Reflection and Transmission of Elastic Waves at the Loosely Bonded Solid-Solid Interface

Neelam Kumari
Assistant Professor, Department of Mathematics,
Ch. Devi Lal University, Sirsa, 125055, India.

Abstract

Reflection and transmission phenomenon of plane waves at a loosely bonded interface between linear isotropic elastic solid half space and fluid saturated incompressible porous solid half space is studied in the present study. Plane wave P or SV- wave incidents on the interface through linear isotropic elastic solid half space. The amplitude ratios of various reflected and transmitted waves to that of incident wave are obtained. These amplitude ratios have been computed numerically for a particular model for different values of bonding parameter. It is observed that these amplitude ratios depend on angle of incidence of the incident wave and material properties of medium and these are affected by the bonding parameter and fluid filled in the pores of fluid saturated incompressible porous half space. A special case is also obtained and discussed from the present study.

Keywords: Porous solid, reflection, transmission, longitudinal wave, transverse wave, amplitude ratios, empty porous solid.

1. Introduction

Elastic waves propagation in fluid saturated porous medium is of great interest due to its importance in various fields such as soil dynamics, hydrology, seismology, earthquake engineering and geophysics. Layers of porous solids, such as sandstone or limestone, saturated with oil or groundwater are often present in the earth's crust. They are of great interest in geophysical exploration. Therefore, it is a matter of interest to study the incompressible fluid saturated poroelastic solid in contact with another incompressible fluid saturated poroelastic solid. To study the mechanical behaviour of a fluid saturated porous medium due to the different motions of the solid and liquid phases and different material properties and the complicated structures of pores, is very complex. So many researchers tried to overcome this difficulty from time to time.

Bowen[1] and de Boer and Ehlers[2-3]developed a theory for incompressible fluid saturated porous medium based on the work of Fillunger model[4]. Based on this theory, many researchers like de Boer and Liu[5-6], Kumar and Hundal [7],de Boer and Didwania [8], Tajuddin and Hussaini [9],Kumar et.al.[10] etc. studied some problems of wave propagation in fluid saturated porous media. In the problems of wave propagation at the interface between two elastic half spaces, the contact between them is normally assumed to be welded. However, in certain situations, there are reasons for expecting that bonding is not complete. Murty [11] discussed a theoretical model for reflection, transmission, and attenuation of elastic waves through a loosely bonded interface between two elastic solid half spaces by assuming that the interface behaves like a dislocation which preserves the continuity of stresses allowing a finite amount of slip. A similar situation occurs at the two different poroelastic solids, as the liquid present in the porous skeleton may cause the two media to be loosely bonded. Vashisth and Gogna [12], Kumar and Singh [13] etc. discussed the problems of reflection and transmission at the loosely bonded interface between two half spaces.

Using de Boer and Ehlers [3] theory for fluid saturated porous medium, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SV-wave) at a loosely bonded interface between linear isotropic elastic solid half space and fluid saturated porous half space is investigated. A special case when fluid saturated porous half spaces reduce to empty porous solid half spaces has been deduced and discussed accordingly. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically.

2. Basic equations and constitutive relations

For M_1 (Fluid Saturated Incompressible Porous Medium)

Following de Boer and Ehlers [3] the equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are

$$\nabla.(\eta^S \dot{u}_S + \eta^F \dot{u}_F) = 0, \tag{1}$$

$$(\lambda^S + \mu^S)\nabla(\nabla.u_S) + \mu^S\nabla^2 - \eta^S\nabla p - \rho^S\ddot{u}_s + S_v(\dot{u}_F - \dot{u}_S) = 0, \tag{2}$$

$$\eta^F\nabla p + \rho^F\ddot{u}_F + S_v(\dot{u}_F - \dot{u}_S) = 0, \tag{3}$$

$$T_E^S = 2\mu^S E_S + \lambda^S(E_S.I)I, \tag{4}$$

$$E_S = \frac{1}{2}(\text{grad}\, u_S + \text{grad}^T u_S), \tag{5}$$

ere $u_i, \dot{u}_i, \ddot{u}_i$, $i = F, S$ denote the displacement, velocities and acceleration of fluid and solid phases respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid respectively. T_E^S is the stress in the solid phase and E_S is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{6}$$

The case of isotropic permeability, the tensor s_v describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990) as

$$S_v = \frac{(\eta^F)^2 \gamma^{FR}}{K^F} \mathbf{I}, \tag{7}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

We assume the displacement vector u_i $(i = F, S)$ as

$$u_i = (u^i, 0, w^i) \text{ where } i = F, S. \tag{8}$$

Using eq. (8) in eqs. (1)-(3), the following equations for fluid saturated incompressible porous medium are obtained as:

