Soft Ideals of a Soft Lattice

E.K.R.Nagarajan [1] and P.Geetha [2]

[1] *Department of Mathematics,*
Pandian Saraswathi Yadav Engineering College,
Arasanoor,
Madurai - 625 021,
Tamil Nadu, INDIA.

[2] *Department of Mathematics,*
V.V.Vanniaperumal College for Women,
Virudhunagar-626 001,
Tamil Nadu, INDIA.

***Abstract*- Soft set theory was introduced by Molodtsov in 1999 as a mathematical tool for dealing with problems that contain uncertainty. Faruk Karaaslan et al.[6] defined the concept of soft lattices, modular soft lattices and distributive soft lattices over a collection of soft sets. In this paper, we define the concept of soft ideals and soft filters over a collection of soft sets, study their related properties and illustrate them with some examples. We also define the maximum and minimum conditions in soft lattice. In addition, we characterize soft modularity and soft distributivity of soft lattices of soft ideals.**

***Key words* - Soft lattices, soft filters , distributive soft lattices, modular soft lattices, soft lattice of soft ideals.**

I. INTRODUCTION

Soft set theory was introduced by Molodtsov [9] in 1999 as a mathematical tool for dealing with uncertainty. Maji et al.[8]defined some operations on soft sets and proved related properties. Irfan Ali et al.[5] studied some new operations in soft set theory. Li [7], Nagarajan et al.[10] defined the soft lattices using soft sets. Faruk Karaaslan et al.[6] defined the concept of soft lattices over a collection of soft sets by using the operations of soft sets defined by Cagman et al.[1]. Nagarajan et al. [11] proved characterization theorems for modular and distributive soft lattices. Serife Yilmaz et al. [12] defined and discussed soft lattices (ideals,filters) using soft set theory. In this paper, we define the concept of soft ideals, soft filters, prime soft ideals, prime soft filters, principal soft ideals and principal soft filters over a collection of soft sets. We study their related properties with some examples. We define the notion of maximum and minimum conditions in soft lattice. We prove that the set of soft ideals is a soft lattice. Further, we prove that the soft lattice L is modular if and only if the soft ideal lattice $I(L)$ is modular. We also prove that the soft lattice L is distributive if and only if the soft ideal lattice $I(L)$ is distributive.

The readers are asked to refer [1,8,9] for basic definitions and results of soft set theory and [6,10,11] for results on soft lattices.

Throughout this work, U refers to the initial universe, $P(U)$ is the power set of U, E is a set of parameters and $A \subseteq E$. $S(U)$ denotes the set of all soft sets over U.

II. SOFT IDEALS AND SOFT FILTERS

In this section we introduce the concept of soft ideals and soft filters with examples. We prove that every soft ideal and soft filter of a soft lattice L is a convex soft sublattice of L and conversely. We also study about prime soft ideals and prime soft filters. Throughout this work, the soft lattice L means the soft lattice $(L,\vee,\wedge)$.

Definition 2.1 *A non - empty soft subset I of a soft lattice L is said to be a soft ideal if*

(I_1) $f_A, f_B \in I$ implies $f_A \vee f_B \in I$

(I_2) $f_A \in I$ implies $f_A \wedge f_X \in I$ for every element f_X of L or equivalently $f_A \in I$ and $f_X \leq f_A$ implies $f_X \in I$.

Definition 2.2 *A non - empty soft subset F of a* *soft lattice L is said to be a soft filter if*

(F_1) $f_A, f_B \in F$ implies $f_A \wedge f_B \in F$

(F_2) $f_A \in F$ implies $f_A \vee f_X \in F$ for every element f_X of L or equivalently $f_A \in F$ and $f_A \leq f_X$ implies $f_X \in F$.

Note 2.3 *Every soft ideal of a soft lattice L is a soft sublattice of L.*

Example 2.4 Let

$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$,

$E = \{e_1, e_2, e_3\}, P = \{e_1\}, Q = \{e_2\}, R = \{e_3\}$,

$S = \{e_1, e_2\}, T = \{e_1, e_3\}, V = \{e_2, e_3\}$,

$W = \{e_1, e_2, e_3\}$

where $P, Q, R, S, T, V, W \subseteq E$ *and* $L = \{f_\varnothing, f_P, f_Q, f_R, f_S, f_T, f_V, f_W\} \subseteq S(U)$ *with the operations* $\tilde{\cup}$ *and* $\tilde{\cap}$.

