Comparison of Two Reliability Models for Two-Unit Cold Standby System with Three Accidental Effects

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ABSTRACT - Present paper investigates a two-unit cold standby system with three accidental effects. On the failure of a unit, a repairman comes to repair it. During its repair, an accident may take place which may or may not affect the repairman. If it affects, it may be minor or major. Two models have been discussed. In Model 1, it is assumed that when the repairman gets minor injury with an accident, he goes for treatment/rest for some time. After recovery, he comes back and resumes the repair. If major injury takes place, another repairman is called to complete the repair. In Model 2, it is assumed that whenever minor or major accident takes place, another repairman is called. In both the models, it is assumed that after accident the rest of the repair is done with more attention so that no further accident takes place. Various measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made.

INTRODUCTION

Almost all the authors in the literature of the reliability have discussed two-unit systems with the assumption, that no harm takes place with the repairman while repairing a failed unit. However, there may be situations when the repairman may meet with an accident while repairing a failed unit. Bhatia, Taneja and Kumar (1999) studied a two-unit system with the concept of accident wherein it is assumed that if an accident takes place, it harms the repairman in the sense that he remains no more able to repair the unit. Then another repairman is called. However, there may be situations when an accident takes place and it may or may not affect the repairman. If it affects, it may be major or minor.

The system is analysed by making use of semi-Markov process and regenerative point technique. Various measures of system effectiveness such as mean time to system failure (MTSF), steady state availability, expected busy period of the repair facility, expected number of visits by the repairmen, expected number of major/minor accidents and expected profit earned by the system are obtained. Graphs are plotted for a particular case. Comparative study between the models is also made through graphs.

NOTATIONS

O	operative unit
cs	cold standby unit
Fr	failed unit under repair before accident.
Fra	failed unit under repair after accident.
Fr_2	failed unit under repair of another repairman after the accident
Fw	failed unit waiting for repair
Fwa	failed unit waiting for repair after the occurrence of an accident
FR	failed unit under repair when repair is continuing from previous state.
FWa	failed unit waiting for repair when repair is continuing from previous state after the occurrence of an accident
FRa	failed unit under repair when repair is continuing from previous state after accident.
FR_2	failed unit under repair by another repairman whenever repair is continuing from previous state.
A	probability that a repairman completes the repair of the unit without meeting with an accident.
b_1	probability that accident takes places which does not affect the repairman i.e. = $(1-a)p_1$, where p_1 is the probability that accident does not affect the repairman.
b_2	probability that accident occurs and it is minor i.e. = $(1 – a)p_2$, where p_2 is the probability that accident is minor.
b_3	probability that accident occurs and it is major i.e. = $(1 – a)p_3$, where p_3 is the probability that accident is major
λ	failure rate of operative unit
$g_i(t)$, $G_i(t)$	p.d.f. and c.d.f. of repair time for Model i ; i = 1, 2

MODEL 1

In this model, it is assumed that when the repairman gets minor injury due to an accident, he goes for treatment/rest for some time. After recovery, he comes back and resumes the repair. If major injury takes place, another repairman is called to complete the repair. The state transition diagram for this model is shown as in Fig. 1

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The epochs of entry into states 0, 1, 2, 3, 4, 9, 10 and 11 regeneration points and thus these are regenerative states. States 5, 6, 7, 8, 9, 10 and 11 are down states.

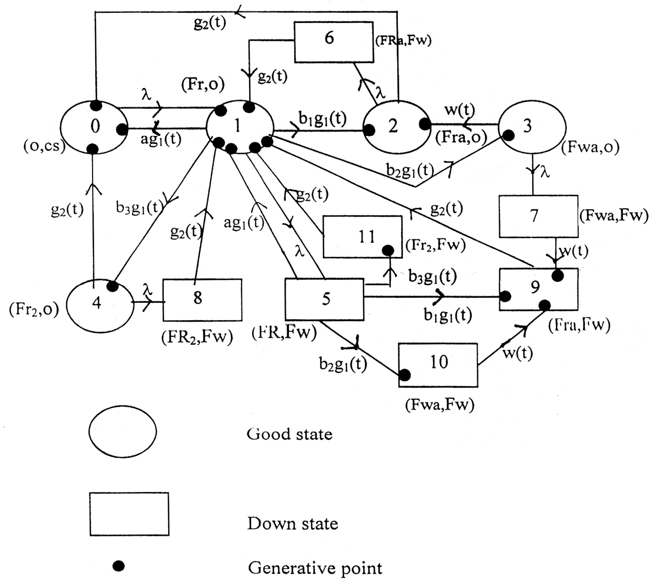

Fig. 1.1

The transition probabilities are

$$dQ_{01}(t)=\lambda e^{-\lambda t}dt$$

$$dQ_{10}(t)=a\,e^{-\lambda t}g_1(t)dt$$

$$dQ_{11}^{(5)}(t)=a\left[\lambda e^{-\lambda t}\copyright 1\right]g_1(t)dt$$

$$dQ_{12}(t)=b_1\,e^{-\lambda t}\,g_1(t)dt$$

$$dQ_{13}(t)=b_2\,e^{-\lambda t}\,g_1(t)dt$$

$$dQ_{14}(t)=b_3\,e^{-\lambda t}\,g_1(t)dt$$

$$dQ_{15}(t)=\lambda e^{-\lambda t}\,\overline{G_1(t)}dt$$

$$dQ_{19}^{(5)}(t)=b_1\left[\lambda e^{-\lambda t}\copyright 1\right]g_1(t)dt$$

$$dQ_{1,10}^{(5)}(t)=b_2\left[\lambda e^{-\lambda t}\copyright 1\right]g_1(t)dt$$

