0.2\},\{0.8\}},\frac{x_2}{\{0.3\},\{0.4\},\{0.5\}},\frac{x_3}{\{0.1\},\{0.3\},\{0.7\}}\right\}\rangle\end{array}\right\}$$

$$(\mathcal{S}_1,\Gamma_1)=\left\{\begin{array}{l}\langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.8\},\{0.6\},\{0.1\}},\frac{x_2}{\{0.7\},\{0.6\},\{0.3\}},\frac{x_3}{\{0.4\},\{0.8\},\{0.1\}}\right\}\rangle\\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.3\}},\frac{x_2}{\{0.7\},\{0.8\},\{0.1\}},\frac{x_3}{\{0.7\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.9\},\{0.6\},\{0.2\}},\frac{x_2}{\{0.8\},\{0.8\},\{0.3\}},\frac{x_3}{\{0.7\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.9\},\{0.8\},\{0.5\}},\frac{x_2}{\{0.6\},\{0.7\},\{0.3\}},\frac{x_3}{\{0.4\},\{0.4\},\{0.4\}}\right\}\rangle\end{array}\right\}$$

$$(\mathcal{S}_1,\Gamma_1)=\left\{\begin{array}{l}\langle(\{a_2\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.6\},\{0.4\}},\frac{x_2}{\{0.6\},\{0.6\},\{0.2\}},\frac{x_3}{\{0.4\},\{0.5\},\{0.2\}}\right\}\rangle\\ \langle(\{a_2\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.2\},\{0.5\}},\frac{x_2}{\{0.6\},\{0.3\},\{0.1\}},\frac{x_3}{\{0.3\},\{0.7\},\{0.8\}}\right\}\rangle\\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.3\},\{0.6\},\{0.2\}},\frac{x_2}{\{0.6\},\{0.2\},\{0.5\}},\frac{x_3}{\{0.7\},\{0.3\},\{0.2\}}\right\}\rangle\\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.5\},\{0.3\},\{0.4\}},\frac{x_2}{\{0.5\},\{0.1\},\{0.3\}},\frac{x_3}{\{0.4\},\{0.5\},\{0.5\}}\right\}\rangle\end{array}\right\}$$

$$(\mathcal{S}_1,\Gamma_1)=\left\{\begin{array}{l}\langle(\{a_2\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.7\},\{0.2\}},\frac{x_2}{\{0.8\},\{0.7\},\{0.2\}},\frac{x_3}{\{0.7\},\{0.6\},\{0.2\}}\right\}\rangle\\ \langle(\{a_2\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.2\}},\frac{x_2}{\{0.7\},\{0.6\},\{0.1\}},\frac{x_3}{\{0.5\},\{0.9\},\{0.4\}}\right\}\rangle\\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.6\},\{0.7\},\{0.1\}},\frac{x_2}{\{0.8\},\{0.5\},\{0.3\}},\frac{x_3}{\{0.8\},\{0.6\},\{0.1\}}\right\}\rangle\\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.8\},\{0.5\},\{0.3\}},\frac{x_2}{\{0.4\},\{0.2\},\{0.2\}},\frac{x_3}{\{0.6\},\{0.6\},\{025\}}\right\}\rangle\end{array}\right\}$$

It is clear that $\bar{\tau_1} \cap \bar{\tau_2}$ is a neutrosophic Superhypersofttopology. But $(\mathcal{S}_1,\Gamma_1) \cup (\mathcal{S}_4,\Gamma_4) \notin \bar{\tau_1} \cup \bar{\tau_2}$ is not a neutrosophic superhypersoft topology.