Assume that, $f_\varnothing = \varnothing$

$f_P = \{(e_1, \{u_1\})\}$

$f_Q = \{(e_2, \{u_2\})\}$

$f_R = \{(e_3, \{u_3\})\}$

$f_S = \{(e_1, \{u_1, u_4\}), (e_2, \{u_2, u_5\})\}$

$f_T = \{(e_1, \{u_1, u_6\}), (e_3, \{u_3, u_7\})\}$

$f_V = \{(e_2, \{u_2, u_8\}), (e_3, \{u_3, u_9\})\}$

$f_W = \{(e_1, \{u_1, u_4, u_6\}), (e_2, \{u_2, u_5, u_8\}), (e_3, \{u_3, u_7, u_9\})\}$

Then $(L, \tilde{\cup}, \tilde{\cap})$ is a soft lattice. The Hasse diagram of it appears in figure 1.

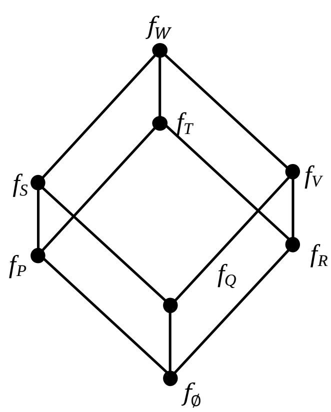

Figure 1.

(1) Consider the soft set $I = \{f_\varnothing, f_P, f_Q, f_S\} \tilde{\subseteq} L$. Clearly $I \neq \varnothing$. It also satisfies the properties I_1 and I_2. Hence I is a soft ideal of L.

(2) Consider the soft set $I = \{f_P, f_S, f_T, f_W\} \tilde{\subseteq} L$. Clearly $I \neq \varnothing$. It satisfies the property I_1 but $f_T \in I$ and $f_R \leq f_T$ implies f_R does not belong to I. Hence I is not a soft ideal of L.

(3) Consider the soft set $F = \{f_Q, f_S, f_V, f_W\} \tilde{\subseteq} L$. Clearly $F \neq \varnothing$. It also satisfies the properties F_1 and F_2. Hence F is a soft filter of L.

(4) Consider the soft set $F = \{f_\varnothing, f_P, f_R, f_T\} \tilde{\subseteq} L$. Clearly $F \neq \varnothing$. It satisfies the property F_1 but $f_P \in F$ and $f_P \leq f_S$ implies f_S does not belong to F. Hence F is not a soft filter of L.

Theorem 2.5 *Every soft ideal and soft filter of a soft lattice L is a convex soft sublattice of L. Conversely, every convex soft sublattice of L is the soft intersection of a soft ideal and a soft filter.*

Proof. Let I be an soft ideal of L. Let $f_X, f_Y \in I$. Then by (I_1), $f_X \vee f_Y \in I$, $f_X \wedge f_Y \leq f_X$, $f_X \in I$ implies $f_X \wedge f_Y \in I$. Therefore I is a soft sublattice of L. Let $f_A, f_B \in I$ and $f_A \leq f_B$. Then $(f_B] = \{f_X \in L \mid f_X \leq f_B\} \tilde{\subseteq} I. f_A \leq f_B$ implies $[f_A, f_B] \tilde{\subseteq} (f_B] \tilde{\subseteq} I$. Therefore $[f_A, f_B] \tilde{\subseteq} I$. Hence I is a convex soft sublattice of L. Similarly, $[f_A, f_B] \tilde{\subseteq} [f_A) \tilde{\subseteq} F$. Hence F is a convex soft lattice of L.

Conversely, let K be a convex soft sublattice of L. Let $I = \{f_X \in L \mid f_X \leq f_V$ for some $f_V \in K\}$. Clearly $f_\varnothing \in I$ and hence I is non - empty. Let $f_X, f_Y \in I$. Then there exist $f_{V_1}, f_{V_2} \in K$ such that $f_X \leq f_{V_1}$ and $f_Y \leq f_{V_2}$. Since $f_{V_1}, f_{V_2} \in K$, $f_{V_1} \vee f_{V_2} \in K$. Also since $f_X \vee f_Y \leq f_{V_1} \vee f_{V_2}$, $f_X \vee f_Y \in I$. Suppose $f_X \in I$ and $f_Y \leq f_X$. Then there exists $f_V \in K$ such that $f_X \leq f_V$. Since $f_Y \leq f_X, f_Y \leq f_V$. Therefore $f_Y \in I$. Hence I is a soft ideal of L. Let $F = \{f_X \in L \mid f_W \leq f_X$ for some $f_W \in K\}$. Clearly $f_\varnothing \in F$ and hence F is non - empty.