$$dQ_{1,11}^{(5)}(t)=b_3\left[\lambda e^{-\lambda t}\copyright 1\right]g_1(t)dt$$

$$dQ_{20}(t)=e^{-\lambda t}g_2(t)dt$$

$$dQ_{21}^{(6)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]g_2(t)dt$$

$$dQ_{26}(t)=\lambda e^{-\lambda t}\,\overline{G_2(t)}dt$$

$$dQ_{32}(t)=e^{-\lambda t}\,w(t)dt$$

$$dQ_{37}(t)=\lambda e^{-\lambda t}\,\overline{W(t)}dt$$

$$dQ_{39}^{(7)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]w(t)dt$$

$$dQ_{40}(t)=e^{-\lambda t}\,g_2(t)dt$$

$$dQ_{48}(t)=\lambda e^{-\lambda t}\,\overline{G_2(t)}dt$$

$$dQ_{41}^{(8)}(t)=\left[\lambda e^{-\lambda t}\copyright 1\right]g_2(t)dt$$

$$dQ_{91}(t)=g_2(t)dt$$

$$dQ_{10,9}(t)=w(t)dt$$

$$dQ_{11,1}(t)=g_2(t)dt$$

The non zero elements p_{ij} of the transition probability matrix for the system are found out as $p_{ij}=\lim_{S\to 0}q_{ij}^{*}(s)\cdot$

Mean sojourn times (μ_1) in regenerative state i are

$$\mu_0=\frac{1}{\lambda},\qquad \mu_1=\frac{1-g_1^{*}(\lambda)}{\lambda},\qquad \mu_2=\frac{1-g_2^{*}(\lambda)}{\lambda}=\mu_4$$

$$\mu_3=\frac{1-w^{*}(\lambda)}{\lambda},\qquad \mu_9=\int_0^{\infty}\overline{G_2(t)}dt=\mu_{11}$$

$$\mu_{10}=\int_0^{\infty}\overline{W(t)}dt$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into the state is mathematically stated as

$$m_{ij}=\int tdQ_{ij}(t)=-\lim_{s\to 0}q_{ij}^{*}{}'(s)$$

Thus,

$$m_{01}=\mu_0$$

$$m_{10}+m_{12}+m_{13}+m_{14}+m_{15}=\mu_1$$

$$m_{10}+m_{12}+m_{13}+m_{14}+m_{19}^{(5)}+m_{1,10}^{(5)}+m_{11}^{(5)}+m_{1,11}^{(5)}=k_1$$

$$m_{20}+m_{21}^{(6)}=k_2$$

$$m_{20}+m_{26}=\mu_2=m_{40}+m_{48}$$

$$\mu_{32}+m_{37}=\mu_3$$

$$m_{32}+m_{39}^{(7)}=k_3=m_{10,9}$$

$$m_{40}+m_{41}^{(8)}=k_2=m_{91}=m_{11,1}$$

where

$$k_1=-g_1^{*'}(0),\quad k_2=-g_2^{*'}(0),\quad k_3=-w^{*'}(0)$$

MEAN TIME TO SYSTEM FAILURE

To determine the MTSF of the system, we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$$\phi_0(t)=Q_{01}(t)(s)\phi_1(t)$$

$$\phi_1(t)=Q_{10}(t)(s)\phi_0(t)+Q_{12}(t)(s)\phi_2(t)$$
$$+Q_{13}(t)(s)\phi_3(t)+Q_{14}(t)(s)\phi_4(t)+Q_{15}(t)$$

$$\phi_2(t)=Q_{20}(t)(s)\phi_0(t)+Q_{26}(t)$$

$$\phi_3(t) = Q_{32}(t)(s)\phi_2(t) + Q_{37}(t)$$
$$\phi_4(t) = Q_{40}(t)(s)\phi_0(t) + Q_{48}(t)$$

Taking Laplace Stieltjes transform of these equations and solving them for $\phi_0^{**}(s)$, The mean time to system failure (MTSF) when the system starts from state 0 is

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 + \mu_2(p_{12} + p_{13}p_{32} + p_{14}) + \mu_3 p_{13}$$

and

$$D = 1 - p_{14}p_{40} - p_{10} - p_{12}p_{20} - p_{13}p_{20}p_{32}$$

AVAILABILITY ANALYSIS

Using the arguments of the theory of regenerative process, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t)\copyright A_1(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)\copyright A_0(t) + q_{12}(t)\copyright A_2(t)$$
$$+q_{13}(t)\copyright A_3(t) + q_{14}(t)\copyright A_4(t)$$
$$+q_{19}^{(5)}(t)\copyright A_9(t) + q_{1,10}^{(5)}(t)\copyright A_{10}(t)$$
$$+q_{1,11}^{(5)}(t)\copyright A_{11}(t) + q_{1,1}^{(5)}(t)\copyright A_1(t)$$
$$A_2(t) = M_2(t) + q_{20}(t)\copyright A_0(t) + q_{21}^{(6)}(t)\copyright A_1(t)$$
$$A_3(t) = M_3(t) + q_{32}(t)\copyright A_2(t) + q_{39}^{(7)}(t)\copyright A_9(t)$$
$$A_4(t) = M_4(t) + Q_{40}(t)\copyright A_0(t) + q_{41}^{(8)}(t)\copyright A_1(t)$$
$$A_9(t) = q_{9,1}(t)\copyright A_1(t)$$
$$A_{10}(t) = q_{10,9}(t)\copyright A_9(t)$$
$$A_{11}(t) = q_{11,1}(t)\copyright A_1(t)$$

