On Hilbert-Schmidt Tuples of Commutative Bounded Linear Operators on Separable Banach Spaces

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Abstract- In this paper, we will study some properties of Tuples that there components are commutative bounded linear operators on a separable Hilbert space H, then we will develop those properties for infinity-Tuples and find some conditions for them to be Hilbert-Schmidt infinity tuple. The infinity-Tuples $T=(T_1,T_2,T_3,...)$ is called Hilbert-Schmidt infinity tuple if for every orthonormal basis $\{\mu_i\}$ and $\{\lambda_i\}$ in H we have had

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left|\left(T_1T_2T_3...\lambda_i,\mu_j\right)\right|^2<\infty$$

Calculation of $T_1T_2T_3...\lambda_i$ doing by supreme over i for i=1, 2, 3...

Keywords: Hypercyclic vector, Hypercyclicity Criterion, Hilbert-Schmidt, Infinity -tuple, Periodic point.

I. INTRODUCTION

Let B be a Banach space and $T_1,T_2,T_3,...$ are commutative bounded linear mapping on B, the infinity-tuple T is an infinity components $T=(T_1,T_2,T_3,...)$ for each $x\in B$ defined

$$T=T_1T_2T_3...(x)=$$
$$Sup_n\{T_1T_2T_3...T_n(x)|n\in \mathbb{N}, n=1,2,3,...\}$$

The set

$$\Omega=\{T_1^{k_1}T_2^{k_2}T_3^{k_3}...|k_i\geq 0, i=1,2,3,...\}$$

is the semi group generated by components of T, for $x\in B$ take

$$Orb(T,x)=\{Sx:S\in\Omega\}=$$
$$\{T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...(x):k_{i,j}\geq 0, i=1,2,3,..., j=1,2,3,...\}$$

The set $Orb(T,x)$ is called orbit of vector x under infinity tuple $T=(T_1,T_2,T_3,...)$.

Definition 1.1

Infinity-Tuple $T=(T_1,T_2,T_3,...)$ is said to be hypercyclic infinity-tuple if $Orb(T,x)$ is dense in B, that is

$$\overline{Orb(T,x)} = \overline{\{Sx : S \in \Omega\}} = \overline{\{T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \ldots (x) : k_{i,j} \geq 0, i = 1,2,3,\ldots, j = 1,2,3,\ldots\}} = B$$

The first example of a hypercyclic operator on a Hilbert space was constructed by Rolewicz in 1969 ([11]). He showed that if B is the backward shift on $\ell^2(N)$, then λB is hypercyclic if and only if $|\lambda| > 1$. Let H be a Hilbert space of functions analytic on a plane domain G such that for each $\lambda \in G$ the linear functional of evaluation at λ given by

$$f \to f(\lambda)$$

is a bounded linear functional on H. By the Riesz representation theorem there is a vector $K_\lambda \in H$ such that

$$f(\lambda) = \langle f, K_\lambda \rangle.$$

The vector $K_\lambda \in H$ is called the reproducing kernel at $\lambda \in G$.

Definition 1.2

Strictly increasing sequence of positive integers $\{m_k\}_{k=1}^{\infty}$ is said to be Syndetic sequence, if

$$Sup_n (m_{n+1} - m_n) < \infty .$$

An infinity tuple $T = (T_1, T_2, T_3, \ldots)$ of operators

$$T_1, T_2, T_3, \ldots$$

on Banach space B is called weakly mixing if for any pair of non-empty open subsets U and V of B and any Syndetic sequences

$$\{m_{k,1}\}_{k=1}^{\infty}, \{m_{k,2}\}_{k=1}^{\infty}, \{m_{k,3}\}_{k=1}^{\infty}, \ldots$$

With

$$Sup_n (m_{n+1,j} - m_{n,j}) < \infty, j = 1,2,3,\ldots$$

there exist

$$m_{k,1}, m_{k,2}, m_{k,3}, \ldots$$

such that

$$T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \ldots (U) \cap V \neq \phi$$

An infinity-tuple $T = (T_1 T_2 T_3 \ldots)$ is called topologically mixing if for any given open subsets U and V subsets of B, there exist positive numbers $K_i, i = 1,2,3,\ldots$, such that

$$T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \ldots (U) \cap V \neq \phi$$

for all $k_{i,j} \geq K_i$ where $j = 1,2,3,\ldots$.