Let $f_X, f_Y \in F$. Then there exist $f_{W_1}, f_{W_2} \in K$ such that $f_{W_1} \le f_X$ and $f_{W_2} \le f_Y$. Since $f_{W_1}, f_{W_2} \in K$, $f_{W_1} \wedge f_{W_2} \in K$. Also since $f_{W_1} \wedge f_{W_2} \le f_X \wedge f_Y$, $f_X \wedge f_Y \in F$. Suppose $f_X \in F$ and $f_X \le f_Y$. Then there exists $f_W \in K$ such that $f_W \le f_X$. Since $f_X \le f_Y$, $f_W \le f_Y$. Therefore $f_Y \in F$. Hence F is a soft filter of L. Let $f_X \in K$. Then $f_X \le f_X$ for some $f_X \in K$, $f_X \in I$ and $f_X \le f_X$ for some $f_X \in K$, $f_X \in F$. Therefore $f_X \in I \tilde{\cap} F$. Hence $K \tilde{\subseteq} I \tilde{\cap} F$. Let $f_X \in I \tilde{\cap} F$. Then $f_X \in I$ and $f_X \in F$. Therefore, there exists $f_V \in K$, $f_W \in K$ such that $f_X \le f_V$ and $f_W \le f_X$. Therefore $f_W \le f_X \le f_V$ for some $f_V, f_W \in K$. Since K is a convex soft sublattice, $[f_W, f_V] \tilde{\subseteq} K$ implies $f_X \in K$. Therefore $I \tilde{\cap} F \tilde{\subseteq} K$. Hence $K = I \tilde{\cap} F$.

Definition 2.6 *A soft ideal I of the soft lattice L is said to be a prime soft ideal if and only if atleast one of an arbitrary pair of elements whose meet is in L is contained in I. That is, $f_X \wedge f_Y \in I$ implies $f_X \in I$ or $f_Y \in I$.*

Definition 2.7 *A soft filter F is said to be a prime soft filter if $f_X \vee f_Y \in F$ implies $f_X \in F$ or $f_Y \in F$.*

Definition 2.8 *Let L be a soft lattice. Let $f_A \in L$. Then $\{f_X \in L \mid f_X \le f_A\}$ is a soft ideal and is called the principal soft ideal generated by f_A and is denoted by $(f_A]$.*

Definition 2.9 *Let L be a soft lattice. Let $f_A \in L$. Then $\{f_X \in L \mid f_A \le f_X\}$ is a soft filter and is called the principal soft filter generated by f_A and is denoted by $[f_A)$.*

Example 2.10 Let

$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$,

$E = \{e_1, e_2, e_3\}, A = \{e_1\}, B = \{e_1, e_2\}$,

$C = \{e_1, e_3\}, D = \{e_1, e_2, e_3\}$

where $A, B, C, D \subseteq E$ *and* $L = \{f_\varnothing, f_A, f_B, f_C, f_D\} \subseteq S(U)$ *with the operations* $\tilde{\cup}$ *and* $\tilde{\cap}$. *Assume that,* $f_\varnothing = \varnothing$

$f_A = \{(e_1, \{u_1, u_2\})\}$

$f_B = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_5, u_6\})\}$

$f_C = \{(e_1, \{u_1, u_2, u_4\}), (e_3, \{u_7, u_8\})\}$

$f_D = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_5, u_6, u_9\}), (e_3, \{u_7, u_8, u_{10}\})\}$

Then $(L, \tilde{\cup}, \tilde{\cap})$ is a soft lattice. The Hasse diagram of it appears in figure 2.

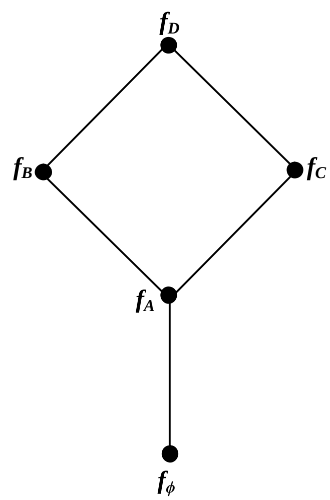

Figure 2.

(1) Consider the soft ideal $I = \{f_\varnothing, f_A, f_B\}$. Now, $f_B \wedge f_C = f_A \in I$ implies $f_B \in I$. Hence I is a prime soft ideal of L.

(2) $(f_B] = \{f_\varnothing, f_A, f_B\}$ is a principal soft ideal generated by f_B.

(3) Consider the soft ideal $I = \{f_\varnothing, f_A\}$. Let $f_B, f_C \in L$. Then, $f_B \wedge f_C = f_A \in I$. Hence I is not a prime soft ideal of L.