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{9}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{10}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{11}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{12}$$

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x\, \partial t} + \frac{\partial^2 w^S}{\partial z\, \partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x\, \partial t} + \frac{\partial^2 w^F}{\partial z\, \partial t}\right] = 0, \tag{13}$$

Also, ${t_{zz}}^S$ and ${t_{zx}}^S$ are the normal and tangential stresses in the solid phase and are

$${t_{zz}}^S = \lambda^S \left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S \frac{\partial w^S}{\partial z}, \tag{14}$$

$${t_{zx}}^S = \mu^S \left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right), \tag{15}$$

where

$$\theta^S = \frac{\partial (u^S)}{\partial x} + \frac{\partial (w^S)}{\partial x}. \tag{16}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{17}$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j = \frac{\partial \phi^j}{\partial x} + \frac{\partial \psi^j}{\partial z}, \quad w^j = \frac{\partial \phi^j}{\partial z} - \frac{\partial \psi^j}{\partial x}, \quad j = F, S. \tag{18}$$

With the help of (18) we obtain the following equations determining $\phi^s, \phi^F, \psi^s, \psi^F$ as:

$$\nabla^2 \phi^S - \frac{1}{C^2}\frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\frac{\partial \phi^S}{\partial t} = 0, \tag{19}$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S, \tag{20}$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{21}$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{22}$$

$$(\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^s}{\partial t^2} - S_v \frac{\partial \phi^s}{\partial t} = 0, \tag{23}$$

where

$$C = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}. \tag{24}$$

For M_2 (Homogeneous Isotropic Elastic Solid Medium)

The equation governing the small motions in a homogeneous isotropic elastic are

$$\mu' \nabla^2 \mathbf{u}' + (\lambda' + \mu')\ \nabla(\nabla . \mathbf{u}') = \rho' \ddot{\mathbf{u}}', \tag{25}$$

where symbols λ', μ' are Lame's constants, ρ' is the density and $\mathbf{u}'$ is the displacement vector. Superposed dots on right hand side of eq. (25) stand for second partial derivative with respect to time.

The stress strain relation in the isotropic elastic medium is given by

$$\sigma_{ij}' = \lambda' e_{kk}' \delta_{ij} + 2\mu' e_{ij}', \tag{26}$$

where

$$e_{ij}' = \frac{1}{2}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right), \tag{27}$$

are the components of the strain tensor, e_{kk}' is the dilatation and σ_{ij}' are the components of stress tensor in the isotropic elastic medium.

For the two dimensional problem, the displacement vector $\mathbf{u}'$ is taken as

$$\mathbf{u}' = (u', 0, w'), \tag{28}$$

The displacement components u' and w' are related to potential functions ϕ' and ψ' as

$$u' = \frac{\partial \phi'}{\partial x} + \frac{\partial \psi'}{\partial z}, \quad w' = \frac{\partial \phi'}{\partial z} - \frac{\partial \psi'}{\partial x}, \tag{29}$$

Using equations (28) and (29) in equation (26), we obtain as

$$\nabla^2 \phi' = \frac{1}{{v_1'}^2}\frac{\partial^2 \phi'}{\partial t^2}, \tag{30}$$

$$\nabla^2 \psi' = \frac{1}{{v_2'}^2}\frac{\partial^2 \psi'}{\partial t^2}, \tag{31}$$

where $v_1' = \sqrt{\frac{\lambda' + 2\mu'}{\rho'}}$ and $v_2' = \sqrt{\frac{\mu'}{\rho'}}$ are the velocities of longitudinal wave (P-wave) and transverse wave (SV-wave) in isotropic elastic medium respectively.

3. Formulation of the problem and its solution. Consider a model consisting of linear isotropic elastic solid half space M_2 $[z < 0]$ and fluid saturated porous medium M_1 $[z > 0]$ (see figure1). We take the z-axis pointing into lower half-space and z=0 is loosely bonded interface. A longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through the fluid saturated porous half space medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M_1 and two transmitted waves P-wave and SV-wave in medium M_2.

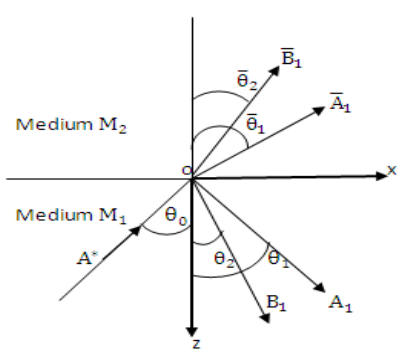


Fig.1 Geometry of the problem

In Medium M_1

The potential function satisfying the equations (19)-(23) can be

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01}\exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1\exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}], \tag{32}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01}\exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + B_1\exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}], \tag{33}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S{\omega_1}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - {\omega_2}^2\rho^F}, \tag{34}$$

and A_{01} and B_{01} are amplitudes of the incident P-wave and SV-wave, respectively. A_1 , B_1 are amplitudes of the reflected P-wave and SV-wave respectively.

