Independent Domination of Splitted Graphs

A. Nellai Murugan*[A] And A. Esakkimuthu*
*PG and Research Department of Mathematics
V O Chidambaram College, Tuticorin-628008,TN, INDIA

***Abstract* -- A dominating set D of a splitted graph S(G) = (V, E) is an independent dominating set if the induced subgraph < D > has no edges. The independent domination number i[S(G)] of a graph S(G) is the minimum cardinality of an independent dominating set.**

***Keywords*: independent Domination, Splitted graph.**
***Subject classification*: AMS 05C69, 05C70.**

I. INTRODUCTION

By a graph, we mean a finite simple , undirected graph and connected graph with neither loops nor multiple edge. The vertex set and edge set of a graph G denoted are by V (G) and E(G) respectively. Let p = | V(G) |, q = | E(G) |. The degree of any vertex u in G is the number of edges incident with u and d is denoted by d(u), The minimum and maximum degree of a graph G is denoted by $\delta(G)$ *and* $\Delta(G)$ respectively. For graph theoretic terminology we follow [4]. For each vertex v of a graph G , take a new vertex u . Join u to those vertices of G adjacent to v . The graph thus obtained is called the splitting graph of G . It is denoted by S(G) . For a graph G , the splitting graph S of G is obtained by adding a new vertex v corresponding to each vertex u of G such that N(u) = N(v) and it is denoted by S(G) .

II. PRELIMINARIES

A set D of vertices in a splitted graph S(G)=(V,E) is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ [S(G)] of S(G) is minimum cardinality of a domination set of S(G)

A dominating set D of a splitted graph S(G) = (V, E) is an independent dominating set if the induced subgraph < D > has no edges. The independent domination number i[S(G)] of a graph S(G) is the minimum cardinality of an independent dominating set.

III. RESULTS

Theorem 3.1. Independent domination number of the splitted graphs S(P_n), then

$$i\,[S(P_n)] = 2\lceil n/3\rceil$$

proof Let S(P_n) be splitted graph of P_n . Let V [S(P_n)] = {u_i , v_i: 1≤ i ≤n }, vertex u_i is adjacent to u_{i-1} , u_{i+1} , v_{i-1} , v_{i+1} where as v_i is adjacent to u_{i-1} *and* u_{i+1} for all 2 ≤ i ≤ n-1. By definition, u_i and v_i are not adjacent and in every triplet of u_i and v_i the middle vertices satisfy the independent dominating set condition.
Hence, i [S(P_n)] = 2 ⌈$n/3$⌉.

For example, i [S(P_7)] = {u_2 , v_2 , u_5 , v_5 , u_7 , v_7} as shown in the figure 3.2

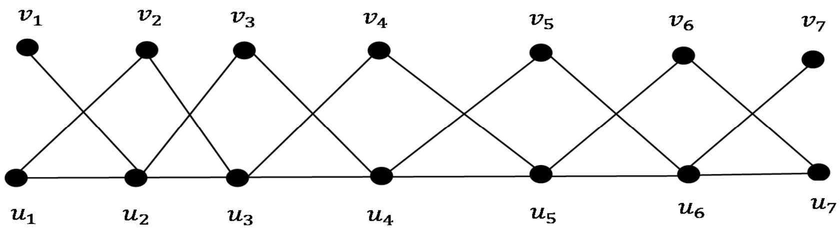


***Figure* 3.2 $S(P_7)$**

Theorem 3.3. Independent domination number of the splitted graphs S(H_n) if n is odd, then

(1) i [S(H_n)] = $\frac{4}{3}(n + 1)$ if n $\cong$ 2 (mod 3)

(2) i [S(H_n)] = $\frac{4}{3}(n + 2)$ if n $\cong$ 1 (mod 3)

(3) i [S(H_n)] = $\frac{4n}{3} + 2$ if n $\cong$ 0 (mod 3)

proof let S(H_n) be splitted graph of H_n if n is odd

case (1) n $\cong$ 2 (mod 3)

