# Detour Distant Divisor Graphs

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**Abstract**

Let $G = (V, E)$ be a $(p, q)$ - graph. The longest path $P$ is called a *detour distant divisor path* if $l(P)$ *is a divisor of* $q$ *. A detour distant divisor graph* $DD(G)$ *of graph* $G = (V, E)$ *has the vertex set* $V = V(G)$ *and two vertices in* $DD(G)$ *are adjacent if they have the detour distant divisor path in G. In this paper, we deal with the detour distant divisor graph of standard graphs such as path, cycle, star, wheel, complete graph, complete bipartite graph and corona of a complete graph etc.*

Key words: Distance, Detour distance, Detour distant divisor path, Detour distant divisor graph, Antipodal graph.
AMS subject classification: 05C12, 05C76.

## 1 Introduction

By a graph, we mean a finite undirected graph without loops and multiple edges. For terms not defined here, we refer to Harary[7].

Distance between the two vertices in a graph is the length of the shortest path between them. Graph Theory [4, 7] has a strong communication with Number Theory [1, 6]. Using the divisibility concept in Number Theory, we introduced a new concept called distant divisor graph[10]. In that paper[10], we found the distant divisor graphs of some standard graphs such as path, cycle, wheel, star, complete graph, complete bipartite graph etc. It was found that there were some properties and characterizations for distant divisor graphs.

Here, the definitions of some Number theoretic functions are given.

**Notations 1.1.** *Let $G$ be a $(p,q)$-graph,*

*(i) $k_1, k_2, \ldots, k_\tau$ denote the positive divisors of $q$ with $k_1 = 1$ and $k_\tau = q$ and $k_1 < k_2 <, \ldots, < k_\tau$. For $q = 12$, $k_1 = 1, k_2 = 2, k_3 = 3, k_4 = 4, k_5 = 6$ and $k_6 = 12$*

**Definition 1.2.** *[3] Two vertices of a graph are said to be antipodal to each other if the distance between them is equal to the diameter of a graph.*

**Definition 1.3.** *[3] The antipodal graph of a graph G, denoted by A(G), has the vertex set as in G and two vertices are adjacent in A(G) if and only if they are antipodal in G.*

**Definition 1.4.** *[8] Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph.*

**Definition 1.5.** *[5] For vertices $u$ and $v$ in a connected graph $G$, the detour distance $D(u,v)$ is the length of the longest $u-v$ path in $G$.*

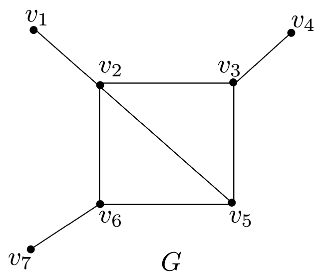
**Theorem 1.6.** *[2] For a cycle $C_p$, $A(C_p) \cong \begin{cases} \frac{p}{2}\, K_2, \text{ if } p \text{ is even.} \\ C_p, \text{ if } p \text{ is odd} \end{cases}$*

In this paper, we are going to find the structure of the detour distant divisor graph of graphs such as path, cycle, star, wheel, complete graph, complete bipartite graph and corona of a complete graph.

# 2 Main Results

**Definition 2.1.** *Let $G = (V, E)$ be a $(p,q)$- graph. The longest path $P$ is called detour distant divisor path if $l(P)$ is a divisor of $q$.*

**Example 2.2.** *Consider the graph,*



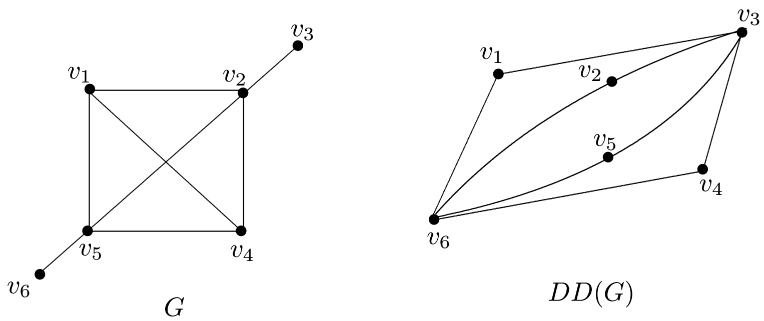

*Here* $q = 8$, *clearly the path* $v_1v_2v_6v_5v_3$ *is a detour distant divisor path.*

Next, we define detour distant divisor graph of a graph.