Proposition 4.2 Let $(\Delta, \bar{\tau}, \Lambda)$ be a neutrosophic Superhypersofttopological space over Δ and $\bar{\tau} = \{(\mathcal{S}_i,\Gamma_i): (\mathcal{S}_i,\Gamma_i) \in NSHS(\Delta,\Lambda)\} = \left\{\left[\alpha, (\mathcal{S}_i(\alpha))\right]_{\alpha\epsilon\Lambda}: (\mathcal{S}_i,\Gamma_i) \in NFHS(\Delta,\Lambda)\right\}$

Where$\$_i(\alpha) = \left\{\langle x, T_{\mathcal{S}_i(\alpha)}(x),\ I_{\mathcal{S}_i(\alpha)}(x),\ F_{\mathcal{S}_i(\alpha)}(x)\rangle: x\epsilon\Delta\right\}$

Then

$$\tau_1 = \left\{\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}$$
$$\tau_2 = \left\{\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}$$
$$\tau_3 = \left\{\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}$$

Define fuzzy soft topologies on Δ.

Proof:

1. $0_{(\Delta_{NSHS},\ \Lambda)}, 1_{(\Delta_{NSHS},\ \Lambda)} \epsilon\ \bar{\tau_1}$
2. Suppose that $\{(\mathcal{S}_i,\Gamma_i) : i \epsilon I\}$ be a family of neutrosophic Superhypersoft sets in $\bar{\tau}$. Then $\left\{\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Gamma}\right\}_{i\,\epsilon\,I}$ is a family of neutrosophic fuzzy soft set sets in τ_1, $\left\{\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Gamma}\right\}_{i\,\epsilon\,I}$ is a family of neutrosophic fuzzy soft set sets in τ_2,

$\left\{\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Gamma}\right\}_{i\,\epsilon\,I}$ is a family of neutrosophic fuzzy soft set sets in τ_3. Since $\bar{\tau}$ is a neutrosophic Superhypersofttopology, then $\cup_{i=I}\ (\mathcal{S}_i, \Gamma_i)\ \epsilon\ \bar{\tau}$. That is,

$$\cup_{i=I}\ (\mathcal{S}_i, \Gamma_i) = \left\{\langle \max\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \max\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \min\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \rangle\right\}_{i\,\epsilon\,I} \epsilon\ \bar{\tau}$$

Therefore

$$\left\{max\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_1$$

$$\left\{max\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_2$$

$\left\{min\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_3$.

3. Suppose that $\{(\mathcal{S}_i, \Gamma_i) : i = \overline{1,k}\}$ be a family of a finite number of neutrosophic hypersoft sets in $\bar{\tau}$. Then $\left\{\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i=\overline{1,k}}$ is a family of neutrosophic fuzzy soft sets in τ_1, $\left\{\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i=\overline{1,k}}$ is a family of neutrosophic fuzzy soft sets in τ_2, $\left\{\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i=\overline{1,k}}$ is a family of neutrosophic fuzzy soft sets in τ_3 .

Since $\bar{\tau}$ is a neutrosophic Superhypersofttopology, then $\cap_{i=1}^{k}(\mathcal{S}_i, \Gamma_i)\ \epsilon\ \bar{\tau}$. That is

$$\cap_{i=1}^{k}\ (\mathcal{S}_i, \Gamma_i) = \left\{\langle \min\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \min\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \max\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}, \rangle\right\}_{i\,\epsilon\,I} \epsilon\ \bar{\tau}$$

Therefore

$$\left\{min\left[T_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_1$$

$$\left\{min\left[I_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_2$$

$\left\{max\left[F_{\mathcal{S}_i(\alpha)}(x)\right]_{\alpha\,\epsilon\,\Lambda}\right\}_{i\,\epsilon\,I} \epsilon\ \tau_3$.

This completes the proof.

Definition 4.3 Assume that$(\mathcal{S}, \Gamma)$is a neutrosophic Superhypersoftset over Δ and that $(\Delta, \bar{\tau}, \Lambda)$is a neutrosophic Superhypersoft topological space over Δ. The union of all neutrosophic Superhypersoft open subsets of $(\mathcal{S}, \Gamma)$ is the neutrosophic Superhypersoft interior, which is therefore represented by the notation $int_{NSHS}(\mathcal{S}, \Gamma)$. It is evident that the largest neutrosophic Superhypersoft open set included in $(\mathcal{S}, \Gamma)$is $int_{NSHS}(\mathcal{S}, \Gamma)$.