A sequence of operators $\{T_n\}_{n=1}^{\infty}$ is said to be a hypercyclic sequence on B, if there exists $x \in B$ such that its orbit under this sequence is dense in B, that is

$$\overline{Orb(\{T_n\}_{n=1}^{\infty}, x)} = \overline{Orb(x, T_1 x, T_2 x, T_3 x, \ldots)} = B .$$

In this case the vector $x \in B$ is called hypercyclic vector for the sequence $\{T_n\}_{n=1}^{\infty}$.

Definition 1.3

Let $T=(T_1T_2T_3...)$ be an infinity tuple of commutative bounded linear mapping c on a separable Hilbert space H, also let $\{\alpha_k\}_{k=1}^{\infty}$ and $\{\beta_j\}_{j=1}^{\infty}$ be orthonormal basis for H, the infinity-tuple $T=(T_1T_2T_3...)$ is said to be Hilbert-Schmidt infinity Tuple, if we have

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left|\left(T_1T_2T_3...\alpha_i,\beta_j\right)\right|^2<\infty$$

Note that all operators in this paper are commutative operator defined on a separable Hilbert space, reader can see [1]-[15] for more information.

I. MAIN RESULT

Theorem 2.1

[The Hypercyclicity Criterion for n-Tuples]

Let B be a separable Banach space and $T=(T_1,T_2,T_3,...,T_n)$ is a tuple of commutative continuous linear mappings on B. If there exist two dense subsets Y and Z in B and strictly increasing sequences

$$\{m_{k,1}\}_{k=1}^{\infty},\ \{m_{k,2}\}_{k=1}^{\infty},\ \{m_{k,3}\}_{k=1}^{\infty},\ ...,\ \{m_{k,n}\}_{k=1}^{\infty}$$

such that

1. $T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...T_n^{k_{n,j}}\to 0$ on Y as $m_{i,j}\to\infty$ for $i=1,2,3,...,n$,
2. There exist function $\{S_k|S_k:Z\to B\}$ such that for every $z\in Z$, $S_kz\to 0$ and $T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...T_n^{k_{n,j}}S_kz\to z$

Then $T=(T_1,T_2,T_3,...,T_n)$ is a hypercyclic tuple.

Note that, if the tuple $T=(T_1,T_2,T_3,...,T_n)$ satisfying the hypothesis of previous theorem, then we say that T satisfying the hypothesis of Hypercyclicity Criterion.

Theorem 2.2

[The Hypercyclicity Criterion for infinity-Tuples]

Let B be a separable Banach space and $T=(T_1,T_2,T_3,...)$ is an infinity tuple of commutative continuous linear mappings on B. If there exist two dense subsets Y and Z in B and strictly increasing sequences

$$\{m_{k,1}\}_{k=1}^{\infty},\ \{m_{k,2}\}_{k=1}^{\infty},\ \{m_{k,3}\}_{k=1}^{\infty},\ ...$$

Such that:

1. $T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...\to 0$ on Y as $m_{i,j}\to\infty$ for $i=1,2,3,...,$
2. There exist function $\{S_k|S_k:Z\to B\}$ such that for every $z\in Z$, $S_kz\to 0$ and $T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...S_kz\to z$

Then $T=(T_1,T_2,T_3,...,T_n)$ is a hypercyclic tuple.