where

$$M_0(t) = e^{-\lambda t}, M_1(t) = e^{-\lambda t}\,\overline{G_2(t)}$$
$$M_2(t) = e^{-\lambda t}\,\overline{G_2(t)},\ M_3(t) = e^{-\lambda t}\,\overline{W(t)}$$
$$M_4(t) = e^{-\lambda t}\,\overline{G_2(t)}$$

Taking Laplace transform of these equations and solving them for $A_0^*(0)$ the steady state availability of the system is given by A_0.

$$A_0 = \lim_{s\to 0} A_0^*(s) = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0\left[1 - p_{13}p_{2,1}^{(6)}p_{32} - p_{11}^{(5)} - p_{21}^{(6)}p_{12} - p_{14}p_{41}^{(8)} - p_{1,11}^{(5)}\right]$$
$$+\mu_1 + \mu_2(p_{12} + p_{14} + p_{13}p_{32}) + \mu_3 p_{13}$$

and

$$D_1 = \mu_0\left[p_{10} + p_{14}p_{40} + p_{12}p_{20} + p_{13}p_{20}p_{32}\right]$$
$$+k_1 + k_2\left(1 - p_{11}^{(5)} - p_{10}\right) + k_3\left(p_{13} + p_{1,10}^{(5)}\right)$$

The other measures of the system effectiveness have been obtained in the similar fashion;

The total fraction of time for which the system is under repair.

$$B_0 = \lim_{s\to 0}\left[s\,B_0^*(s)\right] = \frac{N_2}{D_1}$$

where

$$N_2 = k_1 + k_2\left(1 - p_{10} - p_{1,1}^{(5)}\right)$$

and D_1 is already specified.

In steady state, the number of visit per unit time is given by

$$V_0 = \frac{N_3}{D_1}$$

where $N_3 = p_{10} + p_{14} + p_{1,11}^{(5)} + p_{14}p_{40} + p_{12}p_{20} + p_{13}p_{20}p_{32}$

and D_1 is already specified.

EXPECTED NUMBER OF MAJOR ACCIDENTS

We define $MJ_i(t)$ as the expected number of major accidents in (0, t] given that the system initially starts from regenerative state i. By probabilistic arguments, we have the following recursive relations:

$$MJ_0(t) = Q_{01}(t)(s)MJ_1(t)$$
$$MJ_1(t) = Q_{10}(t)(s)MJ_0(t) + Q_{12}(t)(s)MJ_2(t)$$
$$+Q_{13}(t)(s)MJ_3(t) + Q_{14}(t)(s)\left[1 + MJ_4(t)\right]$$
$$+Q_{19}^{(5)}(t)(s)MJ_9(t) + Q_{1,10}^{(5)}(t)(s)MJ_1(t)$$
$$+Q_{1,11}^{(5)}(t)(s)\left[1 + MJ_{11}(t)\right] + Q_{1,1}^{(5)}(t)(s)MJ_1(t)$$
$$MJ_2(t) = Q_{20}(t)(s)MJ_0(t) + Q_{21}^{(6)}(t)(s)MJ_1(t)$$
$$MJ_3(t) = Q_{32}(t)(s)MJ_2(t) + Q_{39}^{(7)}(t)(s)MJ_9(t)$$
$$MJ_4(t) = Q_{40}(t)(s)MJ_0(t) + Q_{41}^{(8)}(t)(s)MJ_1(t)$$
$$MJ_9(t) = Q_{9,1}(t)(s)MJ_1(t)$$
$$MJ_{10}(t) = Q_{10,9}(t)(s)MJ_9(t)$$
$$MJ_{11}(t) = Q_{11,1}(t)(s)MJ_1(t)$$

Taking Laplace-Stieltjes transform of these equations and solving them for $M_0^{**}(s)$.

In steady state, the number of major accidents per unit time is given by

$$MJ_0 = \lim_{t\to\infty}\left[\frac{MJ_0(t)}{t}\right]$$

$$= \lim_{s \to 0}\left[s\ MJ_0^{**}(s)\right] = \frac{N_4}{D_1}$$

where

$N_4 = p_{14} + p_{1,11}^{(5)}$

and D_1 is already specified.