Let V [S(H_n)] = $\{u_i, v_i, u_i^1, v_i^1 : 1 \le i \le n\}$, vertex u_i is adjacent to u_{i-1}, u_{i+1}, v_{i-1}, v_{i+1} where as v_i is adjacent to u_{i-1} *and* u_{i+1} for all 2 ≤ i ≤ n-1. By definition, u_i and v_i are not adjacent and in every triplet of u_i and v_i the middle vertices satisfy the independent dominating set condition. The same process is application for the set of vertices u_i^1 *and* v_i^1.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 1)$.

For example, i [S(H_5)] = $\{u_2, v_2, u_5, v_5, u_2^1, v_2^1, u_5^1, v_5^1\}$ as shown in the figure 3.4.

case (2) n $\cong$ 1 (mod 3)

Let V [S(H_n)] = $\{u_i, v_i, u_i^1, v_i^1 : 1 \le i \le n\}$, considering a triplet in the case to choose a pair of vertices from u_i 's, (n+2) is alone a multiple of 3. In the same process. It is required pair of vertices from u_i^1 's to the dominating set.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 2)$

For example, i [S(H_7)] = $\{u_2, v_2, u_5, v_5, u_7, v_7, u_2^1, v_2^1, u_5^1, v_5^1, u_7^1, v_7^1\}$ as shown in the figure 3.5.

case (3) n $\cong$ 0 (mod 3)

In addition to the above process. It is not possible to consider both $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$. simultaneously in the dominating set . It is in need to consider the adjacent vertices of either $u_{\frac{n+1}{2}}$ or $v_{\frac{n+1}{2}}$ in the dominating set which result is addition of two more vertices,

Hence, i [S(H_n)] = $\frac{4n}{3} + 2$.

For example, i [S(H_9)] = $\{u_2, v_2, u_5, v_5, u_8, v_8, u_2^1, v_2^1, u_5^1, v_5^1, u_8^1, v_8^1\}$ as shown in the figure 3.6.