Theorem 2.3

Suppose X be an F-sequence space with the unconditional basis $\{e_k\}_{k=1}^{\infty}$ and $T_1,T_2,T_3,...$ are unilateral weighted backward shifts with weight sequences

$$\{a_{k,1}\}_{k=1}^{\infty},\ \{a_{k,2}\}_{k=1}^{\infty},\ \{a_{k,3}\}_{k=1}^{\infty},\ \ldots$$

also $T=(T_1,T_2,T_3,...)$ be an infinity tuple of those operators , then the following assertions are equivalent:

1. Infinity tuple T is chaotic.
2. Infinity tuple T is Hypercyclic and has a non-trivial periodic point.
3. Infinity tuple T has a non-trivial periodic point.
4. The series $\sum_{m=1}^{\infty}\left(\prod_{k=1}^{m}\left(\frac{e_m}{a_{k,i}}\right)\right)$ convergence in X for $i=1,2,3,...$.

Proof:

Proof of the cases $(1)\to(2)$ and $(2)\to(3)$ are trivial. For $(3)\to(4)$, suppose that T has a non-trivial periodic point, and

$$x=\{x_n\}\in X$$

be a non-trivial periodic point for T, that is there are

$$\mu_1,\mu_2,\mu_3,...\in N$$

such that

$$T_1^{\mu_1}T_2^{\mu_2}T_3^{\mu_3}...(x)=x.$$

Comparing the entries at positions $i+kM_\lambda$ for

$$\lambda=1,2,3,...,n,k\in N\cup\{0\}$$

of x and

$$T_1^{\mu_1}T_2^{\mu_2}T_3^{\mu_3}...T_n^{\mu_n}(x)=x$$

we find that

$$x_{j+kM_\lambda}=\left(\prod_{t=1}^{M_\lambda}\left(a_{j+kN+t}\right)\right)x_{j+(k+1)}$$
$$\lambda=1,2,3,...$$

so that we have,

$$x_{j+kM_\lambda}=\left(\prod_{t=j+1}^{j+kM_\lambda}(a_t)\right)^{-1}x_j=c_\lambda\left(\prod_{t=1}^{j+kM_\lambda}(a_t)\right)^{-1}$$
$$k\in N\cup\{0\},\lambda=1,2,3,...$$

with

$$c_\lambda=\left(\prod_{t=1}^{j}\left(m_{j,\lambda}\right)\right)x_j,\ j=1,2,3,...$$

Since $\{e_k\}_{k=1}^{\infty}$ is an unconditional basis and $x=\{x_n\}\in X$ it follows that

$$\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{j+kM_\lambda}\left(a_{j,\lambda}\right)}.e_{j+kM_\lambda}\right)=$$
$$\frac{1}{c_\lambda}\left(\sum_{k=0}^{\infty}\left(x_{j+kM\lambda}.e_{j+kM_\lambda}\right)\right)$$
$$\lambda=1,2,3,...$$

convergence in X. Without loss of generality, assume that $j \geq N$, applying the operators

$$T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...(x)$$

for $j=1,2,3,...Q-1$, with

$$Q=Min\{M_\lambda : \lambda=1,2,3,...\}$$

to this series and note that

$$T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}...(e_n)=a_j e_{n-1}$$

For $n \geq 2$ and $j=1,2,3,...$, we deduce that

$$\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{j+kM_\lambda-\upsilon_\lambda}\left(m_{j,\lambda}\right)}.e_{j+kM_\lambda-\upsilon_\lambda}\right)$$
$$\lambda=1,2,3,...$$

convergence in X. By adding these series, we see that condition (4) holds.