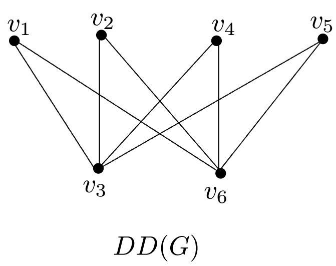
**Definition 2.3.** *Let* $G = (V, E)$ *be a* $(p, q)$ *- graph. The detour distant divisor graph* $DD(G)$ *of the graph* $G$ *has the vertex set* $V = V(G)$ *and two vertices in* $DD(G)$ *are adjacent if they have the detour distant divisor path in* $G$.

The following example illustrates the concept of detour distant divisor graph of a graph.

**Example 2.4.** *The graph* $G$ *and its detour distant divisor graph* $DD(G)$ *are shown below.*




*we redraw the above* $DD(G)$ *as follows.*




*Note that* $DD(G) \cong K_{4,2}$

First, we shall find the detour distant divisor graph of complete graph.

**Theorem 2.5.** *For a graph* $K_p$ , $DD(K_p) \cong \begin{cases} K_p, \text{ if } p \text{ is even} \\ \overline{K_p}, \text{ if } p \text{ is odd} \end{cases}$

*Proof.* In $K_p$ , $q = \frac{p(p-1)}{2}$ and $D(v_i, v_j) = p - 1$ for all $i \neq j$ .

Case $(i)$: Suppose $p$ is even.

Then, $D(v_i, v_j)$ divides $q$ for all $i \neq j$ . Thus, there is a detour distant divisor path between any two vertices of $K_p$ . Hence, in $DD(K_p)$ each vertex is adjacent to every other vertices.

Thus, $DD(K_p) \cong K_p$ .

Case $(ii)$: Suppose $p$ is odd.

Then , $D(v_i, v_j)$ is not a divisor of $q$ , for all $i \neq j$ . Thus, there is no detour distant divisor path between any two vertices of $K_p$ . Hence, in $DD(K_p)$ each vertex is isolated.

Thus, $DD(K_p) \cong \overline{K_p}$ . □

Next, we shall find the detour distant divisor graph of cycles.

**Theorem 2.6.** *For a cycle* $C_p$ , $DD(C_p) \cong \begin{cases} A(C_p), \text{ if } p \text{ is even} \\ \overline{K_p}, \qquad \text{if } p \text{ is odd} \end{cases}$

*Proof.* Case $(i)$: Suppose $p$ is even.

Then $\frac{p}{2} \leq D(v_i, v_j) \leq p - 1$ , for all $i \neq j$ .

Since $\frac{p}{2}$ is the only divisor of $q$ and also in $C_p$ , the number of vertices having detour distance $\frac{p}{2}$ from each $v_i$ is $1$ , $DD(C_p)$ is disconnected and degree of each vertex is one. Thus, $DD(C_p)$ is disconnected graph of $\frac{p}{2}$ components and each component is $K_2$ . Hence, $DD(C_p) \cong \frac{p}{2} K_2$ .

From the theorem 1.6 it follows that $A(C_p) \cong \frac{p}{2} K_2$ , if $p$ is even.

Thus, $DD(C_p) \cong A(C_p)$ .

Case $(i)$: Suppose $p$ is odd.

Then $\frac{p+1}{2} \leq D(v_i, v_j) \leq p - 1$ , for all $i \neq j$ .

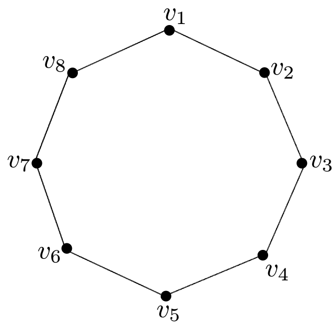
For $i \neq j$ , we observe that $D(v_i, v_j)$ is not a divisor of $q$ .