In Medium M_2

Take the potential function satisfying the equations (30) and (31) as

$$\phi' = \bar{A}_1\exp\{i\bar{k}_1(x\sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \tag{35}$$

$$\psi' = \bar{B}_1\exp\{i\bar{k}_2(x\sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}, \tag{36}$$

where $\bar{k}_1$ and $\bar{k}_2$ are wave numbers of transmitted P-wave and SV-wave, respectively. $\bar{A}_1$ and $\bar{B}_1$ are amplitudes of transmitted P-wave and SV-wave.

4. Boundary conditions. For propagation of plane waves at the loosely bonded interface of an isotropic elastic solid half space and fluid saturated incompressible porous half space, the boundary conditions are

$$t_{zz}^S - p = \sigma_{zz}', \quad t_{zx}^S = \sigma_{zx}', \quad \sigma_{zx}' = K_t(u^s - u'), \quad w^s = w', \tag{37}$$

$$\text{where } K_t = ik\mu\tau \text{ and } \tau = \frac{\gamma}{(1-\gamma)\sin\theta_0}, \tag{37a}$$

γ is bonding constant. $0 \le \gamma \le 1$. $\gamma = 0$ corresponds to smooth surface and $\gamma = 1$ corresponds to a welded interface

In order to satisfy the boundary conditions, the extension of the Snell's law gives

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{v_1'} = \frac{\sin\bar{\theta}_2}{v_2'}, \tag{38}$$

Also

$$k_1V_1 = k_2V_2 = \bar{k}_1v_1' = \bar{k}_2v_2' = \omega, \quad \text{at } z = 0, \tag{39}$$

For P-wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{40}$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{41}$$

For incident longitudinal wave (P-wave), putting $B_{01} = 0$ in equation (33) and for incident transverse wave putting $A_{01} = 0$ in equation (32). Substituting the expressions of potentials given by (32)-(33) and(35)-(36) in equations (14)-(15),(18) and (26) and (29) and using equations (37)-(41), we get a system of four non homogeneous which can be written as

$$\sum_{j=0}^{4} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4) \tag{42}$$

where

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{A_2}{A^*}, \quad Z_3 = \frac{\bar{A}_1}{A^*}, \qquad Z_4 = \frac{\bar{B}_1}{A^*} \tag{43}$$

The components a_{ij} in non dimensional form can be written as

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S k_1{}^2},$$

$$a_{12} = -2\sin\theta_2\cos\theta_2 \frac{k_2{}^2}{k_1{}^2}, \qquad a_{13} = -\frac{\lambda' \bar{k}_1{}^2}{k_1{}^2\mu^S},$$

$$a_{14} = -\frac{\mu' \bar{k}_2{}^2 \sin 2\bar{\theta}_2}{k_1{}^2\mu^S}, \qquad a_{21} = 2\sin\theta_1\cos\theta_1,$$

$$a_{22} = \frac{k_2{}^2(\cos^2\theta_2 - \sin^2\theta_2)}{k_1{}^2}, \qquad a_{23} = \frac{\mu' \bar{k}_1{}^2 \sin 2\bar{\theta}_1}{k_1{}^2\mu^S},$$

$$a_{24} = -\frac{\mu' \bar{k}_2{}^2 \cos 2\bar{\theta}_2}{k_1{}^2\mu^S}, \qquad a_{31} = K_t\, i \sin\theta_1,$$

$$a_{32} = \frac{K_t\, i\, k_2 \cos\theta_2}{k_1},$$

$$a_{33} = -\frac{K_t\, i\, \bar{k}_1}{k_1}\sin\bar{\theta}_1 - \frac{\mu' \bar{k}_1{}^2 \sin 2\bar{\theta}_1}{k_1}$$

$$a_{34} = \frac{K_t\, i\, \bar{k}_2 \cos\bar{\theta}_2}{k_1} + \frac{\mu' \bar{k}_2{}^2 \cos 2\bar{\theta}_2}{k_1},$$

$$a_{41} = i\cos\theta_1, \qquad a_{42} = -\frac{i\, k_2 \sin\theta_2}{k_1},$$

$$a_{43} = \frac{i\, \bar{k}_1 \cos\bar{\theta}_1}{k_1}, \qquad a_{44} = \frac{i\, \bar{k}_2 \sin\bar{\theta}_2}{k_1}, \tag{44}$$