Proof of $(4)\rightarrow(1)$. It follows from theorem 2.1 that under condition (4) the operator T is Hypercyclic. Hence it remains to show that T has a dense set of periodic points. Since $\{e_k\}_{k=1}^{\infty}$ is an unconditional basis, condition (4) implies that for each $j \in N$ and $M,N \in \mathbb{N}$ the series

$$\psi(j,M_\lambda)=\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{k=1}^{j+kM_\lambda}\left(m_{j,\lambda}\right)}.e_{j+kM_\lambda}\right)=$$
$$\left(\prod_{k=1}^{j}\left(m_{k,\lambda}\right)\right)\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{k=1}^{j+kM_\lambda}\left(m_{k,\lambda}\right)}.e_{j+kM_\lambda}\right)$$

converges for $\lambda=1,2,3,...$ and define infinity elements in X. Moreover, if $M \geq i$ then

$$T_1^{k_{j_\lambda,1}}T_2^{k_{j_\lambda,2}}T_3^{k_{j_\lambda,3}}...=\psi_\lambda\left(j_\lambda,M_\lambda\right)$$

for $\lambda=1,2,3,...$, and

$$T_1^{k_{j_\lambda,1}}T_2^{k_{j_\lambda,2}}T_3^{k_{j_\lambda,3}}...=$$
$$\psi_\lambda\left(j_\lambda,M_\lambda\right)T_1^{M_1}T_2^{M_2}T_3^{M_3}...\psi_\lambda\left(j_\lambda,M_\lambda\right)$$

so

$$T_1^{m_{j_\lambda,1}}T_2^{m_{j_\lambda,2}}T_3^{m_{j_\lambda,3}}...\omega((i,j),N)=\omega((i,j),N)$$

Also, if $N \geq j_i$ then

$$T_1^{m_{j,1}}T_2^{m_{j,2}}T_3^{m_{j,3}}...\omega((i,j),N)=\omega((i,j),N)$$

for $m_{j,i} \geq N$ and $i=1,2,3,...$.

So that each $\psi(j,N)$ and $j \leq N$ is a periodic point for T. We shall show that T has a dense set of periodic points. Since $\{e_k\}$ is a basis, it suffices to show that for every element

$$x \in Span\{e_k : k \in \mathbb{N}\}$$

there is a periodic point y arbitrarily close to it. For this, let

$$x = \sum_{j=1}^{m} e_j x_j, \varepsilon > 0.$$

Without lost of generality, for $\lambda = 1,2,3,...$ we can assume that

$$\left| x_j . \prod_{t=1}^{i} a_{t,\lambda} \right| \le 1, i = 1,2,3,...,m_\lambda$$

Since $\{e_k\}_{n=1}^{\infty}$ is an unconditional basis, then condition (4) implies that there are $M, N \ge m_\lambda$ such that

$$\left\| \sum_{k=M_\lambda+1}^{\infty} \left(\varepsilon_{k,i} \frac{1}{\prod_{k=1}^{k} (a_{j,\lambda})} . e_k \right) \right\| < \frac{\varepsilon}{m_\lambda}$$
$$\lambda = 1,2,3,...$$

for every sequence $\{\varepsilon_{k,i}\}, i = 1,2,3,...$ taking values 0 or 1. By (1) and (2) the infinity elements

$$y_t = \sum_{i=1}^{m_t} x_i . \psi(i, M_\varphi)$$
$$\varphi = 1,2,3,...$$

of X is a periodic point for T and we have

$$\|y_\lambda - x\| = \left\| \sum_{i=1}^{m_\lambda} \left(x_i . \psi(i, M_\varphi) - e_i \right)_i \right\|$$

$$= \left\| \sum_{i=1}^{m_\lambda} \left(x_i . \prod_{t=1}^{i} d_t, M_\lambda \right) \sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda \right) \right\|$$

$$\le \sum_{i=1}^{m_\lambda} \left\| \left(x_i . \prod_{t=1}^{i} d_t, M_\lambda \right) \left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda \right) \right) \right\|$$

$$\le \sum_{i=1}^{m_\lambda} \left\| \left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda \right) \right) \right\|$$

$$\le \varepsilon$$

As $\lambda = 1,2,3,...$, so by this, the proof is complete. □

Theorem 2.4

Let X be a topological vector space and $T_1, T_2, T_3,...$ are commutative bounded linear mapping on X, and $T = (T_1, T_2, T_3,...)$ be an infinity tuple of those operators. The following conditions are equivalent:

I. Infinity tuple T is weakly mixing.
II. For any pair of non-empty open subsets U and V in X, and for any Syndetic sequences

$$\{m_{k,1}\}_{k=1}^{\infty}, \{m_{k,2}\}_{k=1}^{\infty}, \{m_{k,3}\}_{k=1}^{\infty}, \ldots$$

there exist

$$m'_1, m'_2, m'_3, \ldots$$

such that

$$T_1^{m'_1}T_2^{m'_2}T_3^{m'_3}...(U)\cap V \neq \phi .$$

III. It suffices in II to consider only those sequences

$$\{m_{k,1}\}_{k=1}^{\infty},\ \{m_{k,2}\}_{k=1}^{\infty},\ \{m_{k,3}\}_{k=1}^{\infty},\ \ldots$$

for which there is some

$$m_1 \geq 1,\ m_2 \geq 1,\ m_3 \geq 1,\ \ldots$$

with

$$m_{k,j} \in \{m_j, 2m_j\}$$

for all $k, j \geq 1$.

Proof $(I \to II)$, Given

$$\{m_{k,1}\}_{k=1}^{\infty},\ \{m_{k,2}\}_{k=1}^{\infty},\ \{m_{k,3}\}_{k=1}^{\infty},\ \ldots$$

and U and V satisfying the hypothesis of condition II, take

$$m_j = Sup_k\{m_{k+1,j} - m_{k,j} : k = 1,2,3,...\}$$

for all j and since infinity product map

$$\overbrace{T \times T \times T \times ... \times T}^{n-times} :$$
$$\overbrace{X \times X \times X \times ... \times X}^{n-times} \to \overbrace{X \times X \times X \times ... \times X}^{n-times}$$

as $n \to \infty$, is transitive, then there is

$$m_{k',1},\ m_{k',2},\ m_{k',3},\ \ldots$$

in $\mathbb{N}$, such that

$$\left(T_1^{m_{k',1}}T_2^{m_{k',2}}T_3^{m_{k',3}}...(U)\right)\cap$$
$$\left(\left(T_1^{m_{k'',1}}\right)^{-1}\left(T_2^{m_{k'',2}}\right)^{-1}\left(T_3^{m_{k'',3}}\right)^{-1}...(V)\right) \neq \phi$$

for $m_{k'',i} = 1,2,3,..., i = 1,2,3,...$ so

$$\left(T_1^{m_{k',1}+m_{k'',1}}T_2^{m_{k',2}+m_{k'',2}}T_3^{m_{k',3}+m_{k'',3}}...(U)\right)\cap(V) \neq \phi$$
$$m_{k'',i} = 1,2,3,..., i = 1,2,3,...$$

By the assumption on

$$\{m_{k,1}\}_{k=1}^{\infty},\ \{m_{k,2}\}_{k=1}^{\infty},\ \{m_{k,3}\}_{k=1}^{\infty},\ \ldots$$

for all j we have

$$\{m_{k,1} : k = 1,2,3,...\}\cap$$
$$\{n + m_1, n + m_2, n + m_3,...\} \neq \phi$$

If for all j we can select

$$m'_{k,j} \in \{m_{k,1} : k = 1,2,3,...\}\cap$$
$$\{n + m_1, n + m_2, n + m_3,...\} \neq \phi$$

then we have

$$T_1^{m'_{k,1}}T_2^{m'_{k,2}}T_3^{m'_{k,3}}...(U)\cap V \neq \phi.$$

By this the proof is completed.

The case $(II \to III)$ is trivial.

