Plane Gravitational Waves with Macro and Micro Matter Fields in Bimetric Relativity

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Abstract: - In this paper, we will study Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational waves either for macro field or for micro matter field or for coupled macro and micro matter represented by perfect fluid and scalar meson matter field respectively do not exist in bimetric theory of gravitation formulated by Rosen[1].Only a vacuum model can be constructed.

Keywords: - Plane gravitational waves, Macro and micro matter field, Bimetric Relativity

AMS Code-83C05 (General relativity)

I Introduction:

The bimetric theory of gravitation was proposed by Rosen[1][2](1940,1973) to modify the Einstein's general theory of relativity,by assuming two metric tensors.

In this theory he has proposed a new formulation of the general relativity by introducing a background

Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric γ_{ij} the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system x^i are

$$ds^2 = g_{ij}dx^i dx^j$$

(1.1)

And $$d\sigma^2 = \gamma_{ij}dx^i dx^j$$

(1.2)

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable.

One can regard it as describing the geometry that would exist if no matter were present.

H Takeno (1961) [3] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results.

A fairly general case of "plane" gravitational wave is represented by the metric

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2$$

(1.3)

both for weak field approximation and for exact solutions of Einstein field equations.

Reformulating Takeno's (1961) [3] definition of plane wave, we will use here, Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational waves by using the line elements

$$ds^2 = -A\left(dx^2 + dy^2\right) - C\left(dz^2 - dt^2\right)$$

(1.4)

Mohseni, Tucker and Wang [4] have studied the motion of spinning test particles in plane gravitational waves. S Kessari, D Singh et al [5], analyzed the motion of electrically neutral massive spinning test particle in the plane gravitational and electromagnetic wave background.

The theory of plane gravitational waves have been studied by many investigators,

H Takeno [6]; Pandey [7]; Lal and Shafiullah [8];Lu Hui qing [9];Bondi, H. et.al.[10],Torre,C.G.[11]; Hogan, P.A.[12];Deo and Ronghe[13],[14] and they obtained the solutions .

In this paper,we will study Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational wave with macro and micro matter field and will observe the result in the context of Bimetric theory of relativity.

II FIELD EQUATIONS IN BIMETRIC RELATIVITY:

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8\pi\kappa T_i^j$$

(2.1)

where

$$N_i^j = \tfrac{1}{2}\gamma^{\alpha\beta}\left[g^{hj} g_{hi}|_\alpha\right]|_\beta$$

(2.2)

$$N = N_\alpha^\alpha \qquad \kappa = \sqrt{\frac{g}{\gamma}}$$

(2.3)

And $g = |g_{ij}| \quad , \gamma = |\gamma_{ij}|$

(2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

And , T_i^j the energy momentum tensor for macro matter field represented by

$$T_i^j = T_i^{j^p} = (\rho + p) u_i u^j - p g_i^j$$

(2.5)

together with $g_i^j u_i u^j = 1$ where u_i is the four-velocity of the fluid having p and ρ as proper pressure and energy density respectively.

III Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational wave with Macro Matter Field:

For Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ plane gravitational waves , we have the line element as

$$ds^2 = -A\left(dx^2 + dy^2\right) - C\left(dz^2 - dt^2\right)$$

(3.1)

where $A = A(Z)$, C=C(Z) and

$$Z = \left(\frac{\sqrt{3}\ t}{x + y + z} \right)$$

Corresponding to the equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -\left(dx^2 + dy^2 + dz^2\right) + dt^2$$

(3.2).

Using equations (2.1) to (2.5) with (3.1) and (3.2),

We get the field equations as

$$D\left(\frac{\bar{c}^2}{c^2} - \frac{\bar{\bar{c}}}{c}\right) = -\frac{16}{3}\pi\kappa p$$

(3.3)

$$D\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = -\frac{16}{3}\pi\kappa p$$

(3.4)

$$D\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = \frac{16}{3}\pi\kappa\rho$$

(3.5)

where $D = \left[\frac{3t^2 - (x + y + z)^2}{(x + y + z)^4}\right]$

and

$$\bar{A} = \frac{\partial A}{\partial Z}, \quad \bar{\bar{A}} = \frac{\partial^2 A}{\partial Z^2},$$

$$\bar{C} = \frac{\partial C}{\partial Z}, \quad \bar{\bar{C}} = \frac{\partial^2 C}{\partial Z^2}$$

Using equation (3.3) to (3.5), we get

$$p + \rho = 0$$

(3.6)

This equation of state is known as false vacuum.In view of reality conditions $p > 0, \rho > 0$

Equation (3.6) immediately implies that $p = 0, \rho = 0$ that is macro matter field like perfect fluid does not exist in

$Z = \left(\frac{\sqrt{3}\ t}{x + y + z} \right)$ plane gravitational waves in Rosen's Bimetric theory of relativity.