Then, there is no detour distant divisor path between any two vertices of $C_p$ .

Hence, $DD(C_p) \cong \overline{K_p}$ . □

The Theorem 2.6 is illustrated by the following examples.

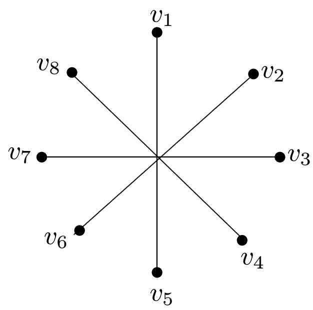
**Example 2.7.** *Case(i): $p$ is even, Consider the cycle $C_8$ ,*




$C_8$

*Here $q = 8$, the divisors of $q$ are $1, 2, 4$ and $8$ and $D(v_1, v_5) = 4$ , $D(v_2, v_6) = 4$ , $D(v_3, v_7) = 4$ , $D(v_4, v_8) = 4$ .*

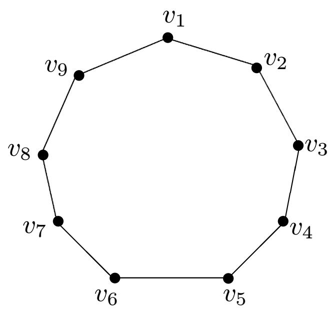
*The detour distant divisor graph of $C_8$ is given below.*




$DD(C_8)$

*Note that $DD(C_8) \cong 4K_2$*

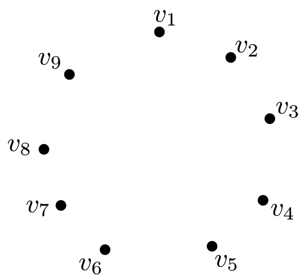
*Case(ii): $p$ is odd. Consider the graph $C_9$ .*




$C_9$

*Here $q =$ $9$ , the divisors of $q$ are $1, 3$ and $9$ but $D(v_i, v_j) \geq 5$ , for all $i \neq j$ .*

*The detour distant divisor graph of $C_9$ is given below.*




$DD(C_9)$

*Note that* $DD(C_p) \cong \overline{K_p}$

Next, we shall find the detour distant divisor graph of a wheel graph.

**Theorem 2.8.** *For a wheel* $W_p$ *,* $DD(W_p) \cong K_p$ *.*

*Proof.* In $W_p$ , $D(v_i, v_j) = p - 1$ , for all $i \neq j$ and $q = 2(p-1)$ .
Since $D(v_i, v_j)$ divides $q$ , for all $i \neq j$ , each vertex in $DD(W_p)$ is adjacent to every other vertices.
Hence, $DD(W_p) \cong K_p$ . □

It is found that the detour distant divisor graph and distant divisor graph of a tree are the same.

**Theorem 2.9.** *For a tree* $T$ *,* $DD(T) \cong D(T)$ *.*

*Proof.* For a tree, $d_T(v_i, v_j) = D_T(v_i, v_j)$ , for all $i \neq j$ , the result follows. □

**Corollary 2.10.** *For a star* $K_{1,p}$ *,* $DD(K_{1,p}) \cong D(K_{1,p})$ *.*

**Corollary 2.11.** *For a path* $P_p$ *,* $DD(P_p) \cong D(P_p)$ *.*

Next, we find the structure of the detour distant divisor graph of complete bipartite graph.

**Theorem 2.12.** *For a graph* $K_{m,n}$ *,* $m < n$ *,* $DD(K_{m,n})$ *is either* $K_m \cup \overline{K_n}$ *or* $K_{m,n}$ *or* $K_n \cup \overline{K_m}$ *or* $K_m + \overline{K_n}$ *or* $K_n + \overline{K_m}$ *or* $K_m \cup K_n$ *or* $K_{m+n}$ *or* $\overline{K_{m+n}}$ *.*

*Proof.* Let $X = \{u_1, u_2, \ldots, u_m\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$ be the bipartition of $V(K_{m,n})$ and $|X| < |Y|$ .
In $K_{m,n}$ , we observe that

*(i)* $D(u_i, u_j) = 2(m-1)$ , for all $i \neq j$ .
*(ii)* $D(v_i, v_j) = 2m$ , for all $i \neq j$ .
*(iii)* $D(u_i, v_j) = 2m - 1$ , for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ .

Note that $K_{m,n}$ has $m + n$ vertices and $mn$ edges.

Then, we check only the numbers $2(m-1)$, $2m$ and $2m-1$ if they are the divisors of $mn$.

Case $(i)$: Suppose $mn \cong 0(mod(2(m-1)))$ and $mn \ncong 0(mod(2m-1))$ and $mn \ncong 0(mod(2m))$.

Then, the divisor of $mn$ is only $2(m-1)$. Since $D(u_i, u_j) = 2(m-1)$, for all $i \neq j$, in $DD(K_{m,n})$, we join each $u_i$ to every $u_j$, and $v_i$'s are isolated.

Thus, $DD(K_{m,n}) \cong K_m \cup \overline{K_n}$.

Case $(ii)$: Suppose $mn \ncong 0(mod(2(m-1)))$ and $mn \cong 0(mod(2m-1))$ and $mn \ncong 0(mod(2m))$.

Then, the divisor of $mn$ is only $2m-1$. Since $D(u_i, v_j) = 2m-1$ for all $i \neq j$, in $DD(K_{m,n})$, we join each $u_i$ to every $v_j$, and $u_i$ is not adjacent to $u_j$ for all $i \neq j$ and also $v_i$ is not adjacent to $v_j$ for all $i \neq j$.

Hence, $DD(K_{m,n}) \cong K_{m,n}$.

Case $(iii)$: Suppose $mn \ncong 0(mod(2(m-1)))$ and $mn \ncong 0(mod(2m-1))$ and $mn \cong 0(mod(2m))$.

Then, the divisor of $mn$ is only $2m$. Since $D(v_i, v_j) = 2m$, for all $i \neq j$, we join each $v_i$ to every $v_j$, and all $u_i$'s are isolated.

Hence, $DD(K_{m,n}) \cong K_n \cup \overline{K_m}$.

Case $(iv)$: Suppose $mn \cong 0(mod(2(m-1)))$ and $mn \cong 0(mod(2m-1))$ and $mn \ncong 0(mod(2m))$.

Then, the divisors of $mn$ are $2(m-1)$ and $2m-1$. Since $D(u_i, u_j) = 2(m-1)$, for all $i \neq j$ and $D(u_i, v_j) = 2m-1$, for all $i \neq j$ in $DD(K_{m,n})$, we join each $u_i$ to every $u_j$ and $v_j$, and no two $v_i$'s are adjacent..

Hence, $DD(K_{m,n}) \cong K_m + \overline{K_n}$.

Case $(v)$: Suppose $mn \ncong 0(mod(2(m-1)))$ and $mn \cong 0(mod(2m-1))$ and $mn \cong 0(mod(2m))$.

Then, the divisors of $mn$ are $2m-1$ and $2m$. Since $D(u_i, v_j) = 2m-1$, for all $i \neq j$ and $D(v_i, v_j) = 2m$, for all $i \neq j$ in $DD(K_{m,n})$, we join each $v_i$ to every $v_j$ and $u_j$, and no two $u_i$'s are adjacent.

Hence, $DD(K_{m,n}) \cong K_n + \overline{K_m}$.

Case $(vi)$: Suppose $mn \cong 0(mod(2(m-1)))$ and $mn \ncong 0(mod(2m-1))$ and $mn \cong 0(mod(2m))$.

Then, the divisor of $mn$ are $2(m-1)$ and $2m$. Since $D(u_i, u_j) = 2(m-1)$, for all $i \neq j$ and $D(v_i, v_j) = 2m$, for all $i \neq j$ in $DD(K_{m,n})$, we join each $u_i$ to every $u_j$ and each $v_i$, to every $v_j$ and no two $u_i$'s and $v_j$'s are adjacent..