Example 2.11 *Let*

$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$,

$E = \{e_1, e_2, e_3\}, A = \{e_1\}, B = \{e_2\}, C = \{e_1, e_2\}$,

$D = \{e_1, e_2, e_3\}$

where $A, B, C, D \subseteq E$ *and* $L = \{f_\varnothing, f_A, f_B, f_C, f_D\} \subseteq S(U)$ *with the operations* $\tilde{\cup}$ *and* $\tilde{\cap}$. *Assume that,* $f_\varnothing = \varnothing$

$f_A = \{(e_1, \{u_1, u_2\})\}$

$f_B = \{(e_2, \{u_3, u_4\})\}$

$f_C = \{(e_1, \{u_1, u_2, u_5\}), (e_2, \{u_3, u_4, u_6\})\}$

$f_D = \{(e_1,\{u_1,u_2,u_5,u_7\}),(e_2,\{u_3,u_4,u_6,u_8\}),$
$(e_3,\{u_9,u_{10}\})\}$

Then $(L,\tilde{\cup},\tilde{\cap})$ is a soft lattice. The Hasse diagram of it appears in figure 3.

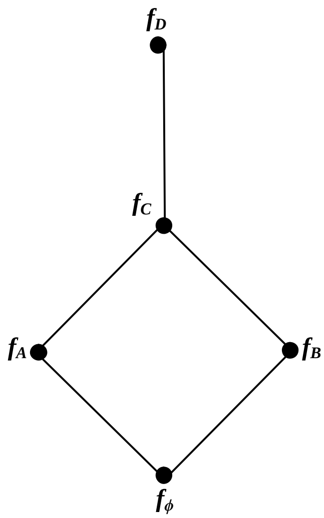

Figure 3.

(1) Consider the soft filter $F = \{f_A, f_C, f_D\}$. Now, $f_A \vee f_B = f_C \in F$ implies $f_A \in F$. Hence F is a prime soft filter.

(2) Consider the soft filter $F = \{f_C, f_D\}$. Let $f_B, f_C \in L$. Then, $f_B \vee f_A = f_C \in F$. Hence F is not a prime soft filter.

(3) $[f_A) = \{f_A, f_C, f_D\}$ is a principal soft filter generated by f_A.

Theorem 2.12 *Every soft lattice has almost one minimal and one maximal element. These elements are at the same time the least and greatest element of that soft lattice.*

Proof. If possible, let there be two minimal elements $f_M, f_N \in L$, then $f_M \wedge f_N \leq f_M$. Since f_M is a minimal element, $f_M \wedge f_N < f_M$ is impossible. Therefore $f_M \wedge f_N = f_M$ and hence $f_M \leq f_N$. Similarly we take $f_M \wedge f_N \leq f_N$, then $f_N \leq f_M$. Therefore $f_M = f_N$. Hence the soft lattice L has atmost one minimal element and it is the least element of the lattice. By the principle of duality, every soft lattice has atmost one maximal element and it is the greatest element of that soft lattice.

Definition 2.13 *An element f_A of a soft lattice L is called a greatest element of the soft lattice L if $f_X \leq f_A$ for all $f_X \in L$. Similarly an element f_A of a soft lattice L is called a least element of L if $f_A \leq f_X$ for all $f_X \in L$.*

III. THE MAXIMUM AND MINIMUM CONDITIONS

In this section, we define the maximum and minimum conditions in soft lattice. We also obtain a necessary and sufficient condition for a soft lattice to satisfy the maximum condition. we also define $\vee$ and $\wedge$ of two soft ideals and we prove that the set of all soft ideals is a soft lattice.

Definition 3.1 *Let C_0 be any element of a poset P in the soft lattice. Let us form the subchain of P in the following way: Let the greatest element of the subchain be C_0. Let $C_k (k \geq 1)$ be an element of P such that $C_k < C_{k-1}$. If each of the chains so formed, commencing at any C_0 is finite, then P is said to satisfy the maximum condition.*

Definition 3.2 *Let the least element of the subchain be C_0. Let $C_k (k \geq 1)$ be an element of P such that $C_{k-1} < C_k$. If each of the chains so formed, commencing at any C_0 is finite, then P is said to satisfy the minimum condition.*

Result 3.3 *If a poset P in a soft lattice satisfies the minimum condition then for any $f_X \in P$, there exists atleast one minimal element f_M of P such that $f_M \leq f_X$.*

Result 3.4 *If a poset P in a soft lattice satisfies the maximum condition then for any $f_X \in P$, there exists atleast one maximal element f_M of P such that $f_X \leq f_M$.*

Corollary 3.5 *Every soft lattice satisfying minimum (maximum)condition has a least (greatest) element.*

Note 3.6 *By a soft ideal chain of a soft lattice L, we shall mean a set of soft ideals in L in which one of every pair of soft ideals includes the other.*

Lemma 3.7 *The soft union of any soft ideal chain of a soft lattice L is itself a soft ideal in L.*

Proof. Let C be a chain of soft ideals of L. Let I denote the soft union of all soft ideals of L in C. Let $f_A, f_B \in I$. Then there exists soft ideals I_1 and I_2 in C such that $f_A \in I_1$ and

$f_B \in I_2$. Since $I_1, I_2 \in C$, either $I_1 \tilde{\subseteq} I_2$ or $I_2 \tilde{\subseteq} I_1$. Let $I_1 \tilde{\subseteq} I_2$. Then $f_A \in I_1 \tilde{\subseteq} I_2$. Therefore $f_A \in I_2$. Since $f_A, f_B \in I_2, f_A \vee f_B \in I_2$. Hence $f_A \vee f_B \in I$. Let $f_X \in L$. Then $f_A \wedge f_X \in I_1 \tilde{\subseteq} I$. Therefore $f_A \wedge f_X \in I$. Hence I is a soft ideal.