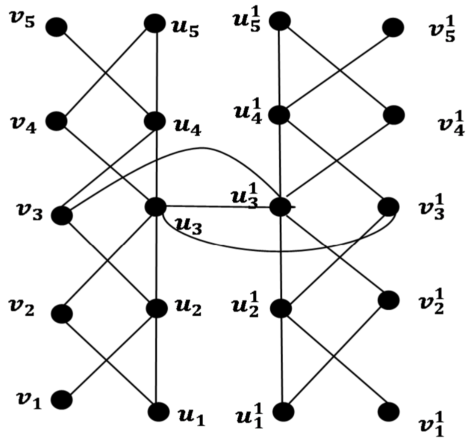


Figure 3.4 $S(H_5)$

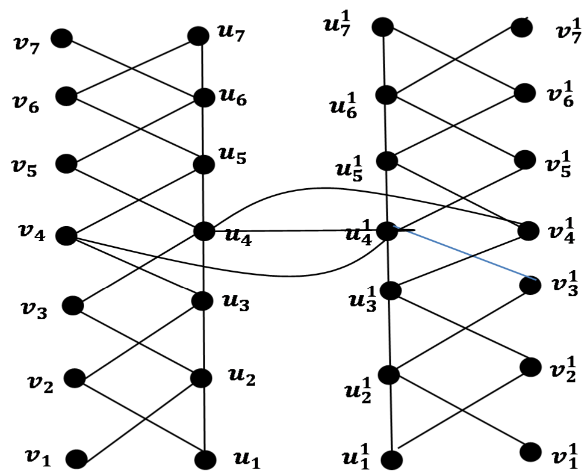


Figure 3.5 $S(H_7)$

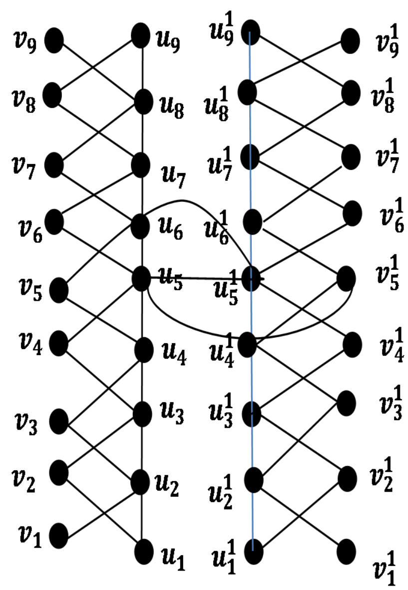


Figure 3.6 $S(H_9)$

Theorem 3.7. Independent domination number of the splitted graphs S(H_n) if n is even, then

(1) i [S(H_n)] = $\frac{4n}{3}$ if n ≅ 0 (mod 12)

(2) i [S(H_n)] = $\frac{4}{3}(n + 1)$ if n ≅ 2 (mod 12)

(3) i [S(H_n)] = $\frac{4}{3}(n + 2)$ if n ≅ 4 (mod 12)

(4) i [S(H_n)] = $\frac{4n}{3}$ if n ≅ 6 (mod 12)

(5) i [S(H_n)] = $\frac{4}{3}(n + 1)$ if n ≅ 8 (mod 12)

(6) i [S(H_n)] = $\frac{4}{3}(n + 2)$ if n ≅ 10 (mod 12).

Proof let S(H_n) be splitted graph of H_n if n is even.

case (1) n ≅ 0 (mod 12)

Let V [S(H_n)] = {u_i , v_i , u_i^1 , v_i^1: 1≤ i ≤n } , vertex u_i is adjacent to u_{i-1} , u_{i+1} , v_{i-1} , v_{i+1} where as v_i is adjacent to u_{i-1} *and* u_{i+1} for all 2 ≤ i ≤ n-1. By definition, u_i and v_i are not adjacent and in every triplet of u_i and v_i the middle vertices satisfy the independent dominating set condition. The same process is application for the set of vertices u_i^1 *and* v_i^1.

Hence , i [S(H_n)] = $\frac{4n}{3}$.

For example, i [S(H_{12})] = {$u_2, v_2, u_5, v_5, u_8, v_8, u_{11}, v_{11}, u_2^1, v_2^1, u_5^1, v_5^1, u_8^1, v_8^1$, u_{11}^1, v_{11}^1 } as shown in the figure 3.8.

case (2) n ≅ 2 (mod 12)

Let V [S(H_n)] = {u_i , v_i , u_i^1 , v_i^1: 1≤ i ≤n } , considering a triplet in the case to choose a pair of vertices from u_i 's, (n+1) is alone a multiple of 3. In the same process. It is required pair of vertices from u_i^1 's to the dominating set.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 1)$

For example,

i [S(H_{14})] = {$u_2, v_2, u_5, v_5, u_8, v_8, u_{11}, v_{11}, u_{14}, v_{14}, u_2^1, v_2^1, u_5^1, v_5^1, u_8^1, v_8^1$, u_{11}^1, v_{11}^1 , u_{14}^1, v_{14}^1 } as shown in the figure 3.9.

case (3) n ≅ 4 (mod 3)

Let V [S(H_n)] = {u_i , v_i , u_i^1 , v_i^1: 1≤ i ≤n } , considering a triplet in the case to choose a pair of vertices from u_i 's, (n+2) is alone a multiple of 3. In the same process. It is required pair of vertices from u_i^1 's to the dominating set.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 2)$

For example, i [S(H_4)] = {u_2, v_2, u_4, v_4 $u_2^1, v_2^1, u_4^1, v_4^1$ } as shown in the figure 3.10

case (4) n ≅ 6 (mod 12)

Let V [S(H_n)] = {u_i , v_i , u_i^1 , v_i^1: 1≤ i ≤n } , vertex u_i is adjacent to u_{i-1} , u_{i+1} , v_{i-1} , v_{i+1} where as v_i is adjacent to u_{i-1} *and* u_{i+1} for all 2 ≤ i ≤ n-1. By definition, u_i and v_i are not adjacent and in every triplet of u_i and v_i the middle vertices satisfy the independent dominating set condition. The same process is application for the set of vertices u_i^1 *and* v_i^1.