Case $(III \to I)$, Suppose that U, V_1 and V_2 are non-empty open subsets of X, then there are

$$m_{k_1,1}, m_{k_1,2},\ m_{k_1,3},\ \ldots, m_{k_1,n}, \ldots$$

in N, such that

$$T_1^{m_{k_1,1}}T_2^{m_{k_1,2}}T_3^{m_{k_1,3}}...T_3^{m_{k_1,n}}...(U)\cap V_1 \neq \phi$$

and

$$T_1^{m_{k_1,1}}T_2^{m_{k_1,2}}T_3^{m_{k_1,3}}...T_3^{m_{k_1,n}}...(U)\bigcap V_2 \neq \phi$$

This will imply that T is weakly mixing. Since (III) is satisfied, then we can take

$$m_{k_2,1}, m_{k_2,2}, m_{k_2,3}, \ldots, m_{k_2,n}, \ldots$$

in $\mathbb{N}$, such that

$$T_1^{m_{k_2,1}}T_2^{m_{k_2,2}}T_3^{m_{k_2,3}}...T_3^{m_{k_2,n}}...(U)\bigcap V_2 \neq \phi$$

By continuity, we can find $\tilde{V}_1 \subset V_1$ open and non-empty such that

$$T_1^{m'_{k,1}}T_2^{m'_{k,2}}T_3^{m'_{k,3}}...\tilde{V}_1 \subset V_2.$$

Also there exist some

$$m'_{k,1},\ m'_{k,2},\ m'_{k,3},\ \ldots,\ m'_{k,n},\ \ldots,$$

in $\mathbb{N}$, such that

$$T_1^{m'_{k,1}+\eta_1}T_2^{m'_{k,2}+\eta_2}T_3^{m'_{k,3}+\eta_3}...T_3^{m'_{k,n}+\eta_n}...U \subset \tilde{V}_2$$

Now we take

$$m_{k,j} = m'_{k,j} + \eta_j$$

for all j, indeed we find strictly increasing sequences of positive integer

$$m_{k,1},\ m_{k,2},\ m_{k,3},\ \ldots,\ m_{k,n},\ \ldots,$$

such that

$$m_{k,j} \in \{m_j, 2m_j\}$$

for all j, and

$$T_1^{m_{k,1}}T_2^{m_{k,2}}T_3^{m_{k,3}}...T_3^{m_{k,n}}...(U)\bigcap \tilde{V}_1 \neq \phi.$$

Now we have

$$(T_1^{m'_{k,1}+\eta_1}T_2^{m'_{k,2}+\eta_2}T_3^{m'_{k,3}+\eta_3}...T_3^{m'_{k,n}+\eta_n}...U)\bigcap \tilde{V}_1 \neq \phi.$$

So the set

$$\left(T_1^{m_{k,1}+\eta_1}T_2^{m_{k,2}+\eta_2}T_3^{m_{k,3}+\eta_3}...T_3^{m_{k,n}+\eta_n}...\right)\bigcap \left(T_1^{m'_{k,1}}T_2^{m'_{k,2}}T_3^{m'_{k,3}}...T_3^{m'_{k,n}}...\right)(U)\bigcap \tilde{V}_1 \neq \phi.$$

is a subset of

$$\left(T_1^{m'_{k,1}+\eta_1}T_2^{m'_{k,2}+\eta_2}T_3^{m'_{k,3}+\eta_3}...T_3^{m'_{k,n}+\eta_n}...(U)\right)\bigcap \left(T_1^{m_{k,1}}T_2^{m_{k,2}}T_3^{m_{k,3}}...T_3^{m_{k,n}}...(\tilde{V}_1)\right).$$

Then we have

$$T_1^{m_{k,1}}T_2^{m_{k,2}}T_3^{m_{k,3}}...T_3^{m_{k,n}}...(U)\bigcap \tilde{V}_1 \neq \phi$$

Also by similarly method we conclude that

$$T_1^{m_{k,1}}T_2^{m_{k,2}}T_3^{m_{k,3}}...T_3^{m_{k,n}}...(U)\bigcap \tilde{V}_2 \neq \phi.$$

This is the end of proof. □

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