For incident longitudinal wave:

$$A^* = A_{01}, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = -a_{31}, Y_4 = a_{41}, \tag{45}$$

For incident transverse wave:

$$A^* = B_{01}, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = -a_{42}, \tag{46}$$

Special case:-

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and hence can be neglected, so the equation (24) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \tag{47}$$

and the coefficients a_{11} in (44) changes to

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1, \tag{48}$$

and all the remaining coefficients in (44) remain same. For this case the problem reduces to the problem of linear isotropic elastic solid half space over empty porous solid half space.

5. Numerical results and discussion

After obtaining the theoretical results in above sections, we have computed them numerically by taking the following values of relevant elastic parameters to study in more detail the behaviour of various amplitude ratios.

In medium M_1, the physical parameters for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu [14] as

$$\eta^s = 0.67, \quad \eta^F = 0.33, \quad \rho^s = 1.34\,\mathrm{Mg/m^3},$$

$$\rho^F = 0.33\,\mathrm{Mg/m^3}, \quad \lambda^s = 5.5833\,\mathrm{MN/m^2}$$

$$K^F = 0.01\,\mathrm{m/s}, \quad \gamma^{FR} = 10.00\,\mathrm{KN/m^3},$$

$$\mu^s = \frac{8.3750\mathrm{N}}{\mathrm{m}^2}, \quad \omega^* = \frac{10}{s}, \tag{49}$$

In medium M_2, the physical parameters for isotropic elastic solid are as follows

$$\rho' = 2.65\frac{Mg}{m^3},\quad \mu' = 2.238\frac{MN}{m^2},\quad \lambda' = 2.238\frac{MN}{m^2}, \tag{50}$$

Using MATLAB, a computer programme has been developed and modulus of amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ for various reflected and transmitted waves have been computed. $|Z_1|$ and $|Z_2|$ represent the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also, $|Z_3|$ and $|Z_4|$ represent the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively. Dashed dotted line represents the variations of the amplitude ratios for bonding constant $\gamma = 0$, dotted line correspond $\gamma = 0.25$, dashed line for $\gamma = 0.5$, solid line for $\gamma = 0.75$ and bold dotted line for $\gamma = 1$ in all the figures (2)-(17) w.r.t. angle of incidence of the incident P or SV-wave. The variations in all the figures are shown for the range $0^0 \le \theta \le 90^0$.

Incident P-wave

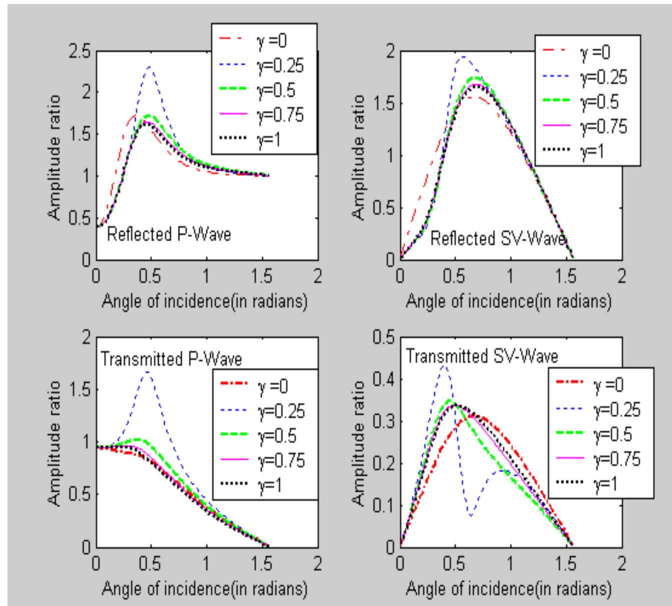
Figures (2)-(5) represent the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave. The behaviour of all these distribution curves for reflected P-wave and for transmitted P-wave is similar. For reflected SV–wave and transmitted SV-wave, the behaviour of all curves is also same i.e. Increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence. Figures (6)-(9) show the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave in special case. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (2)-(5) and (6)-(9). Also in the figures (6)-(9), the values corresponding to bonding parameter $\gamma = 0$, i.e., for smooth interface are large in comparison to other interface parameters.

