Fact Type Results in Half Domination in Graphs

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Abstract **- Let G = (V, E) be a finite graph with n vertices and m edges. A subset D⊆ V of vertices in a graph G is called half dominating set if at least half of the vertices of V(G) are either in D or adjacent to vertices of D. (i.e) $|N[D]| \geq \left\lceil \left|\frac{V(G)}{2}\right| \right\rceil$. In this paper we deals with fact type results in half domination number of G.**

Keywords: **Half Dominating Set, Half Domination Number**

AMS Subject Classification (2010): 05C69

I. INTRODUCTION

By a graph we mean a finite undirected graph without loops or multiple edges. Let G= (V, E) be a finite graph and v be a vertex in V. The closed neighborhood of v is defined by N[v] = N (v) ∪ $\{v\}$ .The closed neighbourhood of a set of vertices D is denoted as N [D] = $\cup_{v\in D} N[v]$. A set D subset of V(G) is said to be a half dominating set if atleast half of the vertices of V(G) are either in D or adjacent to vertices of D. (i.e) $|N[D]| \geq \left\lceil \frac{|V[G]|}{2} \right\rceil$; In other hand $|N[D]| \geq \left\lceil \frac{n}{2} \right\rceil$.

II. FACTS

Result 1: $3\left\lceil \frac{n}{6} \right\rceil \geq \left\lceil \frac{n}{2} \right\rceil$

Proof: Let n ≡ 0 $(mod\ 6)$. $(i.e)$ n = 6r.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r}{6} \right\rceil = 3(r) = 3\left(\frac{n}{6}\right) = \frac{n}{2}$

$(i.e)$ $3\left\lceil \frac{n}{6} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$

Let n ≡ 1 $(mod\ 6)$ $(i.e)$ n=6r + 1.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r+1}{6} \right\rceil$

$= 3(r+1) = 3\left(\frac{n-1}{6} + 1\right)$

$= 3\left(\frac{n+5}{6}\right) = \frac{n+5}{2} > \left\lceil \frac{n}{2} \right\rceil$

Let n ≡ 2 $(mod\ 6)$ $(i.e)$ $n = 6r + 2$.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r+2}{6} \right\rceil = 3(r+1)$

$= 3\left(\frac{n-2}{6} + 1\right) = 3\left(\frac{n+4}{6}\right)$

$= \frac{n+4}{2} > \frac{n}{2}$

Let n ≡ 3 $(mod\ 6)$ $(i.e)$ $n = 6r + 3$.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r+3}{6} \right\rceil = 3(r+1)$

$= 3\left(\frac{n-3}{6} + 1\right) = 3\left(\frac{n+3}{6}\right)$

$= \frac{n+3}{2} > \frac{n}{2}$

Let n ≡ 4 $(mod\ 6)$ $(i.e)$ $n = 6r + 4$.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r+4}{6} \right\rceil = 3(r+1) = 3\left(\frac{n-4}{6} + 1\right)$

$= 3\left(\frac{n+2}{6}\right) = \frac{n+2}{2} > \frac{n}{2}$

Let n ≡ 5 $(mod\ 6)$ $(i.e)$ $n = 6r + 5$.

Then $3\left\lceil \frac{n}{6} \right\rceil = 3\left\lceil \frac{6r+5}{6} \right\rceil = 3(r+1) = 3\left(\frac{n-5}{6} + 1\right)$

$= 3\left(\frac{n+1}{6}\right) = \frac{n+1}{2} > \frac{n}{2}$

In all cases $3\left\lceil\frac{n}{6}\right\rceil \geq \left\lceil\frac{n}{2}\right\rceil$

Result 2: $2\left\lceil\frac{n}{4}\right\rceil \geq \left\lceil\frac{n}{2}\right\rceil$.

Proof: Let n = 0 (mod 4) $(i.e)$ n = 4r.

