Properties of Fuzzy Soft Matrices

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Abstract

The concept of soft set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parameterization tools of soft set theory enhance the flexibility of its applications. In this work, we give definition of fuzzy soft matrix, and their properties and examples. We further give the definition of Cardinality of a fuzzy soft matrix and Degree of Sub matrix hood of fuzzy soft matrix with examples.

Keywords: *Soft Set, Fuzzy Soft Set, Fuzzy Soft Class, Fuzzy soft matrix, Cardinality of a fuzzy soft matrix, Degree of Sub matrix hood.*

INTRODUCTION

Most of our real life problems in medical sciences, engineering, management, environment and social sciences often involve data which are not always all crisp, precise and deterministic in character because of various uncertainties typical for these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets, intutionistic fuzzy sets, interval mathematics and rough sets etc. However, Molodstov [2] has shown that each of the above topics suffers from some inherent difficulties due to inadequacy of their parameterization tools and introduced a concept called "Soft Set Theory" having parameterization tools for successfully dealing with various types of uncertainties. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable. Fuzzy set theory proposed by Professor L.A. Zadeh[1] in 1965 is considered as a special case of the soft sets. In 2003, P.K. Maji, R. Biswas and A.R. Roy[6] studied the theory of soft sets initiated by Molodstov [2] and developed several basic notions of Soft Set Theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao[7] and Chen et al. [5] studied and improved the findings of Maji et al [6]. In 2011, T.J. Neog and D.K. Sut[8] studied the theory of soft sets initiated by Molodstov [2] and developed several basic notions of Soft Set Theory. Recently, Cagman et al. [9] introduced soft matrix and applied it in decision making problems. In one of our earlier work [10], we proposed the idea of 'Properties of Fuzzy Soft Set' in sequel to [10] defining some operations. The present paper aims to define fuzzy soft matrices and establish some results on them. In this paper, we define a sub matrix hood of a soft fuzzy matrices along with several examples.

2. Preliminaries:

Definition 1. [2]

Let X be an initial set and U be a set of parameters. Let P(X) denotes the power set of X, and let A ⊂ U. A pair (F, A) is called a soft set over X, where F is a mapping given by

$$F: A \rightarrow P(X).$$

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\xi \in A$, F (ξ) may be considered as the set of ξ-elements of the soft set (F, A) or as the ξ - approximate elements of the soft set. Clearly, a soft set is not a (crisp) set. To illustrate this idea, let we consider the following example.

Example:

Let us consider a soft set (F, A) which describes the "attractiveness of houses" that Mr. P is considering to purchase.

Suppose that there are six houses in the universe X under consideration,

$$X=\{h_1, h_2, h_3, h_4, h_5, h_6\}$$

and

$$A=\{e_1, e_2, e_3, e_4, e_5\} \subset U$$

is a set of decision parameters, where e_1 stands for the parameters "expensive", e_2 stands for the parameters "beautiful", e_3 stands for the parameters "wooden", e_4 stands for the parameters "cheap", e_5 stands for the parameters "in the green surrounding".

Suppose that
$F(e_1)=\{h_3, h_5\}$, $F(e_2)=\{h_1, h_3, h_5\}$, $F(e_3)=\{h_1, h_4, h_5\}$, $F(e_4)=\{h_1, h_2\}$, $F(e_5)=\{h_5\}$.

Therefore, $F(e_1)$ means "houses (expensive)", whose functional value is the set $\{h_2, h_5\}$. Thus, we can view the soft set (F, A) as a collection of approximations as below

$$(F, A)=\left\{\begin{array}{c} expensive\ houses\ =\ \{h_{2,}h_5\} \\ beautiful\ houses\ =\ \{h_{1,}h_{3,}h_5\} \\ wooden\ houses\ =\ \{h_{1,}h_{4,}h_5\} \\ cheap\ houses\ =\ \{h_{1,}h_{2,}\} \\ in\ green\ surrounding\ houses\ =\ \{h_5\} \end{array}\right\}$$

Definition 2. [6]

A pair (F, A) is called a fuzzy soft set over X where F: A→ $\widetilde{P}(X)$ is a mapping from A into $\widetilde{P}(X)$.