Hence , i [S(H_n)] = $\frac{4n}{3}$.

For example, i [S(H_6)] = $\{u_2, v_2, u_5, v_5, u_2^1, v_2^1, u_5^1, v_5^1\}$ as shown in the figure 3.11.

case (5) n ≅ 8 (mod 12)

Let V [S(H_n)] = $\{u_i , v_i , u_i^1 , v_i^1 : 1 \leq i \leq n\}$, considering a triplet in the case to choose a pair of vertices from u_i 's, (n+1) is alone a multiple of 3. In the same process. It is required pair of vertices from u_i^1 's to the dominating set.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 1)$

For example, i [S(H_8)] = $\{u_2, v_2, u_5, v_5, u_8, v_8, u_2^1, v_2^1, u_5^1, v_5^1, u_8^1, v_8^1\}$ as shown in the figure 3.12.

case (6) n ≅ 10 (mod 3)

Let V [S(H_n)] = $\{u_i , v_i , u_i^1 , v_i^1 : 1 \leq i \leq n\}$, considering a triplet in the case to choose a pair of vertices from u_i 's, (n+2) is alone a multiple of 3. In the same process. It is required pair of vertices from u_i^1 's to the dominating set.

Hence, i [S(H_n)] = $\frac{4}{3}(n + 2)$

For example, i [S(H_{10})] = $\{u_2, v_2, u_5, v_5, u_8, v_8, u_{10}, u_2^1, v_2^1, u_5^1, v_5^1, u_8^1, v_8^1, u_{10}^1, v_{10}^1\}$ as shown in the figure 3.13.

