On Strongly πgr-Irresolute Functions

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Abstract:

The purpose of this paper is to introduce strongly πgr-irresolute functions, strongly regular πgr-irresolute functions and strongly β- πgr-irresolute functions and study some of their basic properties. Also, some new forms of homeomorphism are defined and obtained their characterizations.

Keywords: strongly πgr-irresolute, strongly regular πgr-irresolute, strongly β- πgr-irresolute, strongly πgrc-homeomorphism, Strongly regular πgrc-homeomorphism.

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1.Introduction:

Levine [10]introduced the concept of generalized closed sets in topological spaces and a class of topological space called $T_{1/2}$-space. The concept of π-closed sets in topological spaces was initiated by Zaitsav[21] and the concept of πg-closed set was introduced by Noiri and Dontchev[6]. N.Palaniappan[18] studied and introduced regular closed sets in topological spaces.The notion of homeomorphism has been studied by many topologists[13,16].Maki et al [13]introduced β-homeomorphisms.The strong forms of continuous map have been discussed by Noiri[17], Levine[11], Arya and Gupta[2], Reily, Vamanamurthy[19] and Zorlutuna et.al[22],Munshi and Bassan[15]. Strongly πgα-irresolute functions and its properties were studied by Janaki.C[7].

In this paper, we introduce strongly πgr-irresolute function ,strongly regular πgr-irresolute functions and obtained their characterizations. Throughout this paper (X,τ),(Y,σ),(Z,η) (or simply X,Y,Z) represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2.Preliminaries:

Let (X,τ) or simply X be a topological space and A be a subset of X. The closure and interior of A are denoted by Cl(A) and Int(A) respectively.

Definition: 2.1

A subset A of X is called is called

(i) Pre-open[14] if $A\subset Int(Cl(A))$.
(ii) Regular open [18]if $Int(Cl(A)) = A$.
(iii) β-open [1]if $A\subset Cl(Int(Cl(A)))$.

Finite union of regular open set is π-open[21] and its complement is π-closed.

Definition:2.2

A subset A of X is called πgr-closed [8] if $rcl(A) \subset U$ whenever $A\subset U$ and U is π-open in X.Let πGRO(X) denote the collection of πgr - open set of X and πGRC(X) denote the collection of πgr - closed set of X .

Definition:2.3

A function f: X→Y is called

(i) Continuous [11]if $f^{-1}(V)$ is closed in X for every closed set V of Y.
(ii) g-continuous [11] if $f^{-1}(V)$ is g-closed in X for every closed set V of Y.
(iii) r-continuous[18] if $f^{-1}(V)$ is regular closed in X for every closed set V of Y.
(iv) π-irresolute[6,7] if $f^{-1}(V)$ is π-closed in X for every π-closed set V of Y.
(v) an R-map [3]if $f^{-1}(V)$ is regular closed in X for every regular closed set V of Y.
(vi) πgr-continuous[9] if if $f^{-1}(V)$ is πgr-closed in X for every closed set V of Y.
(vii) πgr-irresolute[9] if if $f^{-1}(V)$ is πgr-closed in X for every πgr-closed set V of Y.
(viii) β-irresolute[12] if $f^{-1}(V)$ is β-open in X for every β-open set V of Y.

Definition:2.4

A topological space X is called

(i) a πgr-$T_{1/2}$-space [8] if every πgr-closed set is regular closed in X.
(ii) a sub maximal space [5]if every dense subset of X is open in X
(iii) extremally disconnected[4] if the closure of each open subset of X is open.
(iv) hyper connected[20] if every open subset of X is dense .

Definition:2.5

A bijection f: X→Y is called
(i) a homeomorphism[13,16] if both f and f^{-1} are continuous.
(ii) a semi-homeomorphism[13,16] if both f and f^{-1} are semi-continuous.
(iii) a gc-homomorphism [13,16]if both f and f^{-1} are g-continuous.