Then $2\left\lceil\frac{n}{4}\right\rceil = 2\left\lceil\frac{4r}{4}\right\rceil = 2(r) = 2 \times \frac{n}{4} = \frac{n}{2}$

Let n ≡ 1 (mod 4) $(i.e)$ n = 4r + 1.

Then, $2\left\lceil\frac{n}{4}\right\rceil = 2\left\lceil\frac{4r+1}{4}\right\rceil = 2(r+1)$

$= 2(\frac{n-1}{4} + 1) = 2(\frac{n+3}{4}) = \frac{n+3}{2} > \frac{n}{2}$

Let n ≡ 2 (mod 4) $(i.e)$ n = 4r + 2.

Then, $2\left\lceil\frac{n}{4}\right\rceil = 2\left\lceil\frac{4r+2}{4}\right\rceil = 2(r+1)$

$= 2(\frac{n-2}{4} + 1) = 2(\frac{n+2}{4}) = \frac{n+2}{2} > \frac{n}{2}$

Let n ≡ 3 (mod 4) $(i.e)$ n = 4r + 3.

Then, $2\left\lceil\frac{n}{4}\right\rceil = 2\left\lceil\frac{4r+3}{4}\right\rceil = 2(r+1)$

$= 2(\frac{n-3}{4} + 1) = 2(\frac{n+1}{4}) = \frac{n+1}{2} > \frac{n}{2}$

In all cases $2\left\lceil\frac{n}{4}\right\rceil \geq \left\lceil\frac{n}{2}\right\rceil$

Result 3: $\left\lceil\frac{n}{2(r+1)}\right\rceil (r+1) \geq \left\lceil\frac{n}{2}\right\rceil$.

Proof: Let n = 2r. Then $\frac{2r}{2(r+1)}(r+1) = r = \frac{n}{2}$.

Let n = 2r + 1. Then $\frac{2r+1}{2(r+1)}(r+1) = r + \frac{1}{2}$

$= \frac{n-1}{2} + \frac{1}{2} = \frac{n}{2}$

Therefore, $\left\lceil\frac{n}{2(r+1)}\right\rceil (r+1) \geq \left\lceil\frac{n}{2}\right\rceil$

Result 4: When n is even, then $\left\lceil\frac{n-1}{2}\right\rceil = \left\lceil\frac{n}{2}\right\rceil$ and When n is odd, then $\left\lceil\frac{n-1}{2}\right\rceil < \left\lceil\frac{n}{2}\right\rceil$.

Result 5:

$$n - \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil = \begin{cases} \frac{n}{4} & \text{if } n \equiv 0 \ (mod\ 4) \\ \frac{n-5}{2} & \text{if } n \equiv 1 \ (mod\ 4) \\ \frac{n-2}{2} & \text{if } n \equiv 2 \ (mod\ 4) \\ \frac{n-3}{2} & \text{if } n \equiv 3 \ (mod\ 4) \end{cases}$$

Proof: Let n ≡ 0 $(mod\ 4)$ $(i.e)$ n = 4r ⟹ r = $\frac{n}{4}$.

$n - \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil = 4r - 2r - r = r = \frac{n}{4}$

Let n ≡ 1 $(mod\ 4)$ $(i.e)$ n = 4r + 1 ⟹ r = $\frac{n-1}{4}$

$n - \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil = 4r + 1 - (\frac{4r+1}{2}) - (\frac{4r+1}{4})$

= 4r+1 – (2r+1)-(r+1)

= 4r+1-2r-1-r-1

= r – 1

$= \frac{n-1}{4} - 1$

$= \frac{n-5}{4}$.

Let n ≡ 2 $(mod\ 4)$ $(i.e)$ n = 4r + 2 ⟹ r = $\frac{n-2}{4}$

$n - \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil = 4r + 2 - (\frac{4r+2}{2}) - (\frac{4r+2}{4})$

= 4r+2 – (2r+1)-(r+1)

= 4r+2-2r-1-r-1

$= r = \frac{n-2}{4}$

Let n ≡ 3 $(mod\ 4)$ $(i.e)$ n = 4r + 3 ⟹ r = $\frac{n-3}{4}$

$n - \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil = 4r + 3 - (\frac{4r+3}{2}) - (\frac{4r+3}{4})$

= 4r+3 – (2r+2)-(r+1)

= 4r+3-2r-2-r-1

= r

$= \frac{n-3}{4}$

Hence the result.