Hence, $DD(K_{m,n}) \cong K_m \cup K_n$.

Case $(vii)$: Suppose $mn \cong 0(mod(2(m-1)))$ and $mn \cong 0(mod(2m-1))$ and $mn \cong 0(mod(2m))$.

Then, the divisors of $mn$ are $2(m-1), 2m-1$ and $2m$. From (i), (ii) and (iii), it is understood that in $DD(K_{m,n})$, each vertex is adjacent to every other vertices.

Hence, $DD(K_{m,n}) \cong K_{m+n}$.

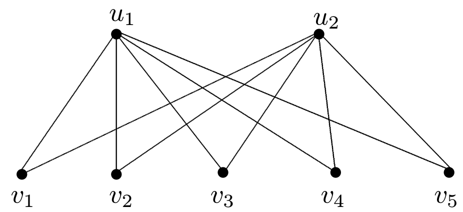
Case $(viii)$: Suppose $mn \ncong 0(mod(2(m-1)))$ and $mn \ncong 0(mod(2m-1))$ and $mn \ncong 0(mod(2m))$.

Then, $mn$ is not divisible by $2(m-1), 2m-1$ and $2m$. So, in $DD(K_{m,n})$, each vertex is isolated.

Hence, $DD(K_{m,n}) \cong \overline{K_{m+n}}$. □

The Theorem 2.12 is illustrated by the following examples.

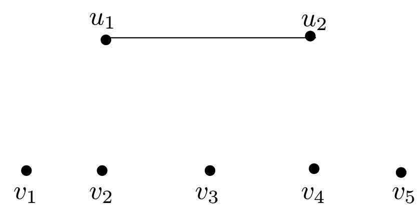
**Example 2.13.** *Case(i): Consider the graph* $K_{2,5}$.




$K_{2,5}$

*The divisors of* $q = 10$ *are* $1, 2, 5$ *and* $10$ *and* $D(u_1, u_2) = 2$, $D(u_i, v_j) = 3$ *and* $D(v_i, v_j) = 4$, *for all* $i \neq j$.

*The detour distant divisor graph of* $K_{2,5}$ *is given below.*




$DD(K_{2,5})$

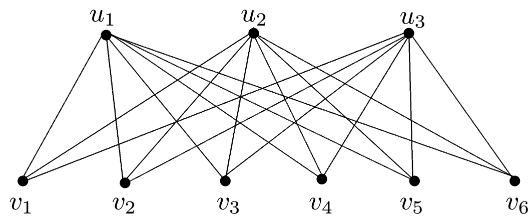
*Note that* $DD(K_{2,5}) \cong K_2 \cup \overline{K_5}$.

*Case(ii): Consider the graph* $K_{3,5}$.

*The divisors of* $q = 15$ *are* $1, 3, 5$ *and* $15$ *and* $D(u_i, u_j) = 4$, $D(u_i, v_j) = 5$ *and* $D(v_i, v_j) = 6$, *for all* $i \neq j$.

*Thus, the detour distant divisor graph of* $K_{3,5}$ *is isomorphic to itself.*

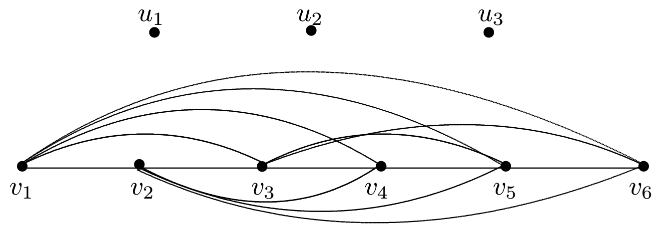
*Case(ii): Consider the graph* $K_{3,6}$.