Example 4.2. We consider the attributes in Example 3.1

Suppose that any $(\mathcal{S}_3, \Gamma_3) \in NSHS(\Delta, \Lambda)$ is defined as follows:

$$(\mathcal{S}, \Gamma) = \left\{\begin{array}{l} \langle(\{a_1\},\{c_1\},\{d_1\})\left\{\frac{x_1}{\{0.9\},\{0.7\},\{0.1\}}, \frac{x_2}{\{0.8\},\{0.7\},\{0.2\}}, \frac{x_3}{\{0.6\},\{0.8\},\{0.1\}}\right\}\rangle \\ \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.9\},\{0.6\},\{0.2\}}, \frac{x_2}{\{0.8\},\{0.9\},\{0.1\}}, \frac{x_3}{\{0.8\},\{0.7\},\{0.1\}}\right\}\rangle \\ \langle(\{a_1\},\{c_2\},\{d_1\})\left\{\frac{x_1}{\{0.9\},\{0.8\},\{0.1\}}, \frac{x_2}{\{0.9\},\{0.8\},\{0.2\}}, \frac{x_3}{\{0.8\},\{0.7\},\{0.2\}}\right\}\rangle \\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.9\},\{0.9\},\{0.3\}}, \frac{x_2}{\{0.7\},\{0.8\},\{0.1\}}, \frac{x_3}{\{0.6\},\{0.7\},\{0.3\}}\right\}\rangle \end{array}\right\}$$

Then

$$0_{(\Delta_{NSHS},\ \Lambda)}, (\mathcal{S}_1, \Gamma_1), (\mathcal{S}_2, \Gamma_2)\ \subseteq\ (\mathcal{S}_3, \Gamma_3)$$

Therefore

$int_{NSHS}(\mathcal{S}_3, \Gamma_3) = 0_{(\Delta_{NSHS},\ \Lambda)} \cup (\mathcal{S}_1, \Gamma_1)\ \cup\ (\mathcal{S}_2, \Gamma_2) = (\mathcal{S}_2, \Gamma_2)$.

Theorem 4.1 Let $(\Delta, \bar{\tau}, \Lambda)$ be a neutrosophic Superhypersofttopological space over Δ and $(\mathcal{S}_1, \Gamma_1), (\mathcal{S}_2, \Gamma_2) \in NSHS(\Delta, \Lambda)$. Then

1. $int_{NSHS}\left(0_{(\Delta_{NSHS},\ \Lambda)}\right) = 0_{(\Delta_{NSHS},\ \Lambda)}$ and $int_{NSHS}\left(1_{(\Delta_{NSHS},\ \Lambda)}\right) = 1_{(\Delta_{NSHS},\ \Lambda)}$
2. $int_{NSHS}\left(int_{NSHS}(\mathcal{S}_1, \Gamma_1)\right) = int_{NSHS}(\mathcal{S}_1, \Gamma_1)$

3. If $(\mathcal{S}_1, \Gamma_1) \subseteq (\mathcal{S}_2, \Gamma_2)$, then $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \subseteq int_{NSHS}(\mathcal{S}_2, \Gamma_2)$
4. $(\mathcal{S}_1, \Gamma_1)$ is a neutrosophic Superhypersoft open set iff $int_{NSHS}(\mathcal{S}_1, \Gamma_1) = (\mathcal{S}_1, \Gamma_1)$
5. $int_{NSHS}[(\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)]$ = then $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2)$.

Proof:

1. It is clear from the definition of interior
2. Let $\left(int_{NSHS}(\mathcal{S}_1, \Gamma_1)\right) = (\mathcal{S}_2, \Gamma_2)$
   Since $(\mathcal{S}_2, \Gamma_2)$ is a neutrosophic Superhypersoft open set $int_{NSHS}(\mathcal{S}_2, \Gamma_2) = (\mathcal{S}_2, \Gamma_2)$, so
   $int_{NSHS}\left(int_{NSHS}(\mathcal{S}_1, \Gamma_1)\right) = int_{NSHS}(\mathcal{S}_1, \Gamma_1)$
3. Let $(\mathcal{S}_1, \Gamma_1) \subseteq (\mathcal{S}_2, \Gamma_2)$. $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \subseteq (\mathcal{S}_1, \Gamma_1)$ and hence $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \subseteq (\mathcal{S}_2, \Gamma_2)$also $int_{NSHS}(\mathcal{S}_1, \Gamma_1)$ is the biggest neutrosophic Superhypersoft open set contained in $(\mathcal{S}_2, \Gamma_2)$ and so $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \subseteq int_{NSHS}(\mathcal{S}_2, \Gamma_2)$
4. Let $(\mathcal{S}_1, \Gamma_1)$ be a neutrosophic Superhypersoftopen set. Since $int_{NSHS}(\mathcal{S}_1, \Gamma_1)$ is the biggest neutrosophic Superhypersoftopen set contained in $(\mathcal{S}_1, \Gamma_1)$, $int_{NSHS}(\mathcal{S}_1, \Gamma_1) = (\mathcal{S}_1, \Gamma_1)$.
   Conversely, suppose that $int_{NSHS}(\mathcal{S}_1, \Gamma_1) = (\mathcal{S}_1, \Gamma_1)$. Since $int_{NSHS}(\mathcal{S}_1, \Gamma_1)$ is a neutrosophic Superhypersoft open set $(\mathcal{S}_1, \Gamma_1)$ is also a neutrosophic Superhypersoft open set.
5. Since $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \subseteq (\mathcal{S}_1, \Gamma_1)$ and $int_{NSHS}(\mathcal{S}_2, \Gamma_2) \subseteq (\mathcal{S}_2, \Gamma_2)$
   Hence $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2) \subseteq (\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)$

   Since the biggest neutrosophic Superhypersoftopen set contained in $(\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)$ is $int_{NFH}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2) \subseteq int_{NSHS}[(\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)]$.
   Conversely, $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2) \subseteq int_{NSHS}(\mathcal{S}_1, \Gamma_1)$
   and $int_{NSHS}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2) \subseteq int_{NSHS}(\mathcal{S}_2, \Gamma_2)$. Hence
   $int_{NSHS}[(\mathcal{S}_1, \Gamma_1) \cap (\mathcal{S}_2, \Gamma_2)] \subseteq int_{NSHS}(\mathcal{S}_1, \Gamma_1) \cap int_{NSHS}(\mathcal{S}_2, \Gamma_2)$.

Definition 4.4 Assume that $(\mathcal{S}, \Gamma)$is a neutrosophic Superhypersoftset over Δ and that Δ, τ, Λ is a neutrosophic Superhypersofttopological space over Δ. The intersection of all neutrosophic Superhypersoft closed supersets of $(\mathcal{S}, \Gamma)$is then defined as the neutrosophic Superhypersoft closure of $(\mathcal{S}, \Gamma)$, and it is denoted by the notation $cl_{NSHS}(\mathcal{S}, \Gamma)$. The smallest neutrosophic Superhypersoft closed set containing $(\mathcal{S}, \Gamma)$is obviously cl_{NSHS}.

Example 4.3 Suppose that any $(\mathcal{S}, \Gamma) \in NSHS(\Delta, \Lambda)$ be defined as follows

$$(\mathcal{S}, \Gamma) = \left\{ \begin{array}{l} \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.1\},\{0.2\},\{0.5\}}, \frac{x_2}{\{0.1\},\{0.2\},\{0.9\}}, \frac{x_3}{\{0.2\},\{0.3\},\{0.8\}}\right\}\rangle \\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.6\},\{0.9\}}, \frac{x_2}{\{0.1\},\{0.2\},\{0.8\}}, \frac{x_3}{\{0.3\},\{0.1\},\{0.6\}}\right\}\rangle \\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.1\},\{0.1\},\{0.8\}}, \frac{x_2}{\{0.1\},\{0.3\},\{0.8\}}, \frac{x_3}{\{0.1\},\{0.2\},\{0.8\}}\right\}\rangle \\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.4\},\{0.9\}}, \frac{x_2}{\{0.1\},\{0.5\},\{0.6\}}, \frac{x_3}{\{0.2\},\{0.2\},\{0.7\}}\right\}\rangle \end{array} \right\}$$