Result 6: $\left\lceil\frac{3n}{4}\right\rceil - K + \left\lceil\frac{n}{4}\right\rceil$ = n - k or n + 1 - K

if n ≡ 0 (mod 4) or not.

Proof: Case (i): Let n ≡ 0 $(mod\ 4)$ $(i.e)$ n ≡ $4r$.

Then $\left\lceil\frac{3n}{4}\right\rceil - K + \left\lceil\frac{n}{4}\right\rceil = \frac{4r\ x\ 3}{4} - K + \frac{4r}{4}$

= 3r – K + r

= 4r – k = n – K

Case (ii): Let n ≡ 1 $(mod\ 4)$ $(i.e)$ n = $4r + 1$.

Then $\left\lceil\frac{3n}{4}\right\rceil - K + \left\lceil\frac{n}{4}\right\rceil = \frac{3(4r+1)}{4} - K + \frac{4r+1}{4}$

$= \frac{12r+3}{4} - K + r + 1$

= 3r+1 - K + r + 1

= 4r+2 – K

$= 4(\frac{n-1}{4})$+2 - K

= n - K+1

Let n ≡ 2 $(mod\ 4)$ $(i.e)$ n = $4r + 2$.

Then $\left\lceil\frac{3n}{4}\right\rceil - K + \left\lceil\frac{n}{4}\right\rceil = \frac{3(4r+2)}{4} - K + \frac{4r+2}{4}$

= 3r+2$-K + r + 1$

= 4r+3 - K

$= 4(\frac{n-2}{4})$ - K+3

= n - K+1

Let n ≡ 3 $(mod\ 4)$ $(i.e)$ n = $4r + 3$.

Then $\left\lceil\frac{3n}{4}\right\rceil - K + \left\lceil\frac{n}{4}\right\rceil = \frac{3(4r+3)}{4} - K + \frac{4r+3}{4}$

= 3r+3$-K + r + 1$

= 4r+4 - K

$= 4(\frac{n-3}{4})$-K+4

= n - K+1

Hence the result.

Result 7:

i) n - k+2$\left\lceil\frac{k}{2}\right\rceil$ = n if n ≡ 0 $(mod\ 4)$.

ii) n+1 - K+2$\left\lceil\frac{k}{2}\right\rceil$ ≥ n if n ≡ 1,2,3 $(mod\ 4)$.

Proof: Case (i):

Let n = 4r $(i.e)$ r = $\frac{n}{4} = \frac{k}{2}$.

Then n – K + 2 $\left\lceil\frac{k}{2}\right\rceil$ = n - $\frac{n}{2} + \frac{n}{2}$ = n

Case (ii):

Let n = 4r + 1 $(i.e)$ r = $\frac{n-1}{4} = \frac{k}{2}$ ⟹ K = $\frac{n-1}{2}$

Then,

n+1 - K+2$\left\lceil\frac{k}{2}\right\rceil$ = n +1 - $\frac{n+1}{2} + \frac{n-1}{2}$ = n + 1 > n

Case (iii):

Let n = 4r + 2 $(i.e)$ r = $\frac{n-2}{4} = \frac{k}{2}$.

Then,

n+1 - K+2$\left\lceil\frac{k}{2}\right\rceil$ = n +1 - $\frac{n+2}{2} + \frac{n-2}{2}$ = n + 1 > n

Case (iv):

Let n = 4r + 3 $(i.e)$ r = $\frac{n-3}{4} = \frac{k}{2}$.

Then,

n+1 - K+2$\left\lceil\frac{k}{2}\right\rceil$ = n +1 - $\frac{n+3}{2} + \frac{n-3}{2}$ = n + 1 > n

Hence the result.

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