EXPECTED NUMBER OF MINOR ACCIDENTS

We define $MN_i(t)$ as the expected number of minor accidents in (0, t], given that the system initially starts from regenerative state i. By probabilistic arguments, we have the following recursive relations:

$$MN_0(t) = Q_{01}(t)(s)MN_1(t)$$

$$MN_1(t) = Q_{10}(t)(s)MN_0(t) + Q_{12}(t)(s)MN_2(t) + Q_{13}(t)(s)(1 + MN_3(t)) + Q_{14}(t)(s)MN_4(t) + Q_{19}^{(5)}(t)(s)MN_9(t) + Q_{1,10}^{(5)}(t)(s)(1 + MN_{10}(t)) + Q_{1,11}^{(5)}(t)(s)MN_{11}(t) + Q_{11}^{(5)}(t)(s)MN_1(t)$$

$$MN_2(t) = Q_{20}(t)(s)MN_0(t) + Q_{21}^{(6)}(t)(s)MN_1(t)$$

$$MN_3(t) = Q_{32}(t)(s)MN_2(t) + Q_{39}^{(7)}(t)(s)MN_9(t)$$

$$MN_4(t) = Q_{40}(t)(s)MN_0(t) + Q_{41}^{(8)}(t)(s)MN_1(t)$$

$$MN_9(t) = Q_{9,1}(t)(s)MN_1(t)$$

$$MN_{10}(t) = Q_{10,9}(t)(s)MN_9(t)$$

$$MN_{11}(t) = Q_{11,1}(t)(s)MN_1(t)$$

Taking Laplace-Stieltjes transform of above equations and solving them for $MN_0^{**}(s)$.

In steady state, the number of minor accidents per unit time is given by

$$MN_0 = \frac{N_5}{D_1}$$

where

$N_5 = p_{13} + p_{1,10}^{(5)}$

and D_1 is already specified.

COST-BENEFIT ANALYSIS

In Steady state, the profit is given by

$P_1 = C_0A_0 - C_1B_0 - C_2V_0 - C_3MN_0 - C_4MJ_0$

where

C_o = revenue per unit uptime of the system.

C_1 = cost per unit time for which the system is under repair

C_2 = cost per visit by the repairman

C_3 = cost per minor accident

C_4 = cost per major accident.

GRAPHICAL STUDY

Let us assume that the repair rate and waiting rate are exponentially distributed as under:

$g_2(t) = \alpha e^{-\alpha t}, \qquad w(t) = \beta e^{-\beta t}$

$g_1(t) = \gamma\ e^{-\gamma t}$

The behaviour of the MTSF and the profit with respect to failure rate (λ) for different values of repair rate (α) is shown as in Figs. 1.3 and 1.4 respectively. It is clear from the graphs that as failure rate increases, the values of the MTSF and the profit both decrease but increase with increase in repair rate (α).

Figs. 1.5 and 1.6 show the behaviour of the MTSF and the profit respectively with respect to probability that accident does not take place (a) for variation in waiting rate (β). It is interpreted from the graphs that the values of the MTSF and the profit increase with increase in probability (a) and also with increase in waiting rate.

Graph between the MTSF versus probability that accident does not affect the repairman (p_1) for different values of repair rate (γ) is shown as in Fig. 1.7. It can be seen from the graph that the MTSF remains same irrespective of the values of p_1. However, it increases with the increase in repair rate (γ). The reason behind the value of the MTSF remaining constant with respect to p_1 is that the operation time is not affected whether the accident harms the repairman or not.

Fig. 1.8 shows the behaviour of the profit with respect to probability (p_1) for different values of repair rate (γ). It is clear from the graph that profit increases with the increase in p_1 and also with the increase in γ. The profit increases with increase in p_1 because if the accident does not harm the repairman then no other repairman is called and hence number of visits does not increase as a result of which cost decreases with the increase in p_1. That is why the profit increases with increase in p_1.

MODEL 2

In this model, another repairman is called immediately whenever an accident affects the repairman, no matter whether it is minor or major. The state transition diagram for the model is shown as in Fig. 1.2

Transition Probabilities and Mean Sojourn Times

The transition probabilities are

$$dQ_{01}(t) = \lambda e^{-\lambda t} dt$$

$$dQ_{10}(t) = a\, e^{-\lambda t} g_1(t) dt$$

$$dQ_{11}^{(4)}(t) = a\left[\lambda e^{-\lambda t} © 1\right] dt$$

$$dQ_{17}^{(4)}(t) = b_1\left[\lambda e^{-\lambda t} © 1\right] g_1(t) dt$$

$$dQ_{18}^{(4)}(t) = (b_2+b_3)\left[\lambda e^{-\lambda t} \copyright 1\right] g_1(t)dt$$
$$dQ_{12}(t) = b_1 e^{-\lambda t} g_1(t)dt$$
$$dQ_{13}(t) = (b_2+b_3)e^{-\lambda t} g_1(t)dt$$
$$dQ_{14}(t) = \lambda e^{-\lambda t}\,\overline{G_1(t)}dt$$
$$dQ_{20}(t) = e^{-\lambda t} g_2(t)dt$$
$$dQ_{21}^{(5)}(t) = \left[\lambda e^{-\lambda t} \copyright 1\right] g_2(t)dt$$
$$dQ_{25}(t) = \lambda e^{-\lambda t}\,\overline{G_2(t)}dt$$
$$dQ_{30}(t) = e^{-\lambda t} g_2(t)dt$$
$$dQ_{31}^{(6)}(t) = \left[\lambda e^{-\lambda t} \copyright 1\right] g_2(t)dt$$
$$dQ_{36}(t) = \lambda e^{-\lambda t}\,\overline{G_2(t)}dt$$
$$dQ_{71}(t) = g_2(t)dt$$
$$dQ_{81}(t) = g_2(t)dt$$