Obviously, $\left(0_{(\Delta_{NFH},\ \Lambda)}\right)^c, \left(1_{(\Delta_{NFH},\ \Lambda)}\right)^c, (\mathcal{S}_1, \Gamma_1)^c, (\mathcal{S}_2, \Gamma_2)^c$ are all neutrosophic Superhypersoft closed sets over $(\Delta, \bar{\tau}, \Lambda)$.

$$(\mathcal{S}_3, \Gamma_3)^c = \left\{ \begin{array}{l} \langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.4\},\{0.2\}}, \frac{x_2}{\{0.2\},\{0.4\},\{0.6\}}, \frac{x_3}{\{0.2\},\{0.5\},\{0.4\}}\right\}\rangle \\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.5\},\{0.8\},\{0.8\}}, \frac{x_2}{\{0.1\},\{0.7\},\{0.6\}}, \frac{x_3}{\{0.8\},\{0.3\},\{0.3\}}\right\}\rangle \\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.4\},\{0.3\}}, \frac{x_2}{\{0.5\},\{0.8\},\{0.6\}}, \frac{x_3}{\{0.2\},\{0.7\},\{0.7\}}\right\}\rangle \\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.4\},\{0.7\},\{0.5\}}, \frac{x_2}{\{0.3\},\{0.9\},\{0.5\}}, \frac{x_3}{\{0.5\},\{0.5\},\{0.4\}}\right\}\rangle \end{array} \right\}$$

$$(\mathcal{S}_4,\Gamma_4)^c=\left\{\begin{array}{l}\langle(\{a_1\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.3\},\{0.4\}},\frac{x_2}{\{0.2\},\{0.3\},\{0.8\}},\frac{x_3}{\{0.2\},\{0.4\},\{0.7\}}\right\}\rangle\\ \langle(\{a_1\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.2\},\{0.5\},\{0.8\}},\frac{x_2}{\{0.1\},\{0.4\},\{0.7\}},\frac{x_3}{\{0.4\},\{0.1\},\{0.5\}}\right\}\rangle\\ \langle(\{a_3\},\{c_1\},\{d_2\})\left\{\frac{x_1}{\{0.1\},\{0.3\},\{0.6\}},\frac{x_2}{\{0.3\},\{0.5\},\{0.8\}},\frac{x_3}{\{0.1\},\{0.4\},\{0.8\}}\right\}\rangle\\ \langle(\{a_3\},\{c_2\},\{d_2\})\left\{\frac{x_1}{\{0.3\},\{0.5\},\{0.8\}},\frac{x_2}{\{0.2\},\{0.8\},\{0.4\}},\frac{x_3}{\{0.2\},\{0.4\},\{0.6\}}\right\}\rangle\end{array}\right\}$$

$$\left(0_{(\Delta_{NFH},\ \Lambda)}\right)^c=1_{(\Delta_{NFH},\ \Lambda)}$$
$$\left(1_{(\Delta_{NFH},\ \Lambda)}\right)^c=0_{(\Delta_{NFH},\ \Lambda)}$$

Then $(\mathcal{S}_4,\Gamma_4)\subseteq\left(0_{(\Delta_{NFH},\ \Lambda)}\right)^c,(\mathcal{S}_1,\Gamma_1)^c,(\mathcal{S}_2,\Gamma_2)^c$
$cl_{NFH}(\mathcal{S},\Gamma)=\left(0_{(\Delta_{NFH},\ \Lambda)}\right)^c\cap(\mathcal{S}_1,\Gamma_1)^c\cap(\mathcal{S}_2,\Gamma_2)^c$
$=(\mathcal{S}_2,\Gamma_2)^c$

Theorem 4.2 Let $(\Delta,\ \bar{\tau},\Lambda)$ be a neutrosophic Superhypersoft topological space over Δ and $(\mathcal{S}_1,\Gamma_1),(\mathcal{S}_2,\Gamma_2)\in NFHS(\Delta,\Lambda)$. Then