Example:

Let X=$\{h_1, h_{2,} h_{3,} h_{4,} h_{5,} h_6\}$ be the set of six houses under consideration and A=$\{e_1, e_{2,} e_{3,} e_{4,} e_5\}\subset$ U is a set of decision parameters,

Consider the mapping

F: A→ $\widetilde{P}$(X).

(F, A)= { F (e_1) =$\{h_2\ /0.9, h_5\ /0.7\}$, F (e_2) =$\{h_1\ /0.6, h_3\ /0.9, h_5\ /0.7\}$,

F(e_3)=$\{h_1\ /0.5, h_4\ /0.9, h_5\ /0.6\}$, F($e_4$)=$\{h_1\ /0.7, h_2\ /0.8, h_5\ /0.9\}$,

F(e_5)=$\{h_5\ /0.8\}$}

is the fuzzy soft set representing the "attractiveness of the houses" which Mr. X is going to buy.

Definition 3. [3]

Let X be a universe and U a set of attributes. Then the pair (X, U) denotes the collection of all fuzzy soft sets on X with attributes from U and is called a fuzzy soft class.

Definition 4. [9]

Let X be a universal set & U be the set of parameters. Let A $\subseteq$ U & (F,A) be a fuzzy soft set in the fuzzy soft class (X, U). Then fuzzy soft set (F, A) in a matrix form as A_{mxn} = $[a_{ij}]_{mxn.}$ Or

A= $[a_{ij}]$ where i=1,2,…..m & j=1,2,…….n

Where $a_{ij} = \begin{cases} \mu_j(x_i) & if\ e_j \in A \\ 0 & if\ e_j \notin A \end{cases}$

$\mu_j(x_i)$ represents the membership of x_i in the fuzzy set.

Example:

Let X=$\{h_1, h_2, h_3, h_4\}$ be the set of four houses under consideration and A=$\{e_1, e_2, e_3, e_4\} \subset$ U is a set of decision parameters,

The matrix representation is,

$$\begin{pmatrix} 0.8 & 0.4 & 0.3 & 0.6 \\ 0 & 1 & 0.8 & 0.9 \\ 0.7 & 0 & 0.8 & 0 \\ 0 & 0.8 & 0.9 & 0.6 \end{pmatrix}$$

Definition 5. [1]

For two fuzzy soft matrices (F, A) and (G, B) in a fuzzy soft class (X, U), we say that (F, A) is a fuzzy soft sub matrix of (G, B), if

(i) A$\subseteq$B

(ii) For all $\xi \in$U, F (ξ) $\subseteq$ G (ξ) and is written as (F, A) $\tilde{\subseteq}$ (G, B).

Example

Let X=$\{h_1, h_2, h_3, h_4\}$ be the set of four houses under consideration and U=$\{e_1$(expensive), e_2 (beautiful), e_3(wooden), e_4(cheap)$\}$ be the set of parameters, A = $\{e_1, e_2, e_3\} \subseteq$ U and B = $\{e_1, e_2, e_3, e_4\} \subseteq$ U. Then

$$(F, A) = \begin{pmatrix} 0.6 & 0.6 & 0.5 & 0 \\ 0.9 & 0.8 & 0.6 & 0 \\ 0.7 & 0.9 & 0.5 & 0 \\ 0.5 & 0.7 & 0.9 & 0 \end{pmatrix}$$

is the fuzzy soft matrix representing the "attractiveness of the house" which Mr. P is going to buy and

$$(G, B) = \begin{pmatrix} 0.8 & 0.6 & 0.6 & 0.2 \\ 0.9 & 0.9 & 0.9 & 0.5 \\ 0.9 & 0.9 & 0.6 & 0.9 \\ 0.7 & 0.8 & 0.9 & 0.9 \end{pmatrix}$$

is the fuzzy soft matrix representing the "attractiveness of the house" which Mr. Q is going to buy.

Here A $\subseteq$ B, and for all $\xi \in$A, F (ξ) $\subseteq$ G (ξ). Thus (F, A) $\tilde{\subseteq}$ (G, B).