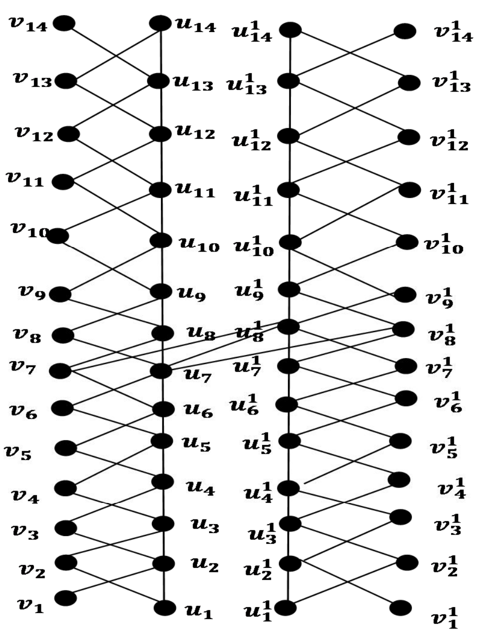


Figure 3.9 $S(H_{14})$

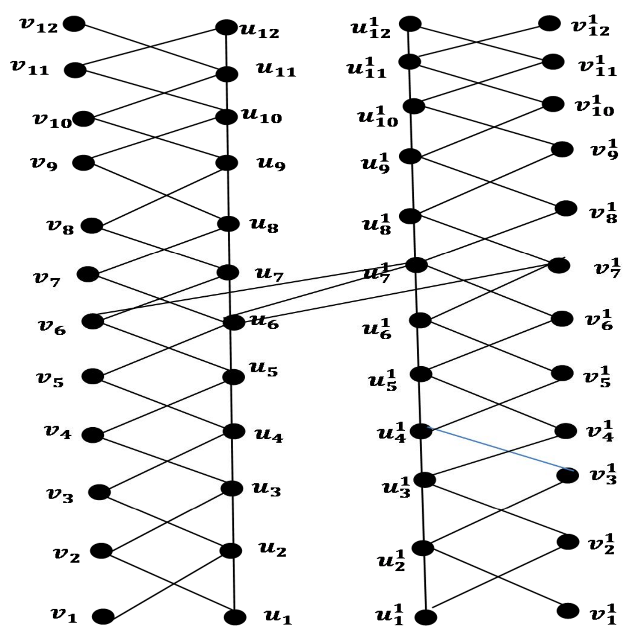
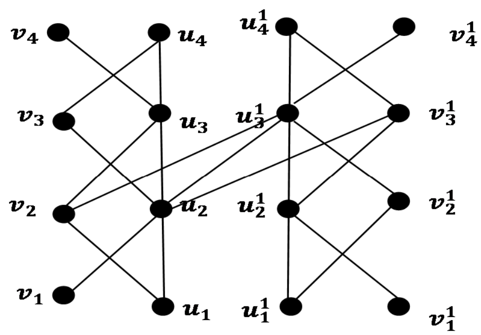
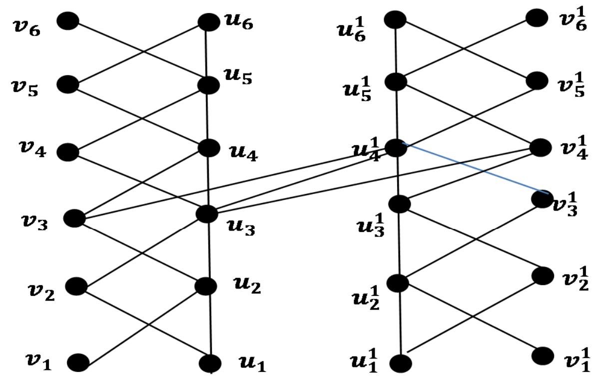