$K_{3,6}$

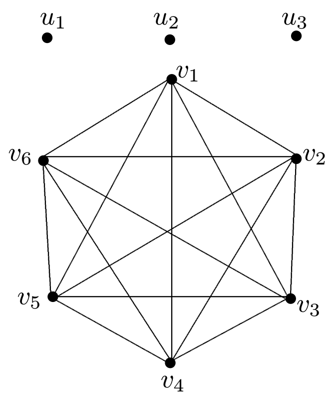
*The divisors of* $q = 18$ *are* $1, 2, 3, 6, 9$ *and* $18$ *and* $D(u_i, u_j) = 4$, $D(u_i, v_j) = 5$ *and* $D(v_i, v_j) = 6$, *for all* $i \neq j$.

*The detour distant divisor graph of* $K_{3,6}$ *is given below.*




$DD(K_{3,6})$

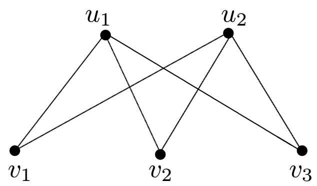
*Now, the above graph is redrawn below.*




$DD(K_{3,6})$

*Note that* $DD(K_{3,6}) \cong K_6 \cup \overline{K_3}$.

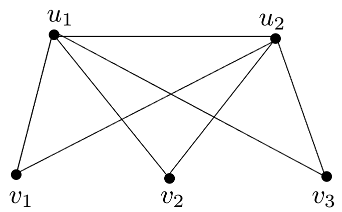
*Case(iv): Consider the graph* $K_{2,3}$.




$K_{2,3}$

*The divisors of* $q=6$ *are* $1,2,3$ *and* $6$ *and* $D(u_1,u_2)=2$, $D(u_i,v_j)=3$ *and* $D(v_i,v_j)=4$, *for all* $i\neq j$.

*The detour distant divisor graph of* $K_{2,3}$ *is given below.*




$DD(K_{2,3})$

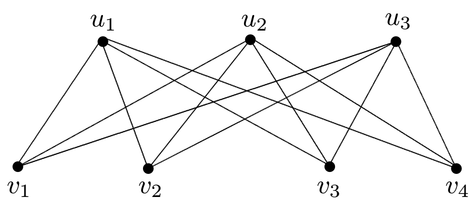
*Note that* $DD(K_{2,3})\cong K_2+\overline{K_3}$.

*Case(v): Consider the graph* $K_{4,14}$.

*The divisors of* $q=56$ *are* $1,2,4,7,8,14,28$ *and* $56$ *and* $D(u_i,u_j)=6$, $D(u_i,v_j)=7$ *and* $D(v_i,v_j)=8$, *for all* $i\neq j$.

*Thus, the detour distant divisor graph of* $K_{4,14}$ *is* $K_{14}+\overline{K_4}$.

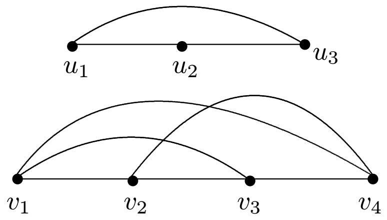
*Case(vi): Consider the graph* $K_{3,4}$.




$K_{3,4}$

*The divisors of* $q=12$ *are* $1,2,3,4,6$ *and* $12$ *and* $D(u_i,u_j)=4$, $D(u_i,v_j)=5$ *and* $D(v_i,v_j)=6$.

*The detour distant divisor graph of* $K_{3,4}$ *is given below.*



$DD(K_{3,4})$

*Note that* $DD(K_{3,4}) \cong K_3 \cup K_4$ .

*Case(vii): Consider the graph* $K_{2,6}$ .

*The divisors of* $q = 12$ *are* $1, 2, 3, 4, 6$ *and* $12$ *and* $D(u_1, u_2) = 2$ , $D(u_i, v_j) = 3$ *and* $D(v_i, v_j) = 4$ , *for all* $i \neq j$ .

*Thus, the detour distant divisor graph of* $K_{2,6}$ *is* $K_{2+6} = K_8$ .

*Case(viii): Consider the graph* $K_{3,9}$ .

*The divisors of* $q = 27$ *are* $1, 3, 9$ *and* $27$ *and* $D(u_i, u_j) = 4$ , $D(u_i, v_j) = 5$ *and* $D(v_i, v_j) = 6$ , *for all* $i \neq j$ .