Theorem 3.8 *A necessary and sufficient condition for a soft ideal I in a soft lattice L to be a principal soft ideal is that the soft lattice L satisfies the maximum condition.*

Proof. Suppose the soft lattice L satisfies the maximum condition. Then it is also satisfied in every soft ideal I of L. By corollary 3.5 the soft ideal I includes a greatest element f_A. Then $I = (f_A]$. Hence every soft ideal of L is a principal soft ideal.Conversely, suppose that every soft ideal is a principal soft ideal. We have to prove soft lattice satisfies the maximum condition. Suppose not, then we can find an infinite subchain of the form $C = C_0 < C_1 < \ldots$. The set $I = \tilde{\cup}_{n=0}^{\infty}(C_n]$ being the soft union of the elements of the soft ideal chain is itself a soft ideal by lemma 3.7. Hence I cannot be a principal soft ideal since every one of its elements is less than the other of its elements.Therefore I has no greatest element which is a contradiction.

Theorem 3.9 *Let I and J be soft ideals of a soft lattice L. Define $I \wedge J = \{f_X \in L \mid f_X \in I \tilde{\cap} J\}$ and $I \vee J = \{f_X \in L \mid f_X \le f_A \vee f_B, f_A \in I, f_B \in J\}$ Then the set of all soft ideals $I(L)$ is a soft lattice.*

Proof. Clearly $I \wedge J \ne \varnothing$ for $f_\varnothing \in I$ and $f_\varnothing \in J$. Let $f_A, f_B \in I \wedge J$. Then, $f_A \in I \tilde{\cap} J$ and $f_B \in I \tilde{\cap} J$. That is, $f_A \in I$ and $f_A \in J$. Also, $f_B \in I$ and $f_B \in J$. Therefore, $f_A \vee f_B \in I$ and $f_A \vee f_B \in J$. Hence $f_A \vee f_B \in I \tilde{\cap} J$ and hence $f_A \vee f_B \in I \wedge J$. Let $f_A \in I \wedge J$ and $f_X \le f_A$. Then $f_A \in I$ and $f_A \in J$. Since $f_X \le f_A \Rightarrow f_X \in I$ and $f_X \in J \Rightarrow f_X \in I \wedge J$. Therefore $I \wedge J$ is a soft ideal .Next we prove that $I \vee J$ is a soft ideal.

Clearly $I \vee J \ne \varnothing$ for if $f_\varnothing \in I$ and so $f_\varnothing \in I \vee J$. Let $f_X, f_Y \in I \vee J$. Then $f_X \le f_{A_1} \vee f_{B_1}, f_Y \le f_{A_2} \vee f_{B_2}$ where $f_{A_1}, f_{A_2} \in I$ and $f_{B_1}, f_{B_2} \in J$. Therefore, $f_X \vee f_Y \le (f_{A_1} \vee f_{B_1}) \vee (f_{A_2} \vee f_{B_2}) = (f_{A_1} \vee f_{A_2}) \vee (f_{B_1} \vee f_{B_2})$. Since $f_{A_1}, f_{A_2} \in I, f_{A_1} \vee f_{A_2} \in I$. Since $f_{B_1}, f_{B_2} \in J, f_{B_1} \vee f_{B_2} \in J$. Hence $f_X \vee f_Y \in I \vee J$. Suppose $f_X \in I \vee J$ and $f_Y \le f_X$. Then $f_X \le f_A \vee f_B$ where $f_A \in I$ and $f_B \in J$. Since $f_Y \le f_X \le f_A \vee f_B, f_Y \le f_A \vee f_B$. Thus, $f_Y \in I \vee J$. Hence $I \vee J$ is a soft ideal. Therefore $I(L)$ is a soft lattice.