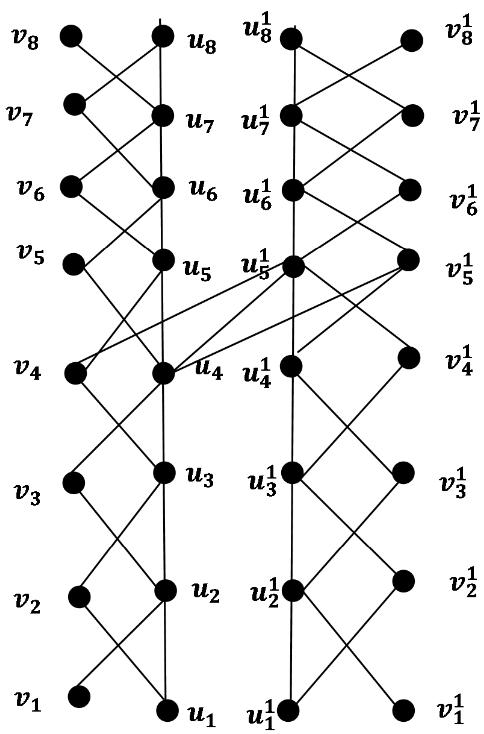
Figure 3.8 $S(H_{12})$



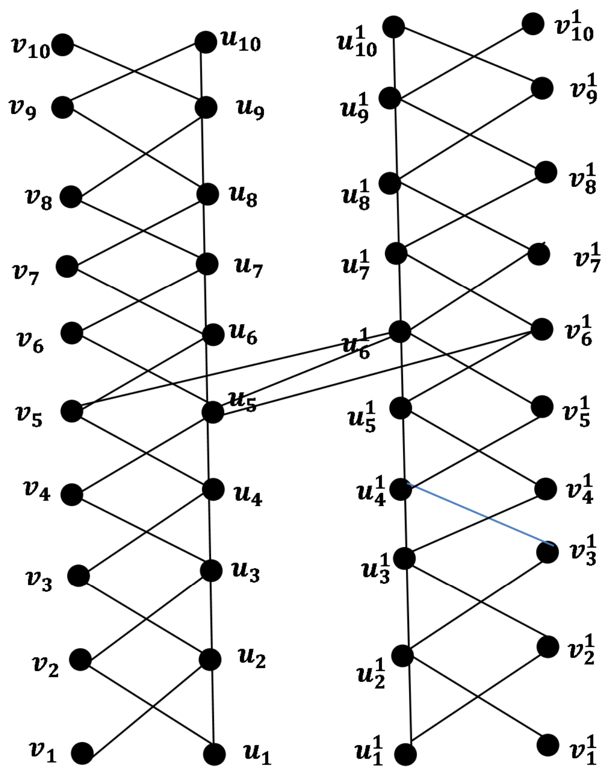
$Figure\ 3.10\ \ S(H_4)$



$Figure\ 3.11\ \ S(H_6)$



$Figure\ 3.12\ \ S(H_8)$



$Figure\ 3.13\ \ S(H_{10})$

Theorem 3.14. Independent domination number of the splitted graphs S(P_n^+) .then
i [S(P_n^+)] = 2n.

Proof. Let V [S(P_n^+)] = {u_i , v_i , u_i^1 , v_i^1: 1≤ i ≤n } , vertex v_i is adjacent to u_{i-1} , u_{i+1} , u_i^1 where as v_i^1 is adjacent to u_i for all 2 ≤ i ≤ n-1. By definition, v_i and v_i^1 are not adjacent and every v_i and v_i^1 ; 1≤ i ≤n , vertices are satisfy the independent dominating set condition.

Hence , i [S(P_n^+)] = 2n.

For example, i [S(P_5^+)] ={$v_1, v_2, v_3, v_1^1, v_2^1, v_3^1$} as shown in the figure 3.15

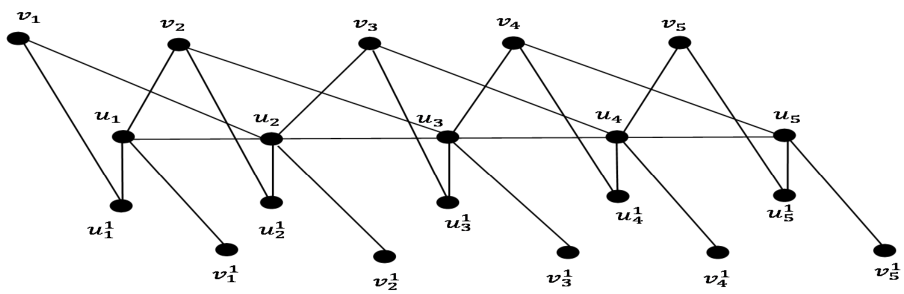


Figure 3.15 S(P_5^+)

Theorem 3.16. Independent domination number of the splitted graphs S($P_n \circ$ NK_1) .then
i [S($P_n \circ$ NK_1)] = n (1+N).

Proof. Let S($P_n \circ$ NK_1) ={ u_i , v_i , u_{ij} , v_{ij} : $1 \leq i \leq n$, $1 \leq j \leq N$ }. vertex v_i is adjacent to u_{i-1} , u_{i+1} and adjacent to u_{ij} ; $1 \leq i \leq n$, $1 \leq j \leq N$ where as v_{ij} is adjacent to u_i for all $1 \leq i \leq n$, $1 \leq j \leq N$. By definition, v_i and v_{ij} are not adjacent and every v_i and v_{ij} ; 1≤ i ≤n 1≤ j ≤ N, vertices are satisfy the independent dominating set condition.

Hence , i [S($P_n \circ$ NK_1)] = n (1+N).