Definition 6. [11]

Let $[\mu_{ij}] \in A_{mxn}$, Then $[\mu_{ij}]$ is called

(i) A zero fuzzy soft matrix, denoted by [0], if μ_{ij}=0 for all i and j.

(ii) A universal fuzzy soft matrix, denoted by [1], if μ_{ij}=1 for all i and j.

(iii) $[\mu_{ij}]$ is a fuzzy soft sub matrix of $[\lambda_{ij}]$, denoted by $[\mu_{ij}] \tilde{\subseteq} [\lambda_{ij}]$, if $\mu_{ij} \leq \lambda_{ij}$

(iv) $[\mu_{ij}]$ and $[\lambda_{ij}]$ are fuzzy soft equal matrices, denoted by $[\mu_{ij}] = [\lambda_{ij}]$, if $\mu_{ij} = \lambda_{ij}$ for all i and j.

Definition 7. [11]

Let $[\mu_{ij}]$, $[\lambda_{ij}] \in A_{mxn}$, ie.)Then $[\gamma_{ij}]$ is called

(i) Union of $[\mu_{ij}]$ and $[\lambda_{ij}]$, denoted by $[\mu_{ij}] \tilde{\cup} [\lambda_{ij}]$ if γ_{ij}= max $\{\mu_{ij}, \lambda ij\}$ for all i and j.

(ii) Intersection of $[\mu_{ij}]$ and $[\lambda_{ij}]$, denoted by $[\mu_{ij}] \tilde{\cap} [\lambda_{ij}]$ if γ_{ij}= max $\{\mu_{ij}, \lambda ij\}$ for all i and j.

(iii) Complement of $[\mu_{ij}]$, denoted by $[\mu_{ij}]^c$, if $[\mu_{ij}]^c = 1 - \mu_{ij}$ for all i and j.

Example

$$\text{Let } [\mu_{ij}] = \begin{pmatrix} 0.6 & 0.8 & 0.5 & 0.2 \\ 0.4 & 0.7 & 0.9 & 0.7 \\ 0.8 & 0.5 & 0.4 & 0.6 \\ 0.7 & 0.8 & 0.1 & 0.3 \end{pmatrix}$$

and $[\lambda_{ij}] = \begin{pmatrix} 0.8 & 0.6 & 0.6 & 0.2 \\ 0.2 & 0.5 & 0.9 & 0.5 \\ 0.9 & 0.9 & 0.6 & 0.9 \\ 0.3 & 0.8 & 0.8 & 0.5 \end{pmatrix}$

$$[\mu_{ij}] \tilde{\cup} [\lambda_{ij}] = \begin{pmatrix} 0.8 & 0.8 & 0.5 & 0.2 \\ 0.4 & 0.7 & 0.9 & 0.7 \\ 0.9 & 0.9 & 0.6 & 0.9 \\ 0.7 & 0.8 & 0.8 & 0.5 \end{pmatrix}$$

$$[\mu_{ij}] \tilde{\cap} [\lambda_{ij}] = \begin{pmatrix} 0.6 & 0.6 & 0.5 & 0.2 \\ 0.2 & 0.5 & 0.9 & 0.5 \\ 0.8 & 0.5 & 0.4 & 0.6 \\ 0.3 & 0.8 & 0.1 & 0.3 \end{pmatrix}$$

$$[\mu_{ij}]^c = \begin{pmatrix} 0.4 & 0.2 & 0.5 & 0.8 \\ 0.6 & 0.3 & 0.1 & 0.3 \\ 0.2 & 0.5 & 0.6 & 0.4 \\ 0.3 & 0.2 & 0.9 & 0.7 \end{pmatrix}$$

3. Cardinality of a Fuzzy Soft Matrix:

For a finite fuzzy soft matrix (F, A), the Cardinality $|(F, A)|$ is defined as,

$$|(F,A)| = \sum_{x \in U} \mu_{ij}(x) = \sum(F, A).$$

And its relative cardinality $\|(F, A)\|$ is,

$$\|(F,A)\| = \frac{|(F,A)|}{10}$$

Set Cardinality of a Fuzzy Soft Matrix:

We define for a finite fuzzy soft matrix (F, A), the Set Cardinality $|(F, A)|$ is defined as,

$$|(F,A)| = \sum_{e_j \in A} \exp F(e_j)$$

Example:

Let X=$\{h_1, h_2, h_3, h_4\}$ be the set of houses under consideration and U = {expensive (e_1), near by city (e_2), cheap (e_3), beautiful (e_4)} be the set of parameters. Consider the fuzzy soft matrices (F, A) and (G, B) where A = $\{e_1, e_2, e_3\} \subseteq$ U. Then

$$(F, A) = \begin{pmatrix} 0.6 & 0.6 & 0.5 & 0 \\ 0.9 & 0.8 & 0.6 & 0 \\ 0.7 & 0.9 & 0.5 & 0 \\ 0.5 & 0.7 & 0.9 & 0 \end{pmatrix}$$

$|(F, A)| = \sum_{e_{ij} \in A} \exp F(e_{ij})$ = 3.3.

Degree of Sub Matrix hood:

Let X be a Universal, U be a set of parameters and let (F, A) and (G, B) are two fuzzy soft matrices of X. Then the degree of sub matrix hood denoted by $\mathbf{S}$(A, B) is defined as,

$$\mathbf{S}(A, B) = \frac{1}{|(F,A)|}\{|(F, A)| - \sum \max\{0, (F, A) - (G, B)\}\}$$

Where $|(F, A)| = \sum_{e_{ij} \in A} \exp F(e_{ij})$

and

$$\mathbf{S}(B, A) = \frac{1}{|(G,B)|}\{|(G, B)| - \sum \max\{0, (G, B) - (F, A)\}\}.$$

Example:

Let X=$\{h_1, h_2, h_3, h_4\}$ be the set of houses under consideration and U = {expensive (e_1), near by city (e_2), cheap (e_3), beautiful (e_4)} be the set of parameters. Consider the fuzzy soft matrices (F, A) and (G, B) where A, B $\subseteq$ U.

Then,

$$(F, A) = \begin{pmatrix} 0.8 & 0.8 & 0 & 0.6 \\ 0.9 & 0.7 & 0.4 & 0.5 \\ 0.8 & 0.5 & 0.2 & 0.6 \\ 0.7 & 0.6 & 0.1 & 0.4 \end{pmatrix}$$

and

$$(G, B) = \begin{pmatrix} 0.8 & 0.9 & 0.6 & 0.7 \\ 0.9 & 0.8 & 0.8 & 0.6 \\ 0.9 & 0.8 & 0.4 & 0.9 \\ 0.8 & 0.6 & 0.8 & 0.7 \end{pmatrix}$$

$$|(F, A)| = \sum_{e_{ij} \in A} \exp F(e_{ij}) = 3.2$$

$$|(G, B)| = \sum_{e_{ij} \in A} \exp G(e_{ij}) = 3.5$$

(F, A) – (G, B) = (-0.1, 0, -0.1, -0.1)

$$\mathbf{S}(A, B) = \frac{1}{3.2}\{3.2 - \sum \max\{0, (-0.1,\ 0,\ -0.1,\ -0.1)\}\}$$

$$= 1$$ and

$$\mathbf{S}(B, A) = \frac{1}{3.5}\{3.5 - \sum \max\{0, (0.1,\ 0,\ 0.1,\ 0.1)\}\}$$

$$= 0.9143 \cong 0.9.$$

4. Conclusion:

In this article, we have explained a very important consistency principle to Degree of Sub matrix hood of the fuzzy soft matrix. In this paper the basic concept of a vague soft matrix is recalled. We have introduced the concept of sub matrix hood of the soft fuzzy matrix as an extension to the fuzzy soft matrix. The basic properties on soft fuzzy matrix are also presented. The null, union, intersection, sub matrix and sub matrix hood as far as future direction are concerned. It is hoped that our findings will help enhancing this study on fuzzy soft matrices for the researchers.

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