1. $cl_{NFH}\left(0_{(\Delta_{NFH},\ \Lambda)}\right)=0_{(\Delta_{NFH},\ \Lambda)}$ and $cl_{NFH}\left(1_{(\Delta_{NFH},\ \Lambda)}\right)=1_{(\Delta_{NFH},\ \Lambda)}$
2. $cl_{NFH}\left(cl_{NFH}(\mathcal{S}_1,\Gamma_1)\right)=cl_{NFH}(\mathcal{S}_1,\Gamma_1)$
3. If $(\mathcal{S}_1,\Gamma_1)\subseteq(\mathcal{S}_2,\Gamma_2)$, then $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}(\mathcal{S}_2,\Gamma_2)$
4. $(\mathcal{S}_1,\Gamma_1)$ is a neutrosophic Superhypersoft open set iff $cl_{NFH}(\mathcal{S}_1,\Gamma_1)=(\mathcal{S}_1,\Gamma_1)$
5. $cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cap(\mathcal{S}_2,\Gamma_2)]$ = then $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\cap cl_{NFH}(\mathcal{S}_2,\Gamma_2)$.

Proof:

1. It is clear from the definition of closure
2. Let $\left(cl_{NFH}(\mathcal{S}_1,\Gamma_1)\right)=(\mathcal{S}_2,\Gamma_2)$

   Since $(\mathcal{S}_2,\Gamma_2)$ is a neutrosophic Superhypersoft closed set $cl_{NFH}(\mathcal{S}_2,\Gamma_2)=(\mathcal{S}_2,\Gamma_2)$, so $cl_{NFH}\left(cl_{NFH}(\mathcal{S}_1,\Gamma_1)\right)=cl_{NFH}(\mathcal{S}_1,\Gamma_1)$.
3. It is known that $(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}(\mathcal{S}_1,\Gamma_1)$and $cl_{NFH}(\mathcal{S}_2,\Gamma_2)\subseteq(\mathcal{S}_2,\Gamma_2)$ and so $(\mathcal{S}_1,\Gamma_1)\subseteq(\mathcal{S}_2,\Gamma_2)\subseteq cl_{NFH}(\mathcal{S}_2,\Gamma_2)$. Since $cl_{NFH}(\mathcal{S}_1,\Gamma_1)$is the smallest neutrosophic Superhypersoft closed set containing $(\mathcal{S}_1,\Gamma_1)$ then $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}(\mathcal{S}_2,\Gamma_2)$.
4. Let $(\mathcal{S}_1,\Gamma_1)$ be a neutrosophic Superhypersoft closed set. We have $(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}(\mathcal{S}_1,\Gamma_1)$. Since $cl_{NFH}(\mathcal{S}_1,\Gamma_1)$ is the smallest neutrosophic Superhypersoft closed set over Δ which contains $(\mathcal{S}_1,\Gamma_1)$, then $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\subset(\mathcal{S}_1,\Gamma_1)$. Hence $cl_{NFH}(\mathcal{S}_1,\Gamma_1)=(\mathcal{S}_1,\Gamma_1)$.