For example, i [S($P_4 \circ 2K_1$)] ={$v_1, v_2, v_3, v_4, v_{11}, v_{12}, v_{21}, v_{22}, v_{31}, v_{32}, v_{33}, v_{23}, v_{13}$} as shown in the figure.3.17

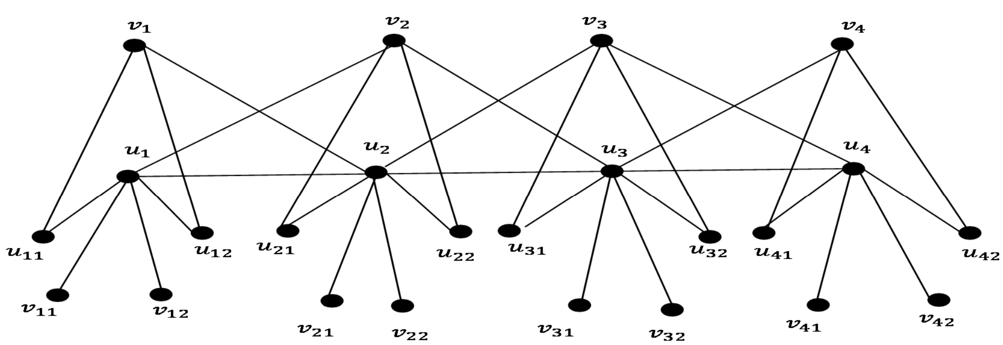


Figure 3.17 S($P_4 \circ 2K_1$)]

Theorem 3.18 For any connected Graph S(G), i [S(G)] $\leq q - \Delta[S(G)]$.
Proof. Let S(G) be a splitted graph. Let D $\subseteq S(G)$ such that the subgraph < D > is independent. Clearly , D is an i-set of S(G). It follows that | D | $\leq q - \Delta[S(G)]$. Hence i [S(G)] $\leq q - \Delta[S(G)]$.

Observation 3.19. Any splitted graph $p \leq q$.

Observation 3.20. For any splitted graph S(G), (i) $\Delta[S(G)] \leq i\,[\mathrm{S(G)}]$
(ii) $\Delta[S(G)] > i\,[\mathrm{S(G)}]$ for S(P_2) and S(P_3) .

Theorem 3.21. For any splitted graph S(G),

$$\gamma\,[S(G)] \leq i\,[S(G)]$$

and this bound is sharp.
Proof. clearly, every independent dominating set is a dominating set of G. Thus $\gamma\,[S(G)] \leq i\,[S(G)]$. For the splitted graph S(P_2) and S(P_3) the bound is sharp $\gamma\,[\mathrm{S}(P_2)] = i\,[\mathrm{S}(P_2)] = 2$ and $\gamma\,[\mathrm{S}(P_3)] = i\,[\mathrm{S}(P_3)] = 2$.

Theorem 3.22. For any splitted graph S(G),

$$\gamma\,[S(G)] + i\,[S(G)] \leq p$$

and this bound is sharp.
Proof. Form the definition, the proof follow and bound is sharp for S(P_2).

Theorem 3.23. For any splitted graph S(G),

$$i\,[S(G)] \leq \left\lfloor \frac{P\Delta[S(G)]}{(\Delta[S(G)]+1)} \right\rfloor$$

and this bund is sharp.
Proof. By theorem 3.22, $i\,[S(G)] \leq p - \gamma\,[S(G)]$.
since $\left\lceil \frac{p}{(\Delta[S(G)]+1)} \right\rceil \leq \gamma\,[S(G)]$.
$i\,[S(G)] \leq p - \left\lceil \frac{p}{(\Delta[S(G)]+1)} \right\rceil$.
Hence, $i\,[S(G)] \leq \left\lfloor \frac{P\Delta[S(G)]}{(\Delta[S(G)]+1)} \right\rfloor$.
For the splitted graph S(P_2), the bound is sharp.

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