*Thus, the detour distant divisor graph of* $K_{3,9}$ *is* $\overline{K_{3+9}} = \overline{K_{12}}$ .

**Corollary 2.14.** *For a graph* $K_{m,m}$ , $DD(K_{m,m}) \cong \overline{K_{2m}}$ .

*Proof.* Let $X = \{u_1, u_2, \ldots, u_m\}$ and $Y = \{v_1, v_2, \ldots, v_m\}$ be the bipartition of $V(K_{m,n})$ and $|X| = |Y|$ .

In $K_{m,m}$ , we observe that

$(i)$ $D(u_i, u_j) = 2(m-1)$ , for all $i \neq j$ .

$(ii)$ $D(v_i, v_j) = 2(m-1)$ , for all $i \neq j$ .

$(iii)$ $D(u_i, v_j) = 2m - 1$ , for all $i \neq j$ . .

Note that $K_{m,m}$ has $2m$ vertices and $m^2$ edges and we observe that $q$ is not divisible by both $2(m-1)$ and $2m-1$ . Thus, there is no detour distant divisor path between any two vertices of $K_{m,m}$ . Hence, in $DD(K_{m,m})$ each vertex is isolated.

Hence, $DD(K_{m,m}) \cong \overline{K_{2m}}$ . □

Next, we find the structure of the detour distant divisor graph of corona of a complete graph.

**Theorem 2.15.** *For a graph* $K_p^+$ , $DD(K_p^+)$ *is either* $K_p^+$ *or* $K_{p,p}$ *or* $K_p + \overline{K_p}$ .

*Proof.* Let $u_1, u_2, \ldots, u_p$ be the vertices of $K_p$ and $v_1, v_2, \ldots, v_p$ be the pendent vertices attached to $u_1, u_2, \ldots, u_p$ respectively. Note that $K_p^+$ has $2p$ vertices and $\frac{p(p+1)}{2}$ edges.

In $K_p^+$, we observe that

$(i)$ $\quad D(u_i, u_j) = p-1$, for all $i \neq j$.

$(ii)$ $\quad D(v_i, v_j) = p+1$, for all $i \neq j$.

$(iii)$ $D(u_i, v_j) =$ either $1$ or $p$, for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, p$.

Then, we check only the numbers $1$, $p$, $p-1$ and $p+1$ if they are divisors of $\frac{p(p+1)}{2}$. The following table gives the number of vertices having the detour distance $1$ or $p$ or $p-1$ or $p+1$ from the vertices of $K_p^+$.

| Vertex $v$ | Detour Distance $D$ | Number of vertices having detour distance $D$ from $v$ |
|---|---|---|
| $u_i$ | $1$ | $1$ |
| | $p$ | $p-1$ |
| | $p-1$ | $p-1$ |
| | $p+1$ | $0$ |
| $v_i$ | $1$ | $1$ |
| | $p$ | $p-1$ |
| | $p-1$ | $0$ |
| | $p+1$ | $p-1$ |

From the table, it is found that in $K_p^+$, the number of vertices having detour distance $1$ from $u_i$ is $1$ and that of detour distance $p$ from $u_i$ is $p-1$ and that of detour distance $p-1$ from $u_i$ is $p-1$ and that of detour distance $p+1$ from $u_i$ is $0$.

The number of vertices having detour distance $1$ from $v_i$ is $1$ and that of detour distance $p$ from $v_i$ is $p-1$ and that of detour distance $p-1$ from $v_i$ is $0$ and that of detour distance $p+1$ from $v_i$ is $p-1$.

Since $q$ is divisible by $1$, all pendent edges of $K_p^+$ are detour distant divisor paths.

Now, we deal this proof with either $p = 3$ or $p > 3$.

Suppose $p = 3$, then $q$ is divisible by both $p$ and $p-1$ but not divisible by $p+1$.

So, in this case $DD(K_p^+)$ is $K_p + \overline{K_p}$.

Suppose $p > 3$, then $q$ is not divisible by $p-1$ and also not divisible by both $p$ and $p+1$. So, $q$ is divisible by either $p$ or $p+1$.