Theorem 3.10 *The set $I_0(L)$ of all principal soft ideals of a soft lattice L is a soft sublattice of $I(L)$, soft isomorphic to L.*

Proof. We claim that $(f_A] \vee (f_B] = (f_A \vee f_B]$ and $(f_A] \wedge (f_B] = (f_A \wedge f_B]$ holds for every pair of elements f_A, f_B of L. First to prove $(f_A] \vee (f_B] = (f_A \vee f_B]$. Let $f_X \in (f_A] \vee (f_B] = (f_A] \tilde{\cup} (f_B] \Rightarrow f_X \in (f_A]$ or $f_X \in (f_B] \Rightarrow f_X \le f_A$ or $f_X \le f_B \Rightarrow f_X \le f_A \vee f_B \Rightarrow f_X \in (f_A \vee f_B]$. Thus $(f_A] \vee (f_B] \tilde{\subseteq} (f_A \vee f_B]$. Let $f_X \in (f_A \vee f_B]$. Then $f_X \le f_A \vee f_B$ where $f_A \in (f_A], f_B \in (f_B]$ (Since $f_X \le f_A$ and $f_A \in (f_A], f_X \in (f_A]$ and since $f_X \le f_B$ and $f_B \in (f_B], f_X \in (f_B]$.) $\Rightarrow f_X \in (f_A] \vee (f_B]$. Hence $(f_A \vee f_B] \tilde{\subseteq} (f_A] \vee (f_B]$. Thus $(f_A] \vee (f_B] = (f_A \vee f_B]$. Next to prove $(f_A] \wedge (f_B] = (f_A \wedge f_B]$. Let $f_X \in (f_A] \wedge (f_B] = (f_A] \tilde{\cap} (f_B] \Rightarrow f_X \in (f_A]$ and $f_X \in (f_B] \Rightarrow f_X \le f_A$ and $f_X \le f_B \Rightarrow f_X \le f_A \wedge f_B \Rightarrow f_X \in (f_A \wedge f_B]$. Therefore $(f_A] \wedge (f_B] \tilde{\subseteq} (f_A \wedge f_B]$. Let $f_X \in (f_A \wedge f_B]$. Then $f_X \le f_A \wedge f_B \le f_A$ and $f_X \le f_A \wedge f_B \le f_B \Rightarrow f_X \le f_A$ and $f_X \le f_B \Rightarrow f_X \in (f_A]$ and

Therefore $(f_A \wedge f_B] \tilde{\subseteq} (f_A] \wedge (f_B]$. Hence $(f_A] \wedge (f_B] = (f_A \wedge f_B]$. Let us define $\eta : L \to I_0(L)$ by $\eta(f_X) = (f_X]$. Suppose $\eta(f_X) = \eta(f_Y)$. Then $(f_X] = (f_Y]$. Since $f_X \in (f_X] = (f_Y] \Rightarrow f_X \in (f_Y] \Rightarrow f_X \le f_Y$ and since $f_Y \in (f_Y] = (f_X] \Rightarrow f_Y \in (f_X] \Rightarrow f_Y \le f_X$.

Therefore $f_X = f_Y$ and hence η is one-one. For every $(f_X] \in I_0(L)$, there exist an element $f_X \in L$ such that $\eta(f_X) = (f_X]$. Therefore η is onto.To prove η is a soft lattice homomorphism.

$$\eta(f_X \vee f_Y) = (f_X \vee f_Y] = (f_X] \vee (f_Y]$$
$$= \eta(f_X) \vee \eta(f_Y) \text{ and}$$
$$\eta(f_X \wedge f_Y) = (f_X \wedge f_Y] = (f_X] \wedge (f_Y]$$
$$= \eta(f_X) \wedge \eta(f_Y).$$

Therefore η is a soft homomorphism.

Hence the map $\eta : L \to I_0(L)$ is a soft isomorphism and $L \cong I_0(L) \tilde{\subseteq} I(L)$.

Theorem 3.11 *The soft lattice L is modular if and only if the soft ideal lattice $I(L)$ is modular.*