Definition:2.6

A collection {Ai; i ∈Λ} of πgr-open sets in a topological space X is called πgr-open cover [9] of a subset B of X if B⊂ ∪ {Ai ; i ∈Λ } holds.

Definition:2.7
A topological space X is πGR-compact [9] if every πgr-open cover of X has a finite sub cover.

Definition:2.8
A subset B of a topological space X is said to be πGR-compact[9]relative to X if, for every collection {Ai ; i ∈Λ }of πgr-open subsets of X such that B⊂∪{Ai ; i∈Λ },there exists a finite subset Λo of Λ such that B ⊂∪{Ai ; i∈ Λo}

Definition :2.9
A subset B of a topological space X is said to be πGR-compact [9] if B is πGR-compact as a subspace of X .

3. Strongly πgr-irresolute functions.

Definition:3.1

A function f: X→Y is said to be strongly πgr-irresolute if $f^{-1}(V)$ is open in X for every πgr-open set V of Y.

Definition:3.2

A function f: X→Y is said to be strongly regular irresolute(strongly r-irresolute)if $f^{-1}(V)$ is open in X for every regular open set V of Y.

Let us denote strongly regular irresolute function as strongly r-irresolute.

Theorem:3.3

If f: X→Y is a strongly πgr-irresolute ,then f is strongly r-irresolute.

Proof: Let V be regular open set in Y and hence V is πgr-open in Y. Since f is strongly πgr-irresolute, then $f^{-1}(V)$ is open in X. Therefore $f^{-1}(V)$ is open in X for every regular open set V in Y. Hence f is strongly r- irresolute.

Theorem:3.4

If f: X→Y is a continuous and Y is a πgr-$T_{1/2}$-space , then f is strongly πgr-irresolute.

Proof: Let V be πgr-open in Y. Since Y is πgr-$T_{1/2}$-space, V is regular open in Y and hence open in Y.Since f is continuous, $f^{-1}(V)$ is open in X. Thus,$f^{-1}(V)$ is open in X for every πgr-open set V in Y. Hence f is strongly πgr- irresolute.

Theorem:3.5

If f: X→Y is a πgr-irresolute, X is a πgr-$T_{1/2}$-space , then f is strongly πgr-irresolute.

Proof: Let V be πgr-open in Y. Since f is πgr-irresolute, $f^{-1}(V)$ is πgr-open in X. Since X is a πgr-$T_{1/2}$-space, $f^{-1}(V)$ is regular open in X and hence open in X. Thus, $f^{-1}(V)$ is open in X for every πgr-open set V in Y. Hence f is strongly πgr- irresolute.

Theorem:3.6

Let f: X→Y and g:Y→Z be any functions. Then

(i) $g \circ f$: X→Z is πgr-irresolute if f is πgr-continuous and g is strongly πgr-irresolute.
(ii) $g \circ f$: X→Z is strongly πgr-irresolute if f is strongly πgr-irresolute and g is πgr-irresolute.

Proof:(i)Let V be a πgr-open set in Z. Since g is strongly πgr-irresolute, $g^{-1}(V)$ is open in Y. Since f is πgr-continuous, $f^{-1}(g^{-1}(V))$ is πgr-open in X.

$\Rightarrow (g \circ f)^{-1}(V)$ is πgr-open in X for every πgr-open set V in Z.

$\Rightarrow (g \circ f)$ is πgr-irresolute.

(ii) Let V be a πgr-open set in Z. Since g is πgr-irresolute, $g^{-1}(V)$ is πgr-open in Y. Since f is strongly πgr-irresolute, $f^{-1}(g^{-1}(V))$ is open in X.

$\Rightarrow (g \circ f)^{-1}(V)$ is open in X for every πgr-open set V in Z.

$\Rightarrow (g \circ f)$ is strongly πgr-irresolute.

Theorem:3.7

The following are equivalent for a function f: X→Y:

(i) f is strongly πgr-irresolute.
(ii) For each $x \in X$ and each πgr-open set V of Y containing f(x), there exists an open set U in X containing x such that $f(U) \subset V$.
(iii) $f^{-1}(V) \subset int(f^{-1}(V))$ for each πgr-open set V of Y.
(iv) $f^{-1}(F)$ is closed in X for every πgr-closed set F of Y.