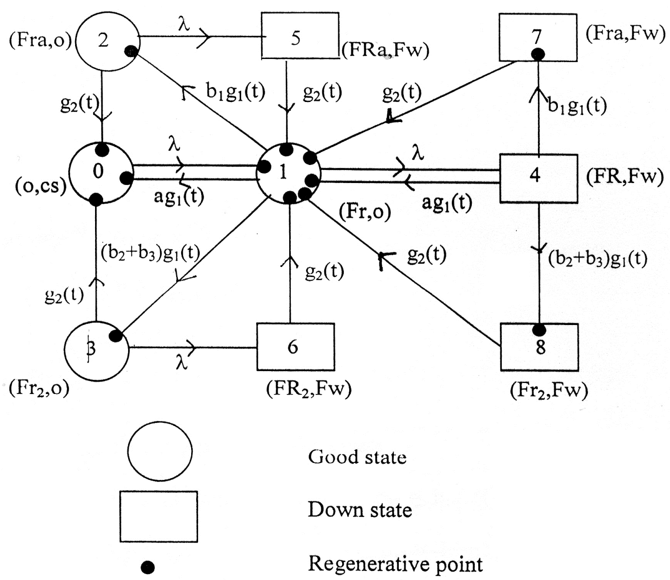

Fig. 1.2

The non-zero elements p_{ij} of the transition probability matrix for the system are found out as $p_{ij} = \lim_{s\to\infty} q_{ij}^{*}(s)$.

The mean sojourn time in state i, its value for various states are obtained by using the formula.

$$\mu_i = \int_0^\infty \Pr[T_i > t]dt \cdot$$

Hence,

$$\mu_0 = \frac{1}{\lambda}$$

$$\mu_1 = \frac{1-g_1^{*}(\lambda)}{\lambda}, \quad \mu_2 = \frac{1-g_2^{*}(\lambda)}{\lambda} = \mu_3$$

$$\mu_7 = \mu_8 = \int_0^\infty \overline{G_2(t)}\,dt$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it (time) is counted from the epoch of entrance into the state i, is mathematically states as

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t)$$

Thus

$$m_{01} = \mu_0$$
$$m_{10} + m_{12} + m_{13} + m_{14} = \mu_1$$
$$m_{10} + m_{12} + m_{13} + m_{11}^{(4)} + m_{17}^{(4)} + m_{18}^{(4)} = -g_1^{*'}(0) = k_1(\text{say})$$
$$m_{20} + m_{25} = \mu_2$$
$$m_{20} + m_{21}^{(5)} = -g_2^{*'}(0) = k_2(\text{say})$$
$$m_{30} + m_{31}^{(6)} = k_2$$
$$m_{30} + m_{36} = \mu_3$$
$$m_{71} = m_{81} = k_2$$

MEAN TIME TO SYSTEM FAILURE

$$\phi_0(t) = Q_{01}(t)(s)\phi_1(t)$$
$$\phi_1(t) = Q_{10}(t)(s)\phi_0(t) + Q_{12}(t)(s)\phi_2(t) + Q_{13}(t)(s)\phi_3(t) + Q_{14}(t)$$
$$\phi_2(t) = Q_{20}(t)(s)\phi_0(t) + Q_{25}(t)$$
$$\phi_3(t) = Q_{30}(t)(s)\phi_0(t) + Q_{36}(t)$$

Taking Laplace-Stieltjes transform of above equations and solving them for $\phi_0^{**}(s)$.

The MTSF when the system starts from state 0, is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 + \mu_2 p_{12} + p_{13}\mu_3$$

and

$$D = 1 - (p_{10} + p_{12}p_{20} + p_{13}p_{30})$$

AVAILABILITY ANALYSIS

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) \copyright A_1(t)$$
$$A_1(t) = M_1(t) + q_{10}(t) \copyright A_0(t) + q_{13}(t) \copyright A_3(t) + q_{17}^{(4)}(t) \copyright A_7(t) + q_{18}^{(4)}(t) \copyright A_8(t)$$

$$+q_{11}^{(4)}(t)\copyright A_1(t)+q_{12}(t)\copyright A_2(t)$$
$$A_2(t)=M_2(t)+q_{20}(t)\copyright A_0(t)+q_{21}^{(5)}(t)\copyright A_1(t)$$
$$A_3(t)=M_3(t)+q_{30}(t)\copyright A_0(t)+q_{31}^{(6)}(t)\copyright A_1(t)$$
$$A_7(t)=q_{71}(t)\copyright A_1(t)$$
$$A_8(t)=q_{81}(t)\copyright A_1(t)$$

where

$$M_0(t)=e^{-\lambda t}, M_1(t)=e^{-\lambda t}\,\overline{G_1(t)}$$
$$M_2(t)=e^{-\lambda t}\,\overline{G_2(t)},\ M_3(t)=e^{-\lambda t}\,\overline{G_2(t)}$$

Now taking Laplace-Stieltjes transform of above equations and solving them for $A_0^*(s)$

The steady state availability is given by

$$A_0=\lim_{s\to 0} s\ A_0^*(s)=\frac{N_1}{D_1}$$

where

$$N_1=\mu_0\left(1-p_{11}^{(4)}-p_{12}p_{21}^{(5)}-p_{13}p_{31}^{(6)}-p_{18}^{(4)}-p_{17}^{(4)}\right)+\mu_1+\mu_2\left(p_{12}+p_{13}\right)$$

and

$$D_1=\mu_0\left(p_{10}+p_{12}p_{20}+p_{13}p_{30}\right)+k_1+k_2\left(p_{12}+p_{13}+p_{17}^{(4)}+p_{18}^{(4)}\right)$$

The total fraction of the time for which the system is under repair is given by

$$B_0=\lim_{s\to 0}\left(s\ B_0^*(s)\right)=\frac{N_2}{D_1}$$

where $N_2=k_1+k_2\left(p_{12}+p_{13}+p_{17}^{(4)}+p_{18}^{(4)}\right)$

and D_1 is already specified.