   Conversely, suppose that $cl_{NFH}(\mathcal{S}_1,\Gamma_1)=(\mathcal{S}_1,\Gamma_1)$. Since $cl_{NFH}(\mathcal{S}_1,\Gamma_1)$ is a neutrosophic Superhypersoft closed set, then $(\mathcal{S}_1,\Gamma_1)$ is closed.
5. Since $(\mathcal{S}_1,\Gamma_1)\subseteq(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)$ and $(\mathcal{S}_2,\Gamma_2)\subseteq(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)$, then $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)]$ and $cl_{NFH}(\mathcal{S}_2,\Gamma_2)\subseteq cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)]$ and so $cl_{NFH}(\mathcal{S}_1,\Gamma_1)\cup cl_{NFH}(\mathcal{S}_2,\Gamma_2)\subseteq cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cap(\mathcal{S}_2,\Gamma_2)]$.
   Conversely, since $(\mathcal{S}_1,\Gamma_1)\subseteq cl_{NFH}(\mathcal{S}_1,\Gamma_1)$ and $(\mathcal{S}_2,\Gamma_2)\subseteq cl_{NFH}(\mathcal{S}_2,\Gamma_2)$, then
   $$(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)\subseteq cl_{NFH}(\mathcal{S}_1,\Gamma_1)\cup cl_{NFH}(\mathcal{S}_2,\Gamma_2)$$
   Besides$cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)]$ is the smallest neutrosophic Superhypersoft closed set that contains $(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)$.
   Therefore $cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)]\subseteq cl_{NFH}(\mathcal{S}_1,\Gamma_1)\cup cl_{NFH}(\mathcal{S}_2,\Gamma_2)$
   Thus $cl_{NFH}[(\mathcal{S}_1,\Gamma_1)\cup(\mathcal{S}_2,\Gamma_2)]=cl_{NFH}(\mathcal{S}_1,\Gamma_1)\cup cl_{NFH}(\mathcal{S}_2,\Gamma_2)$.

Theorem 4.3 Let $(\Delta,\ \bar{\tau},\Lambda)$ be a neutrosophic Superhypersoft topological space over Δ and $(\mathcal{S}_1,\Gamma_1),(\mathcal{S}_2,\Gamma_2)\in NFHS(\Delta,\Lambda)$. Then

1. $\left(cl_{NFH}(\mathcal{S},\Gamma)\right)^c=int_{NFH}((\mathcal{S},\Gamma)^c)$
2. $\left(int_{NFH}(\mathcal{S},\Gamma)\right)^c=cl_{NFH}((\mathcal{S},\Gamma)^c)$

Proof:

1. $cl_{NFH}(\mathcal{S},\Gamma) = \cap\{(\mathcal{S},\Gamma) \in \bar{\tau}^c : (\mathcal{S}_2,\Gamma_2) \subseteq (\mathcal{S},\Gamma)\}$

$$\left(cl_{NFH}(\mathcal{S},\Gamma)\right)^c = (\cap\{(\mathcal{S},\Gamma) \in \bar{\tau}^c : (\mathcal{S}_2,\Gamma_2) \subseteq (\mathcal{S},\Gamma)\})^c$$
$$= \cup\{(\mathcal{S},\Gamma)^c \in \bar{\tau} : (\mathcal{S},\Gamma)^c \subseteq (\mathcal{S}_2,\Gamma_2)^c\}$$
$$= int_{NFH}((\mathcal{S},\Gamma)^c)$$

2. $int_{NFH}(\mathcal{S},\Gamma) = \cup\{(\mathcal{S},\Gamma) \in \bar{\tau} : (\mathcal{S},\Gamma) \subseteq (\mathcal{S}_2,\Gamma_2)\}$

$$\left(int_{NFH}(\mathcal{S},\Gamma)\right)^c = (\cup\{(\mathcal{S},\Gamma) \in \bar{\tau} : (\mathcal{S},\Gamma) \subseteq (\mathcal{S}_2,\Gamma_2)\})^c$$
$$= \cap\{(\mathcal{S},\Gamma)^c \in \bar{\tau}^c : (\mathcal{S}_2,\Gamma_2)^c \subseteq (\mathcal{S},\Gamma)^c\}$$
$$= cl_{NFH}((\mathcal{S},\Gamma)^c).$$

Definition 4.5 Let $(\Delta, \bar{\tau}, \Lambda)$ be a neutrosophic Superhypersofttopological space over Δ and $\bar{\mathfrak{B}} \subseteq \bar{\tau}$. $\bar{\mathfrak{B}}$ is called a neutrosophic Superhypersoft basis for the neutrosophic Superhypersoft topology $\bar{\tau}$ if every element of $\bar{\tau}$ can be written as the neutrosophic Superhypersoft union of elements of $\bar{\mathfrak{B}}$.