Proof: (i) $\Rightarrow$ (ii):

Let $x \in X$ and V be a πgr-open set in Y containing f(x).By hypothesis,$f^{-1}(V)$ is open in X and contains x.

Set $U=f^{-1}(V)$. Then U is open in X and $f(U) \subset V$.

(ii) $\Rightarrow$ (iii):

Let V be a πgr-open set in Y and $x \in f^{-1}(V)$.

By assumption, there exists an open set U in X containing x such that $f(U) \subset V$.

Then $x \in U \subset int(U)$

$\subset int(f^{-1}(V))$.

Then $f^{-1}(V) \subset int(f^{-1}(V))$

(iii) $\Rightarrow$ (iv):

Let F be a πgr-closed set in Y. Set V= Y – F. Then V is πgr-open in Y.

By (iii), $f^{-1}(V) \subset int(f^{-1}(V))$.

Hence $f^{-1}(F)$ is closed in X.

(iv) $\Rightarrow$ (i):

Let V be πgr-open set in Y. Let F = Y – V. That is F is πgr-closed set in Y. Then $f^{-1}(F)$ is closed in X,(by (iv)). Then $f^{-1}(V)$ is open in X. Hence f is strongly πgr-irresolute.

Theorem:3.8

A function f: X→Y is strongly πgr-irresolute if A is open in X, then f/A: A→Y is strongly πgr-irresolute.

Proof: Let V be a πgr-open set in Y. By hypothesis, $f^{-1}(V)$ is open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is open in A and hence f/A is strongly πgr-irresolute.

Theorem:3.9

Let f: X→Y be a function and $\{A_i: i \in \Lambda\}$ be a cover of X by open sets of (X,τ). Then f is strongly πgr-irresolute if $f/A_i : (A_i, \tau/A_i) \to (Y,\sigma)$ is strongly πgr-irresolute for each $i \in \Lambda$.

Proof: Let V be a πgr-open set in Y. By hypothesis, $(f/A_i)^{-1}(V)$ is open in A_i. Since A_i is open in X, $(f/A_i)^{-1}(V)$ is open in X for every $i \in \Lambda$.

$$f^{-1}(V) = X \cap f^{-1}(V)$$

$$= \bigcup \{A_i \cap f^{-1}(V): i \in \Lambda\}$$

$$= \bigcup \{(f/A_i)^{-1}(V): i \in \Lambda\} \text{ is open in X.}$$

Hence f is strongly πgr-irresolute.

Theorem:3.10

Let f: X→Y be a strongly πgr-irresolute surjective function. If X is compact, then Y is πGRO-compact.

Proof: Let $\{A_i: i \in \Lambda\}$ be a cover of πgr-open sets of Y. Since f is strongly πgr-irresolute and X is compact, we get $X \subset \bigcup \{f^{-1}(A_i): i \in \Lambda\}$.Since f is surjective, $Y = f(X) \subset \bigcup \{A_i: i \in \Lambda\}$.Hence Y is πGRO-compact.

Theorem:3.11

If f:X → Y is strongly πgr-irresolute and a subset B of X is compact relative to X, then f(B) is πGRO-compact relative to Y.

Proof: Obvious.

Definition: 3.12

A function f: X→Y is said to be

(i) a strongly regular πgr-irresolute function if $f^{-1}(V)$ is regular open in X for every πgr-open set V in Y.

(ii) a strongly β- πgr-irresolute function if $f^{-1}(V)$ is β- open in X for every πgr-open set V in Y.

Theorem:3.13

(i) If f: X→Y is strongly regular πgr-irresolute , then f is strongly πgr-irresolute.

(ii) If f: X→Y is strongly regular πgr-irresolute , then f is strongly β-πgr-irresolute.