The number of visits per unit time of the expert repairman is given by

$$V_0=\lim_{s\to 0} s\,V_0^{**}(s)=\frac{N_3}{D_1}$$

where $N_3=1-p_{11}^{(4)}-p_{17}^{(4)}-p_{12}p_{21}^{(5)}+p_{13}p_{30}$

and D_1 is already specified.

EXPECTED NUMBER OF MAJOR/MINOR ACCIDENTS

$$MA_0(t)=q_{01}(t)(s)MA_1(t)$$
$$MA_0(t)=q_{10}(t)(s)MA_0(t)+q_{12}(t)(s)MA_2(t)$$
$$+q_{13}(t)(s)\left[1+MA_3(t)\right]$$
$$+Q_{17}^{(4)}(t)(s)MA_7(t)+q_{18}^{(4)}(t)(s)\left[1+MA_8(t)\right]+Q_{11}^{(4)}(t)(s)MA_1(t)$$
$$MA_2(t)=q_{20}(t)(s)MA_0(t)+q_{21}^{(5)}(t)(s)MA_1(t)$$
$$MA_3(t)=q_{30}(t)(s)MA_0(t)+q_{31}^{(6)}(t)(s)MA_1(t)$$
$$MA_7(t)=q_{71}(t)(s)MA_1(t)$$
$$MA_8(t)=q_{81}(t)(s)MA_1(t)$$

Taking Laplace-Stieltjes transform of these equations and solving them for $MA_0^{**}(s)$.

In steady state,

$$MA_0=\lim_{s\to 0} s\,MA_0^{**}(s)=\frac{N_4}{D_1}$$

where

$N_4=p_{13}+p_{18}^{(4)}$

and D_1 is already specified.

COST-BENEFIT ANALYSIS

In steady state, the profit of the system is given by

$P_2=C_0A_0-C_1B_0-C_2V_0-C_3MA_0$

where

C_0 = revenue per unit uptime of the system.

C_1 = the cost per unit time for which the repairman is engaged in repairing the system under repair

C_2 = the cost per visit by the repairman

C_3 = the average cost per major/minor accident

GRAPHICAL STUDY

Let us assume that the repair times follow exponential distribution i.e.

$g_2(t)=\alpha\,e^{-\alpha t},\ g_1(t)=\gamma\ e^{-\gamma t}$

The behaviour of the MTSF and the profit with respect to failure rate (λ) for different values of repair rate (α) is shown as in Figs. 1.9 and 1.10 respectively. It is clear from the graphs that as failure rate increases, the value of MTSF and the profit both decrease but increase with increase in repair rate (α).

Graph between the MTSF versus probability that accident does not affect the repair (p_1) for different values of repair rate (γ) is shown as in Fig. 1.11. It can be seen from the graph that the MTSF remains same irrespective of the values of p_1. However, it increases with the increase in repair rate (γ). The reason behind the value of the MTSF remaining constant with respect to p_1 is that the operation time is not affected whether the accident harms the repairman or not since if the accident does not harm the repairman, the same repairman does the rest of the repair and if the accident harms the repairman, another repairman comes to do the rest of the repair.

Fig. 1.12 shows the behaviour of the profit with respect to probability (p_1) for different values of repair rate (γ). It is clear from the graph that the profit increases with the increase in p_1 and also with the increase in γ. The profit increase with increase in p_1 because if the accident does not harm the repairman then no other repairman is called and hence number of visits does not increase as a result of which cost decreases with the increase in p_1. That is why the profit increases with increase in p_1.

COMPARISON BETWEEN MODEL 1 AND MODEL 2

We can observe from fig. 1.7 and 1.11 that the MTSF in case of Model 1 is greater than that of Model 2, whatever the value of waiting rate (β) is. Hence, if cost factor is not taken into account then Model 1 is better than Model 2.

However, if cost factor is taken into consideration then either of the models may be better than the other depending upon the value of waiting rate (β). Fig. 1.13 depicts the difference of profits $(P_1 - P_2)$ with respect to cost per visit (C_2) for different values of waiting rate (β). The difference $P_1 - P_2$ increases with the increase in C_2 since profit (P_1) increases more rapidly than profit (P_2) with the increase in C_2. This is because the number of visits in Model 1 is lesser than that in case of Model 2. We further conclude the following from Fig. 1.13.

(i) $P_1 - P_2$ increases as the waiting rate (β) increases.

(ii) If $\beta = 5$, $P_1 - P_2 < 0$ or ≥ 0 according as $C_2 < 75$ or ≥ 75. So, for this case Model 1 is better than Model 2 if $C_2 < 75$. Both the models are equally good if $C_2 = 75$.