Case $(i)$: Suppose $q \cong 0 (mod p)$

Then, in $DD(K_p^+)$, two vertices are adjacent, if the detour distance between them in $K_p^+$ is $1$ or $p$. Since $D(u_i, u_j) = p-1$, for all $i \neq j$ and $D(v_i, v_j) = p+1$, for all $i \neq j$, then in $DD(K_p^+)$, each $u_i$ is not adjacent to $u_j$ and each $v_i$ is not adjacent to $v_j$, for all $i \neq j$.

From the table it is clear that in $K_p^+$, the number of vertices having detour distance 1 from $u_i$ is 1 and that of detour distance $p$ from $u_i$ is $p-1$. So, in $DD(K_p^+)$, each vertex $u_i$ is adjacent to $p$ vertices. Similarly in $DD(K_p^+)$, each vertex $v_i$ is adjacent to $p$ vertices.

Thus, in $DD(K_p^+)$, degree of each $u_i$ is $p$ and degree of each $v_i$ is $p$, and also all $v_i$'s and $u_j$'s are not adjacent .

Hence, $DD(K_p^+) \cong K_{p,p}$.

Case $(ii)$: Suppose $q \cong 0(mod\ p+1)$

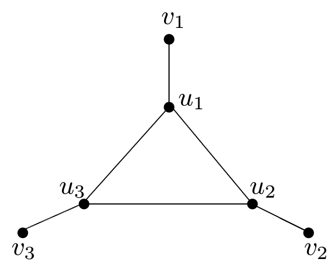
Then, in $DD(K_p^+)$, two vertices are adjacent if the detour distance between them in $K_p^+$ is 1 or $p+1$. Since $D(u_i, u_j) = p-1$, for all $i \neq j$ and $D(v_i, v_j) = p+1$, for all $i \neq j$, then in $DD(K_p^+)$, each $u_i$ is not adjacent to $u_j$ and each $v_i$ is adjacent to $v_j$, for all $i \neq j$.

From the table, it is clear that in $K_p^+$, the number of vertices having detour distance 1 from $u_i$ is 1 and that of detour distance $p+1$ from $u_i$ is 0. So, in $DD(K_p^+)$, each vertex $u_i$ is adjacent to only one $v_i$. Similarly from the table, it is clear that in $K_p^+$, the number of vertices having detour distance 1 from $v_i$ is 1 and that of detour distance $p+1$ from $v_i$ is $p-1$. So, in $DD(K_p^+)$, each $v_i$ is adjacent to exactly $p$ vertices. Thus, in $DD(K_p^+)$, degree of each $u_i$ is 1 and that of $v_i$ is $p$.

Hence, $DD(K_p^+) \cong K_p^+$. $\square$

The theorem 2.15 is illustrated by the following example.

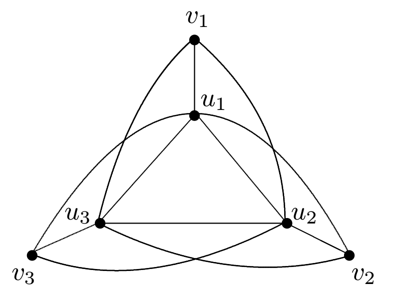
**Example 2.16.** *Consider the graph* $K_3^+$ *,*




$K_3^+$

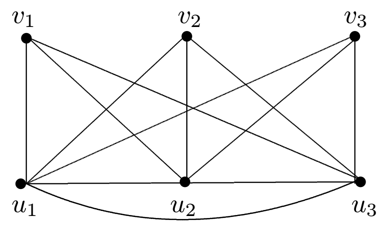
*The divisors of* $q = 6$ *are* $1, 2, 3$ *and* $6$ *and* $D(u_i, u_j) = 2$, $D(u_i, v_j) =$ *either* 1 *or* 3 *and* $D(v_i, v_j) = 4$.

*The detour distant divisor graph of* $K_3^+$ *is given below.*




$DD(K_3^+)$

*The above graph is redrawn below.*




$DD(K_3^+)$

*Note that* $DD(K_3^+) \cong K_3 + \overline{K_3}$ .

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