Proof:(i)Let f be a strongly regular πgr-irresolute function and let V be a πgr-open set in Y. Then $f^{-1}(V)$ is regular open in X and hence open in X.

⇒ $f^{-1}(V)$ is open in X for every πgr-open set V in Y.

Hence f is strongly regular πgr-irresolute.

(ii) Let f be a strongly regular πgr-irresolute function and let V be a πgr-open set in Y. Then

$f^{-1}(V)$ is regular open in X and hence open in X.

⇒ $f^{-1}(V)$ is open in X for every πgr-open set V in Y.

⇒ $f^{-1}(V)$ is β-open in X for every πgr-open set V in Y.

Hence f is strongly β- πgr-irresolute.

Remark: 3.14

Converse of the above need not be true as seen in the following examples.

Example: 3.15

(i)Let X= Y = {a,b,c} , τ= {φ, X,{a},{b},{a,b},{a,c}} and σ ={φ, Y,{a},{b},{a,b}}.

Let f:X→Y be an identity map. Here for every πgr-open set V in Y, $f^{-1}(V)$ is open and β-open in X. Hence f is strongly πgr-irresolute and strongly β-πgr-irresolute.

But for every πgr-open set V in Y, $f^{-1}(V)$ is not regular open in X. Thus, f is not strongly regular πgr-irresolute .Hence strongly πgr-irresolute function need not be strongly regular πgr-irresolute function and strongly β-πgr-irresolute function .

Theorem:3.16

If f:X→Y and g:Y→Z , then g∘f: X→Z is

(i) strongly πgr-irresolute if f is strongly regular πgr-irresolute and g is πgr-irresolute.

(ii) strongly β-πgr-irresolute if f is strongly πgr-irresolute and g is πgr-irresolute.

Proof: Let V be an πgr-open set in Z. Since g is πgr-irresolute, $g^{-1}(V)$ is πgr-open in Y. Since f is strongly regular πgr-irresolute , $f^{-1}(g^{-1}(V))$ is regular open in X.

⇒ $(g\circ f)^{-1}(V)$ is regular open in X and hence open in X.

Hence (g∘f) is strongly πgr-irresolute.

(i) Let V be an πgr-open set in Z. Since g is πgr-irresolute, $g^{-1}(V)$ is πgr-open in Y. Since f is strongly πgr-irresolute , $f^{-1}(g^{-1}(V))$ is open in X and hence β-open in X.

⇒ $(g\circ f)^{-1}(V)$ is β- open in X for every πgr-open set V in Z.

Hence (g∘f) is strongly β- πgr-irresolute.

Theorem:3.17

If f:X→Y and g:Y→Z , then g∘f: X→Z is

(i) strongly regular πgr-irresolute if f is regular irresolute and g is strongly regular πgr-irresolute.
(ii) strongly regular πgr-irresolute if f is regular continuous and g is strongly πgr-irresolute.
(iii) strongly β- πgr-irresolute if f is continuous and g is strongly πgr-irresolute.

Proof: Let V be a πgr-open set in Z. Since g is strongly regular πgr-irresolute, $g^{-1}(V)$ is regular open in Y. Since f is regular irresolute , $f^{-1}(g^{-1}(V))$ is regular open in X.

⇒ $(g\circ f)^{-1}(V)$ is regular open in X .

Hence (g∘f) is strongly regular πgr-irresolute.

(i) Let V be an πgr-open set in Z. Since g is strongly πgr-irresolute, $g^{-1}(V)$ is open in Y. Since f is regular continuous , $f^{-1}(g^{-1}(V))$ is regular open in X.

⇒ $(g\circ f)^{-1}(V)$ is regular open in X .

Hence (g∘f) is strongly regular πgr-irresolute.

(ii) Let V be an πgr-open set in Z. Since g is strongly πgr-irresolute, $g^{-1}(V)$ is open in Y.

Since f is continuous , $f^{-1}(g^{-1}(V))$ is open in X.

$\Rightarrow (g \circ f)^{-1}(V)$ is open in X and hence β-open in X.