(iii) If $\beta = 10$, $P_1 - P_2 < 0$ or ≥ 0 according as $C_2 < 69$ or ≥ 69. So, for this case Model 1 is better than Model 2 if $C_2 > 69$ whereas Model 2 is better than Model 1 if $C_2 < 69$. Both the models are equally good if $C_2 = 69$.

(iv) If $\beta = 20$, $P_1 - P_2 < 0$ or $\geq$ according as $C_2 < 61$ or ≥ 61. So, for this case Model 1 is better than Model 2 if $C_2 > 61$ whereas Model 2 is better than Model 1 if $C_2 < 61$. Both the models are equally good if $C_2 = 61$.

MTSF Versus FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE (α)

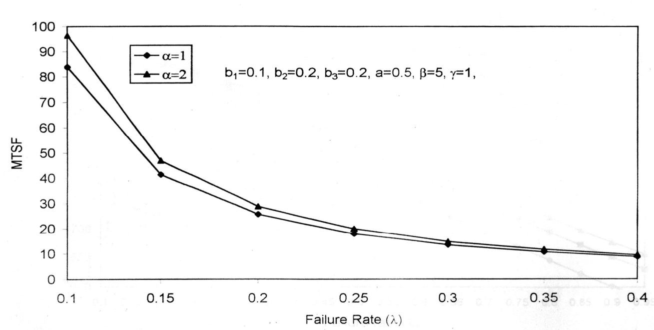

Figure 1.3

PROFIT Versus FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE (α)

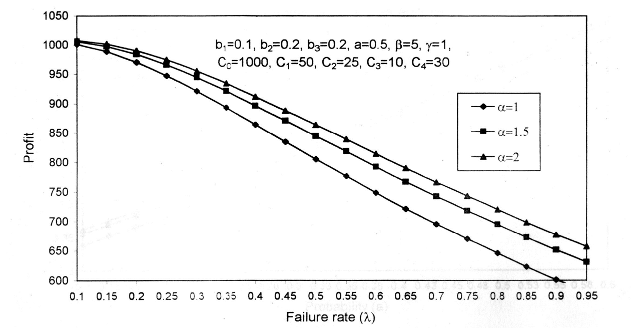

Figure 1.4

MTSF Versus PROBABILITY (a) FOR DIFFERENT VALUES OF WAITING RATE (β)

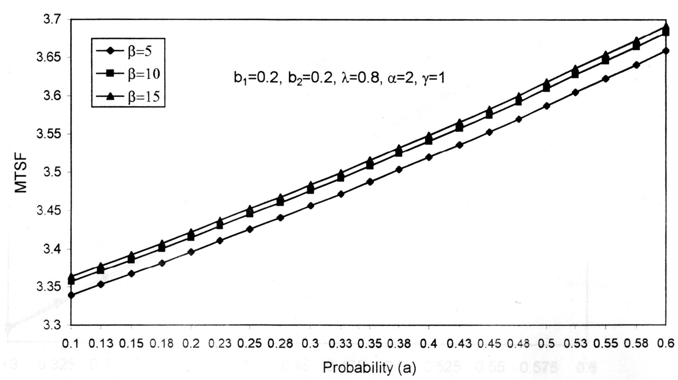

Figure 1.5

PROFIT Versus PROBABILITY (a) FOR DIFFERENT VALUES OF WAITING RATE (β)

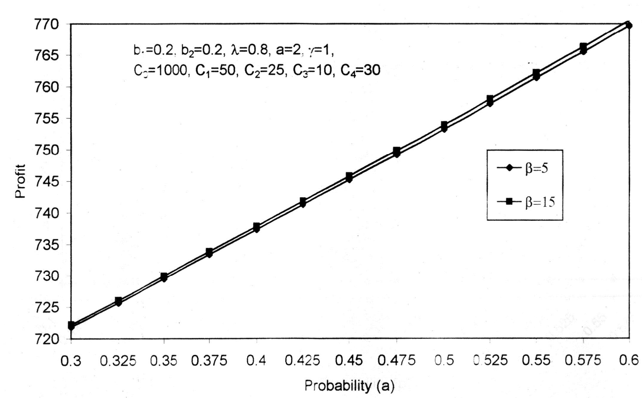

Figure 1.6

MTSF versus PROBABILITY (p_1) FOR DIFFERENT VALUES OF REPAIR RATE (γ)

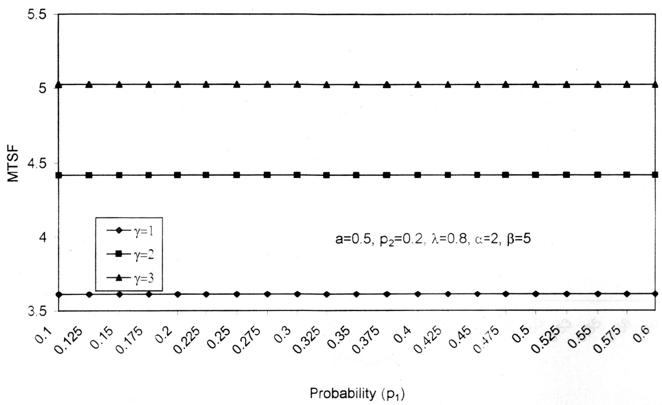

Figure 1.7

PROFIT versus PROBABILITY (p_1) FOR DIFFERENT VALUES OF REPAIR RATE (γ)