Incident SV-wave

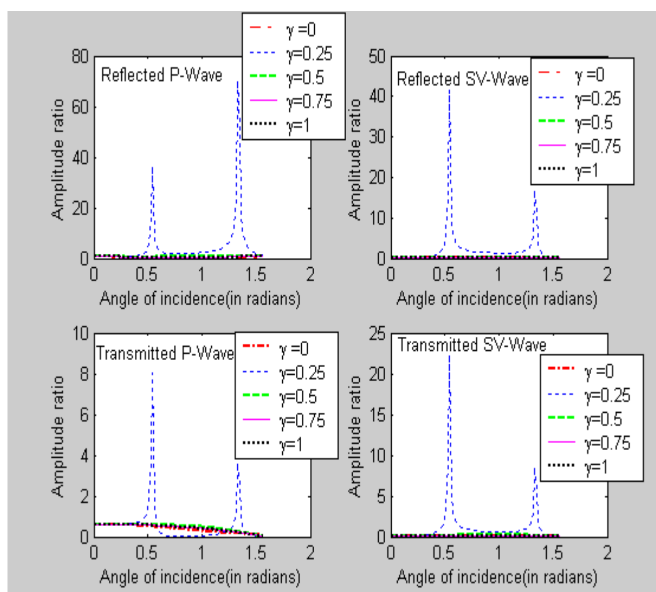
Figures (10)-(17) show the variations of the amplitude ratios for reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of the incident SV-wave. The behaviour of all these curves in figures (10)-(17) is same i.e. they oscillates. In all the figures (10)-(17), the amplitude ratios for the bonding parameter $\gamma = 0.25$ are maximum except in figure (14). The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (10)-(13) and (14)-(17).

6. Conclusion

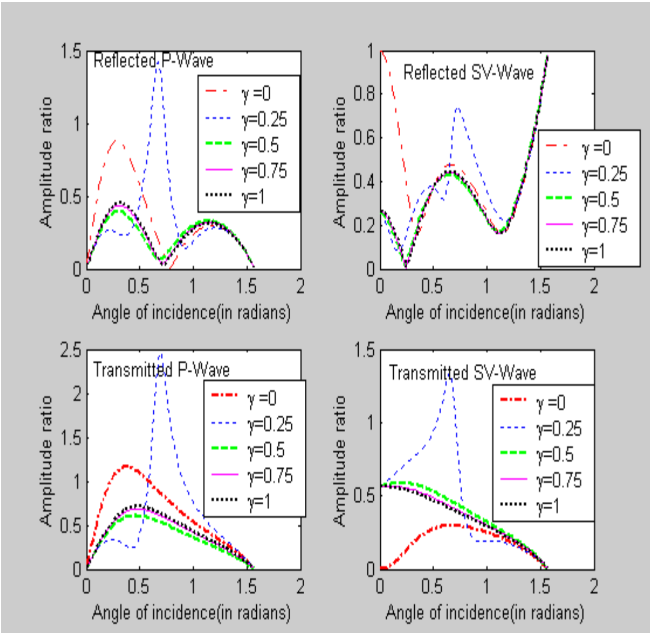
Reflection and transmission phenomenon of incident elastic waves at a loosely interface between linear elastic solid half space and fluid saturated porous half space has been studied when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios. Effect of bonding parameter is observed on amplitude ratios.



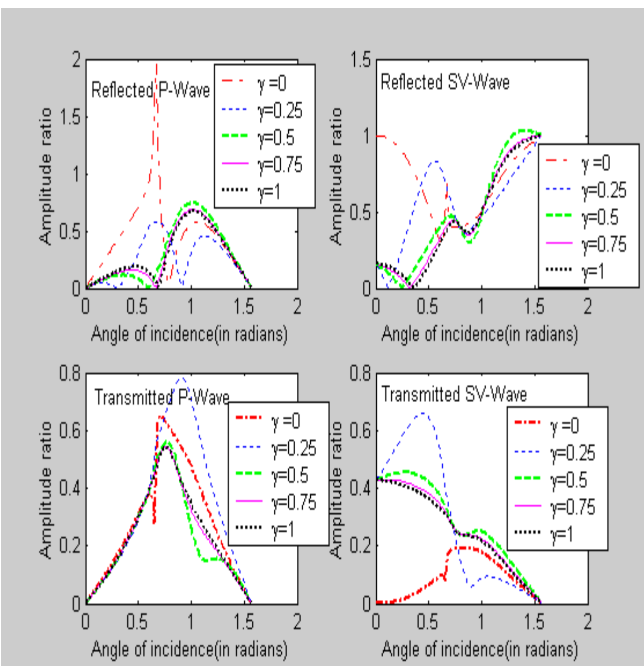
Figs. 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave.



Figs. 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solids.



Figs. 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



Figs. 14-17. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solids.

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