Hence for vacuum case $p = 0 = \rho$, the field equation reduced to

$$\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) D = 0$$

i.e. $$\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = 0$$

(3.7)

Solving equations (3.7), we have

$$A = R_1 e^{S_1 Z}$$

(3.8)

On solving (3.3), we

$$C = R_2 e^{S_2 Z}$$

(3.9)

where R_1 , S_1 and R_2, S_2 are the constant of integration.

Thus substituting the value of (3.8) and (3.9) in (3.1), we get the vacuum line element as

$$ds^2 = -R_1 e^{S_1 Z}\left(dx^2 + dy^2\right) - R_2 e^{S_2 Z}\left(dz^2 - dt^2\right)$$

(3.10)

Thus, it is found that in plane gravitational wave $Z = \left(\frac{\sqrt{3}\ t}{x+y+z}\right)$, the macro matter field like perfect fluid does not survive in Bimetric theory of relativity and only vacuum model can be constructed

IV $Z = \left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational wave with Micro Matter Field:

In this section, we consider the region of the space-time filled with micro matter represented by a massive scalar field whose energy momentum tensor is given by

$$T_i^j = T_i^{j^s} = V_i V^j - \frac{1}{2} g_i^j \left(V_k V^k - m^2 V^2\right), \quad (4.1)$$

together with $\sigma = g_i^{\ j} V_{;i}^{j} + m^2 V$, where m is the mass parameter and σ is the source density of the meson field.Here afterwards the suffix(,) and semicolon (;) after a field variable represent ordinary and covariant differentiation with respect to t and $g_i^{\ j}$ resp.

Using equations (2.1) to (2.5) with (3.1) and (3.2) with energy momentum tensor (4.1) are obtained as

$$D\left(\frac{\bar{c}^2}{c^2} - \frac{\bar{\bar{c}}}{c}\right) = -\frac{8}{3}\pi\kappa\left(V_4^{\ 2} - m^2 V^2\right) \quad (4.2)$$

$$D\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = -\frac{8}{3}\pi\kappa\left(V_4^{\ 2} - m^2 V^2\right) \quad (4.3)$$

$$D\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = \frac{8}{3}\pi\kappa\left(V_4^{\ 2} + m^2 V^2\right) \quad (4.4)$$

Using (4.3) and (4.4), we get

$$\frac{16}{3}\pi\kappa V_4^{\ 2} = 0$$

ie $V_4 = 0$ ie V = constant. Thus for the space-time(3.1) the micro matter field with or without mass parameter does not survive in Bimetric theory of relativity.In both cases source density becomes constant.

V Coupling of Macro And Micro Matter Field:

The energy momentum tensor for a mixture of macro and micro fields representing perfect fluid and scalar meson field together is given by

$$T_i^{\ j} = T_i^{\ j^p} + T_i^{\ j^s} \quad (5.1)$$

By the use of co-moving co-ordinate system,the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

$$D\left(\frac{\bar{c}^2}{c^2} - \frac{\bar{\bar{c}}}{c}\right) = -\frac{16}{3}\pi\kappa\left\{p + \frac{1}{2}\left(V_4^{\ 2} - m^2 V^2\right)\right\} \quad (5.2)$$

$$D\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right) = -\frac{16}{3}\pi\kappa\left\{p + \frac{1}{2}\left(V_4^{\ 2} - m^2 V^2\right)\right\} \quad (5.3)$$

$$D\left(\frac{\overline{A}^2}{A^2}-\frac{\overline{\overline{A}}}{A}\right)=\frac{16}{3}\pi\kappa\left\{\rho+\frac{1}{2}\left(V_4^{\,2}+m^2V^2\right)\right\}$$

(5.4)

Using (5.3) and (5.4) ,we obtain

$$2\left(\rho+p\right)+V_4^{\,2}=0$$

(5.5)

In view of the reality conditions i.e. $p>0,\rho>0$, the above equation implies that $p=0,\rho=0$ and V= constant.

VI CONCLUSION:

In the study of **Z =** $\left(\frac{\sqrt{3}\,t}{x+y+z}\right)$ type plane gravitational waves, there is nil contribution of Macro and Micro matter field in Bimetric theory of relativity respectively.It is observed that the matter fields either in macro or micro level cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists.The conclusion arrived at viz.,V = constant, σ = constant, p = 0 , ρ = 0 are invariant statements and hold in all coordinate systems even though we have derived these in co-moving coordinate system.

ACKNOWLEDMENT:-The authors are thankful to Dr.R.D.Giri, Prof. Emeritus,P.G.T.D. (Mathematics), R. T. M. N. U., Nagpur, India for his constant inspiration.

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