Hence $(g \circ f)$ is strongly β-πgr-irresolute.

Theorem :3.18

The following are equivalent for a function f: X→Y:

(i) f is strongly regular πgr-irresolute.
(ii) For each $x \in X$ and each πgr-open set V of Y containing f(x), there exists a regular open set U in X containing x such that $f(U) \subset V$.
(iii) $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ for each πgr-open set V of Y.
(iv) $f^{-1}(F)$ is regular closed in X for every πgr-closed set F of Y.

Proof: Similar to that of Theorem 3.7

Theorem:3.19

The following are equivalent for a function f: X→Y:

(i) f is strongly β- πgr-irresolute.
(ii) For each $x \in X$ and each πgr-open set V of Y containing f(x), there exists a β- open set U in X containing x such that $f(U) \subset V$.
(iii) $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ for each πgr-open set V of Y.
(iv) $f^{-1}(F)$ is β-closed in X for every πgr-closed set F of Y.

Proof: Similar to that of Theorem 3.7.

Lemma: 3.20

If f: X→Y is strongly regular πgr-irresolute and A is a regular open subset of X, then f/A : A→Y is strongly regular πgr-irresolute.

Proof:

Let V be a πgr-open in Y . By hypothesis, $f^{-1}(V)$ is regular open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is regular open in A. Hence f/A is strongly regular πgr-irresolute.

Theorem:3.21

Let f: X→Y and $\{A_\lambda : \lambda \in \Lambda\}$ be a cover of X by regular open set of (X,τ). Then f is a strongly regular πgr-irresolute function if f/A_λ: $A_\lambda \to Y$ is strongly regular πgr-irresolute for each $\lambda \in \Lambda$.

Proof: Let V be any πgr-open set in Y. By hypothesis, $(f/A_\lambda)^{-1}(V)$ is regular open in A_λ. Since A_λ is regular open in X, it follows that $(f/A_\lambda)^{-1}(V)$ is πgr-open in X for each $\lambda \in \Lambda$.

$f^{-1}(V) = X \cap f^{-1}(V)$

$= \bigcup \{A_\lambda \cap f^{-1}(V): \lambda \in \Lambda\}$

$= \bigcup \{(f/A_\lambda)^{-1}(V): \lambda \in \Lambda\}$ is regular open in X.

Hence f is strongly regular πgr-irresolute.

Lemma:3.22

If f: X→Y is strongly β- πgr-irresolute and A is a regular-open subset of X, then f/A : A→Y is strongly β- πgr-irresolute.

Proof: Let V be a πgr-open in Y . By hypothesis, $f^{-1}(V)$ is β-open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is β- open in A.Hence f/A is strongly β- πgr-irresolute.

Theorem:3.23

Let f: X→Y and $\{A_\lambda : \lambda \in \Lambda\}$ be a cover of X by β- open sets of (X,τ). Then f is a strongly β- πgr-irresolute function if f/A_λ: $A_\lambda \to Y$ is strongly β- πgr-irresolute for each $\lambda \in \Lambda$.

Proof: Let V be any πgr-open set in Y. By hypothesis, $(f/A_\lambda)^{-1}(V)$ is β- open in A_λ. Since A_λ is β- open in X, it follows that $(f/A_\lambda)^{-1}(V)$ is β-open in X for each $\lambda \in \Lambda$.

$f^{-1}(V) = X \cap f^{-1}(V)$

$= \bigcup \{A_\lambda \cap f^{-1}(V): \lambda \in \Lambda\}$

$= \bigcup \{(f/A_\lambda)^{-1}(V): \lambda \in \Lambda\}$ is β- open in X.

Hence f is strongly β- πgr-irresolute.

Theorem:3.24

If a function f: X→Y is strongly β-πgr-irresolute , then $f^{-1}(B)$ is β-closed in X for any nowhere dense set B of Y.

Proof: Let B be any nowhere dense subset of Y. Then Y−B is regular in Y and hence πgr-open in Y. By hypothesis, $f^{-1}(Y-B)$ is β-open in X. Hence $f^{-1}(B)$ is β-closed in X.