Proof. Suppose $I(L)$ is a modular soft lattice. Then the set of all principal soft ideals $I_0(L)$ of a soft lattice L is a soft sublattice of $I(L)$ and is soft isomorphic to L. That is $L \cong I_0(L) \tilde{\subseteq} I(L)$.$I(L)$ is a modular soft lattice implies its soft sublattice $I_0(L)$ is a modular soft lattice. $I_0(L)$ is a modular soft lattice implies its soft isomorphic copy L is a modular soft lattice. Hence $I(L)$ is a modular soft lattice implies that L is a modular soft lattice.Conversely, let L be a modular soft lattice. To prove that $I(L)$ is a modular soft lattice.Let I, J, K be soft ideals of L such that $I \le K$. Clearly $I \vee (J \wedge K) \le (I \vee J) \wedge K$. It is enough to prove that $(I \vee J) \wedge K \le I \vee (J \wedge K)$. Let $f_X \in (I \vee J) \wedge K$. Then $f_X \in I \vee J$ and $f_X \in K$. Since $f_X \in I \vee J, f_X \le f_A \vee f_B$ where $f_A \in I, f_B \in J$. Since $f_A \in I$ and $I \le K, f_A \in K$. Therefore $f_A \vee f_X \in K$. Let $f_C = f_A \vee f_X$. Then $f_C \in K$. Also $f_X \le f_A \vee f_X$. Therefore $f_X \le (f_A \vee f_B) \wedge (f_A \vee f_X) \le (f_A \vee f_B) \wedge f_C$. Since L is a modular soft lattice, $f_A \le f_C \Rightarrow (f_A \vee f_B) \wedge f_C = f_A \vee (f_B \wedge f_C)$. Therefore $f_X \le f_A \vee (f_B \wedge f_C)$. Since $f_B \in J$ and $f_C \in K, f_B \wedge f_C \in J \wedge K$. Therefore $f_X \le f_A \vee (f_B \wedge f_C)$ where $f_A \in I, f_B \wedge f_C \in J \wedge K$. Thus $f_X \in I \vee (J \wedge K)$. Therefore $(I \vee J) \wedge K \le I \vee (J \wedge K)$. Hence $I(L)$ is a modular soft lattice.

Theorem 3.12 *The soft lattice L is distributive if and only if the soft ideal lattice $I(L)$ is distributive.*

Proof. Suppose $I(L)$ is a distributive soft lattice. Then the set of all principal soft ideals $I_0(L)$ is a soft sublattice of $I(L)$ and is soft isomorphic to L. That is $L \cong I_0(L) \tilde{\subseteq} I(L)$.$I(L)$ is a distributive soft lattice implies its soft sublattice $I_0(L)$ is a distributive soft lattice. $I_0(L)$ is a distributive soft lattice implies its soft isomorphic copy L is a distributive soft lattice. Hence $I(L)$ is a distributive soft lattice implies that L is a distributive soft lattice.

Conversely, let L be a distributive soft lattice. To prove that $I(L)$ is a distributive soft lattice.Let I, J, K be a soft ideals of L. Clearly $(I \wedge J) \vee (I \wedge K) \le I \wedge (J \vee K)$. It is enough to prove $I \wedge (J \vee K) \le (I \wedge J) \vee (I \wedge K)$. Let $f_X \in I \wedge (J \vee K)$. Then $f_X \in I$ and $f_X \in J \vee K \Rightarrow f_X \in I$ and $f_X = f_B \vee f_C$ where $f_B \in J, f_C \in K$. Now

$$f_X = f_X \wedge f_X = f_X \wedge (f_B \vee f_C)$$
$$= (f_X \wedge f_B) \vee (f_X \wedge f_C)$$

(Since L is a distributive soft lattice). Since $f_X \in I, f_B \in J, f_X \wedge f_B \in I \wedge J$ and since $f_X \in I, f_C \in K, f_X \wedge f_C \in I \wedge K$. Therefore

$$(f_X \wedge f_B) \vee (f_X \wedge f_C) \in (I \wedge J) \vee (I \wedge K)$$

and hence $f_X \in (I \wedge J) \vee (I \wedge K)$. Thus $I \wedge (J \vee K) \leq (I \wedge J) \vee (I \wedge K)$. Hence $I(L)$ is a distributive soft lattice.

Example 3.13 *Consider the soft lattice* $L = \{f_\varnothing, f_A, f_B, f_C, f_D\} \subseteq S(U)$ *under the operations* $\tilde{\cup}$ *and* $\tilde{\cap}$ *whose Hasse diagram is given in figure 4.*

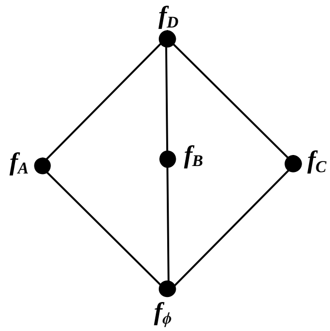

Figure 4.

which is a modular soft lattice. The soft ideal lattice $I(L)$ of L is given in figure 5.

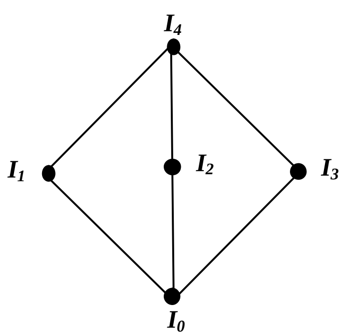

Figure 5.

Now $I(L) = \{I_0, I_1, I_2, I_3, I_4\}$ where
$I_0 = \{f_\varnothing\} = (f_\varnothing], I_1 = \{f_\varnothing, f_A\} = (f_A],$
$I_2 = \{f_\varnothing, f_B\} = (f_B], I_3 = \{f_\varnothing, f_C\} = (f_C],$
$I_4 = \{f_\varnothing, f_A, f_B, f_C, f_D\} = (f_D]$

which is also a modular soft lattice under the operations $\tilde{\cup}$ and $\tilde{\cap}$.