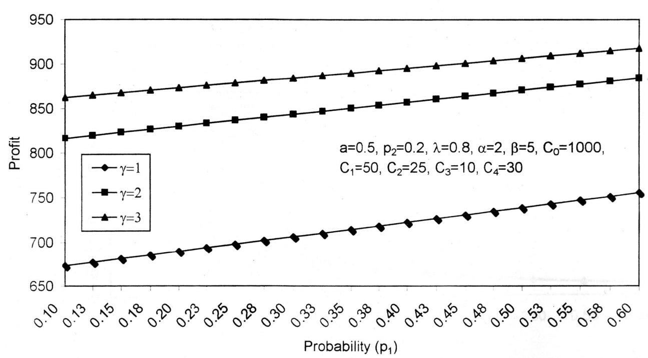

Figure 1.8

MTSF versus FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE(α)

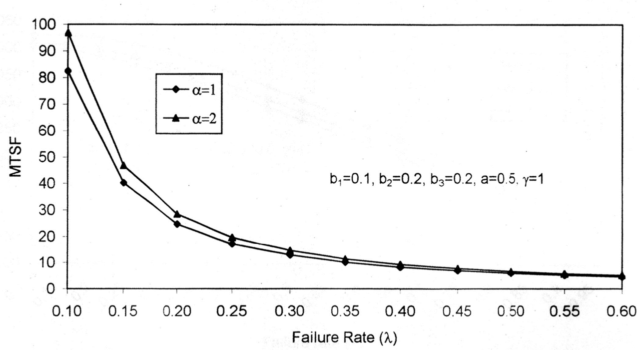

Figure 1.9

PROFIT versus FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE(α)

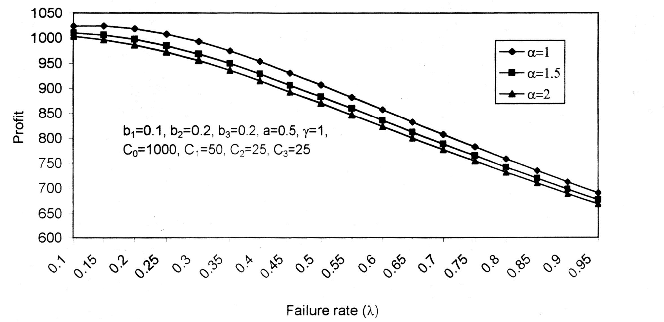

Figure 1.10

MTSF versus PROBABILITY(p_1) FOR DIFFERENT VALUES OF REPAIR RATE (γ)

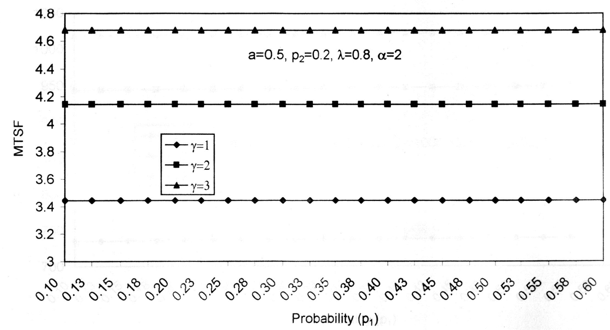

Figure 1.11

PROFIT versus PROBABILITY(p_1) FOR DIFFERENT VALUES OF REPAIR RATE (γ)

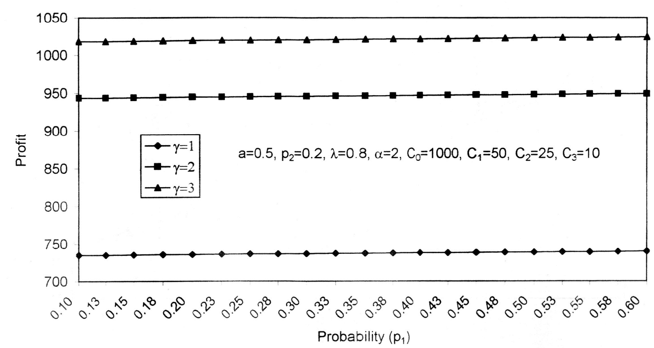

Figure 1.12

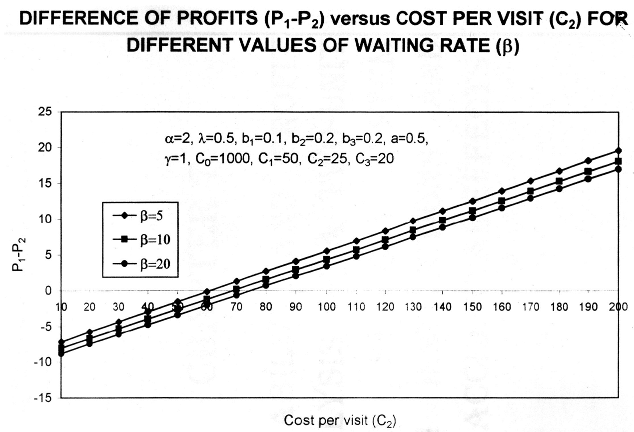

Figure 1.13

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