Theorem:3.25

If a function f: X→Y and g: Y→Z, then $g \circ f$:X→Z is strongly β-πgr-irresolute if

a) f is strongly β-πgr-irresolute and g is πgr-irresolute.

b) f is an R-map and g is strongly regular πgr-irresolute.
c) f is β-irresolute and g is strongly β-πgr-irresolute.
d) f is β-irresolute and g is strongly πgr-irresolute.
e) f is β-continuous and g is strongly πgr-irresolute.
f) f is β-irresolute and g is strongly regular πgr-irresolute.

Proof: Follows from the definitions.

Theorem:3.26

Let X be a sub maximal and extremally disconnected space. Then the following are equivalent for a function f: X→Y. Then the following are equivalent:

a) f is strongly regular πgr-irresolute.
b) f is strongly πgr-irresolute.
c) f is strongly β- πgr-irresolute.

Proof:

If X is sub maximal and extremally disconnected , then τ=RO(X)=βO(X) and hence the result follows.

Definition:3.27

A bijection f: X→Y is

(i) a πgr-homeomorphism if both f and f^{-1} are πgr-continuous.
(ii) a πgrc-homeomorphism if both f and f^{-1} are πgr-irresolute.
(iii) a strongly πgrc-homeomorphism if both f and f^{-1} are strongly πgr-irresolute.
(iv) a strongly regular πgrc-homeomorphism if both f and f^{-1} are strongly regular πgr- irresolute.
(v) a strongly β-πgrc- homeomorphism if both f and f^{-1} are strongly β- πgr-irresolute.

Theorem:3.28

If a bijective function f: X→Y is strongly regular πgrc-homeomorphism, then

1) f is πgrc-homeomorphism.
2) f is strongly πgr-homeomorphism.

Proof: (1)Since a bijection f is strongly regular πgrc-homeomorphism, f and f^{-1} are strongly regular πgr-irresolute. Every strongly regular πgr-irresolute function is πgr-irresolute.(Since every regular open set is πgr-open) .Therefore, both f and f^{-1} are πgr-irresolute functions and hence f is a πgrc-homeomorphism.

(2)Since every strongly regular πgr-irresolute function is strongly πgr-irresolute and hence the result follows.

Proposition:3.29

(i) Every strongly regular πgrc-homeomorphism is a strongly πgrc-homeomorphism and a strongly β-πgrc- homeomorphism.
(ii) Every strongly regular πgrc-homeomorphism is a strongly β-πgrc- homeomorphism.

Proof: Follows from the definitions.

Remark:3.30

The family of all strongly πgrc-homeomorphism from (X,τ) onto itself is denoted by Stπgrch(X,τ)

Theorem:3.31

If f:X→Y and g:Y→Z are strongly regular πgrc-homeomorphisms, then g ∘ f: X→Z is a strongly πgrc-homeomorphism.

Proof: Let V be a πgr-open set in Z. Then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(U)$, where $U = g^{-1}(V)$.Since g is strongly regular πgrc-homeomorphism, g is strongly regular πgr-irresolute and $g^{-1}(V)$ is regular open in Y for every πgr-open set V in Z. Hence $U=g^{-1}(V)$ is πgr-open in Y.Since every regular open set is πgr-open. Also, since f is strongly regular πgr-irresolute, $f^{-1}(U)$ is regular open in X and hence open in X. Therefore, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in X. Hence (g ∘ f) is strongly πgr-irresolute.

Now, (g ∘ f)(A) =g(f(A)) =g(B), where B= f(A). Since f is strongly regular πgrc-homeomorphism. f(A) is regular open in Y and hence πgr-open in Y. Now g is strongly regular πgr-homeomorphism implies g(B) is regular open in Z and hence open in Z. Hence $(g\circ f)^{-1}$ is strongly πgr-irresolute.

⇒(g ∘ f) is a strongly πgrc-homeomorphism.

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