Example 4.14 *Consider the soft lattice* $L = \{f_\varnothing, f_A, f_B, f_C, f_D\} \subseteq S(U)$ *under the operations* $\tilde{\cup}$ *and* $\tilde{\cap}$ *whose Hasse diagram is given in figure 6.*

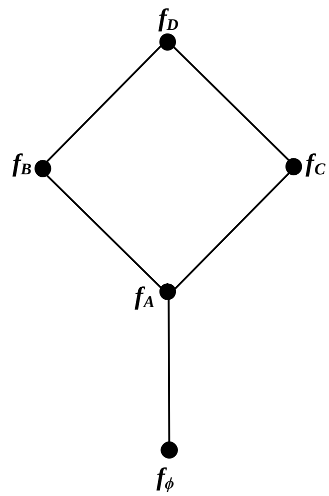

Figure 6.

which is a distributive soft lattice. The soft ideal lattice $I(L)$ of L is given in figure 7.

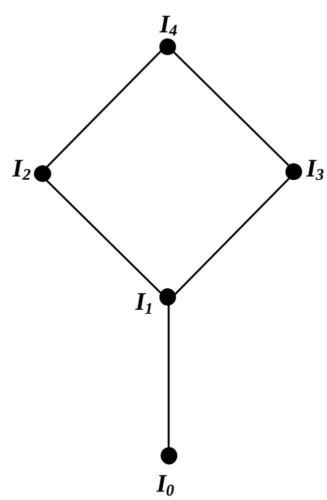

Figure 7.

Now $I(L) = \{I_0, I_1, I_2, I_3, I_4\}$ where
$I_0 = \{f_\varnothing\} = (f_\varnothing], I_1 = \{f_\varnothing, f_A\} = (f_A],$
$I_2 = \{f_\varnothing, f_A, f_B\} = (f_B], I_3 = \{f_\varnothing, f_A, f_C\} = (f_C],$
$I_4 = \{f_\varnothing, f_A, f_B, f_C, f_D\} = (f_D]$
which is also a distributive soft lattice under the operations $\tilde{\cup}$ and $\tilde{\cap}$.

I V. CONCLUSION

In this paper, we defined soft ideals and soft filters, discussed their properties and illustrated them with some examples. We have shown that the set of soft ideals is a soft lattice. We have established the theorem that the principal soft ideal lattice $I_0(L)$ is soft isomorphic to the soft lattice L. We proved that the soft lattice L is modular if and only if the soft ideal lattice $I(L)$ is modular. We also proved that the soft lattice L is distributive if and only if the soft ideal lattice $I(L)$ is distributive. We are studying about these soft lattices and are expected to give some more results.

REFERENCES

[1] Cagman N., and Enginoglu S., *Soft set theory and uni-int decision making*, Eur. J. Oper. Res. 207/2 (2010), 848-855.

[2] Davey B.A., and Priestley H.A. *Introduction to lattices and order*, Cambridge University Press, India, 2009.

[3] Gabor Szasz, *Introduction to Lattice Theory*, Academic Press, New York and London, 1963.

[4] George Gratzer, *General Lattice Theory*, Second edition, Birkhauser verlag, Boston, 1998.

[5] Irfan Ali M., Feng Feng, Xiaoyan Liu, Won Keun Min and Shabir M., *On some new operations in soft set theory*, Comput. Math. Appl. 57 (2009), 1547-1553.

[6] Karaaslan F., Cagman N., and Enginoglu S., *Soft Lattices*, J. New. Res. Sci. 1 (2012),5-17 .

[7] Li F., *Soft Lattices*, Glob. J. Sci. Front. Res.10/4 (2010), 56-58.

[8] Maji P.K., Biswas R., and Roy A.R. *Soft set theory*, Comput. Math. Appl. 45 (2003), 555-562.

[9] Molodtsov D.A., *Soft set theory-First Results*, Comput. Math. Appl.37 (1999), 19-31.

[10] Nagarajan E.K.R., and Meenambigai G., *An Application of Soft Sets to Soft Lattices*, Kragujevac J. Math. 35/1 (2011), 75-87.

[11] Nagarajan E.K.R., and Geetha P., *Characterization Theorems for Modular and Distributive Soft Lattices*, J. Innov. Res. in Sci. Eng. and Tech. 2/11 (2013), 6019-6031.

[12] Serife Yilmaz and osman Kazanchi., *Soft Lattices(Ideals,Filters) Related to Fuzzy Point*, U.P.B.Sci. Bull., Series A. 75/3 (2013), 75-90.