Proposition 4.3 Let $(\Delta, \bar{\tau}, \Lambda)$ be a neutrosophic Superhypersoft topological space over Δ and $\bar{\mathfrak{B}}$ be a neutrosophic Superhypersoft basis of $\bar{\tau}$. Then $\bar{\tau}$ equals the collection of neutrosophic fuzzy hypersoft union of elements of $\bar{\mathfrak{B}}$.

Proof: It is clear from the definition of a neutrosophic Superhypersoft basis.

Theorem 4.4 Let $(\Delta, \bar{\tau}, \Lambda)$ be a neutrosophic Superhypersoft topological space over Δ and $(\$,\Gamma)$neutrosophic Superhypersoft set over Δ. Then the collection
$\bar{\tau}_{(\mathcal{S},\Gamma)} = \{(\mathcal{S},\Gamma) \cap (\alpha,\beta) : (\alpha,\beta) \in \bar{\tau}\}$ is a neutrosophic Superhypersoft topology on the neutrosophic Superhypersoft subset $(\$,\Gamma)$ relative parameter set Γ.

Proof:

1. $0_{(\Delta_{NFH},\ \Lambda)}, 1_{(\Delta_{NFH},\ \Lambda)} \in \bar{\tau}_{(\$,\Gamma)}$
2. Consider $(\mathcal{S}_1,\Gamma_1), (\mathcal{S}_2,\Gamma_2) \in \bar{\tau}_{(\$,\Gamma)}$.Then there exists $(\alpha_i,\beta_i) \in \bar{\tau}$ for each *i=1,2,..* such that $(\mathcal{S}_i,\Gamma_i) = (\mathcal{S},\Gamma) \cap (\alpha_i,\beta_i)$. Then

$$(\mathcal{S}_1,\Gamma_1) \cap (\mathcal{S}_2,\Gamma_2) = [(\mathcal{S},\Gamma) \cap (\alpha_1,\beta_1)] \cap [(\mathcal{S},\Gamma) \cap (\alpha_2,\beta_2)]$$
$$= (\mathcal{S},\Gamma) \cap [(\alpha_1,\beta_1) \cap (\alpha_2,\beta_2)]$$

Since $(\alpha_1,\beta_1) \cap (\alpha_2,\beta_2) \in \bar{\tau}$, we have $(\mathcal{S}_1,\Gamma_1), (\mathcal{S}_2,\Gamma_2) \in \bar{\tau}_{(\$,\Gamma)}$.

3. Let $\{(\alpha,\beta)_j : j \in J\}$ be a subfamily of $\bar{\tau}_{(\mathcal{S},\Gamma)}$. Then, for each $j \in J$ there is a neutrosophic soft set. $(\alpha_2,\beta_2)_j$ of $\bar{\tau}$ such that $(\alpha_2,\beta_2)_j = (\mathcal{S},\Gamma) \cap (\alpha_2,\beta_2)_j$. Then we have,

$$\cup_{j\in k}(\alpha,\beta)_j = \cup_{j\in J}\left[(\mathcal{S},\Gamma) \cap (\alpha_2,\beta_2)_j\right]$$
$$= (\mathcal{S},\Gamma) \cap \left[\cup_{j\in J}(\alpha_2,\beta_2)_j\right]$$

Since $\cup_{j\in J}(\alpha_2,\beta_2)_j \in \bar{\tau}$, we have $\cup_{j\in J}(\alpha_2,\beta_2)_j \in \bar{\tau}_{(\mathcal{S},\Gamma)}$.

5. Conclusion

The notion of neutrosophic Superhypersoft topology and its basic characteristics were initially discussed in this paper, along with a few examples. The present paper may serve as a foundation for further research on neutrosophic Superhypersoft topology. Specifically, by employing neutrosophic Superhypersoft functions, it is possible to investigate neutrosophic Superhypersoft continuity, separation axioms, compact spaces, and connected spaces. Furthermore, since information systems and hypersoft sets have compact ties, the conclusions drawn from the neutrosophic Superhypersoft topological spaces can be utilized to reinforce these kinds of connections.

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