

# "Decompositions of Some Types of Supra Soft Sets and Soft Continuity"

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## Abstract

In this paper we introduce the notion of supra soft topological spaces. We extend the notion of  $\gamma$ -operation, pre-open soft sets,  $\alpha$ -open soft sets, semi-open soft sets and  $\beta$ -open soft sets to such spaces and study their properties and the relations between them. Also, we introduce the concepts of supra pre (resp.  $\alpha$ -, semi-,  $\beta$ -) continuous soft functions on these spaces and study some of their properties. We show that a mapping between two soft topological spaces is supra  $\alpha$ -continuous soft if and only if it is supra pre-continuous soft and supra semi-continuous soft. The importance of this approach is that, the class of supra soft topological spaces is wider and more general than the class of soft topological spaces.

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## 1 Introduction

The concept of soft sets was first introduced by Molodtsov [17] in 1999 as a general mathematical tool for dealing with uncertain objects. In [16, 17], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

After presentation of the operations of soft sets [14], the properties and applications of soft set theory have been studied increasingly [3, 11, 16, 20]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 2, 6, 12, 13, 14, 15, 16, 18, 25]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [7].

Recently, in 2011, Shabir and Naz [21] initiated the study of soft topological spaces. They defined soft topology on the collection  $\tau$  of soft sets over  $X$ . Consequently, they defined basic notions of soft topological spaces such as open soft and closed sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [8] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Kandil, Tantawy, El-Sheikh and Abd El-Latif [9] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notion of  $\gamma$ -operation. In section 4, 5, we extend these different types of subsets of soft topological spaces to supra soft topological spaces and study the decompositions of some forms of supra soft continuity in section 6.

## 2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

**Definition 2.1** [17] Let  $X$  be an initial universe and  $E$  be a set of parameters. Let  $P(X)$  denote the power set of  $X$  and  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  denoted by  $F_A$  is called a soft set over  $X$ , where  $F$  is mapping given by  $F : A \rightarrow P(X)$ . In other words, a soft set over  $X$  is a parametrized family of subsets of the universe  $X$ . For a particular  $e \in A$ ,  $F(e)$  may be considered the set of  $e$ -approximate elements of the soft set  $(F, A)$  and if  $e \notin A$ , then  $F(e) = \phi$  i.e

$F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$ . The family of all these soft sets denoted by  $SS(X)_A$ .

**Definition 2.2** [14] Let  $F_A, G_B \in SS(X)_E$ . Then  $F_A$  is soft subset of  $G_B$ , denoted by  $F_A \tilde{\subseteq} G_B$ , if

$$A \subseteq B, \text{ and}$$

$$F(e) \subseteq G(e), \forall e \in A.$$

In this case,  $F_A$  is said to be a soft subset of  $G_B$  and  $G_B$  is said to be a soft superset of  $F_A$ ,  $G_B \tilde{\supseteq} F_A$ .

**Definition 2.3** [14] Two soft subset  $F_A$  and  $G_B$  over a common universe set  $X$  are said to be soft equal if  $F_A$  is soft subset of  $G_B$  and  $G_B$  is soft subset of  $F_A$ .

**Definition 2.4** [3] The complement of a soft set  $(F, A)$ , denoted by  $(F, A)'$ , is defined by  $(F, A)' = (F', A)$ ,  $F' : A \rightarrow P(X)$  is mapping given by  $F'(e) = X - F(e)$ ,  $\forall e \in A$  and  $F'$  is called the soft complement function of  $F$ .

Clearly  $(F')'$  is the same as  $F$  and  $((F, A)')' = (F, A)$ .

**Definition 2.5** [21] The difference of two soft sets  $(F, E)$  and  $(G, E)$  over the common universe  $X$ , denoted by  $(F, E) - (G, E)$  is the soft set  $(H, E)$  where for all  $e \in E$ ,  $H(e) = F(e) - G(e)$ .

**Definition 2.6** [21] Let  $(F, E)$  be a soft set over  $X$  and  $x \in X$ . We say that  $x \in (F, E)$  read as  $x$  belongs to the soft set  $(F, E)$  whenever  $x \in F(e)$  for all  $e \in E$ .

**Definition 2.7** [14] A soft set  $(F, A)$  over  $X$  is said to be a NULL soft set denoted by  $\tilde{\phi}$  or  $\phi_A$  if for all  $e \in A$ ,  $F(e) = \phi$  (null set).

**Definition 2.8** [14] A soft set  $(F, A)$  over  $X$  is said to be an absolute soft set denoted by  $\tilde{A}$  or  $X_A$  if for all  $e \in A, F(e) = X$ . Clearly we have  $X'_A = \phi_A$  and  $\phi'_A = X_A$ .

**Definition 2.9** [14] The union of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe  $X$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$H(e) = \begin{cases} F(e), e \in A - B, \\ G(e), e \in B - A, \\ F(e) \cup G(e), e \in A \cap B \end{cases} .$$

**Definition 2.10** [14] The intersection of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe  $X$  is the soft set  $(H, C)$ , where  $C = A \cap B$  and for all  $e \in C, H(e) = F(e) \cap G(e)$ . Note that, in order to efficiently discuss, we consider only soft sets  $(F, E)$  over a universe  $X$  in which all the parameter set  $E$  are same. We denote the family of these soft sets by  $SS(X)_E$ .

**Definition 2.11** [26] Let  $I$  be an arbitrary indexed set and  $L = \{(F_i, E), i \in I\}$  be a subfamily of  $SS(X)_E$ .

The union of  $L$  is the soft set  $(H, E)$ , where  $H(e) = \bigcup_{i \in I} F_i(e)$  for each  $e \in E$ . We write  $\bigcup_{i \in I} (F_i, E) = (H, E)$ .

The intersection of  $L$  is the soft set  $(M, E)$ , where  $M(e) = \bigcap_{i \in I} F_i(e)$  for each  $e \in E$ . We write  $\bigcap_{i \in I} (F_i, E) = (M, E)$ .

**Definition 2.12** [21] Let  $\tau$  be a collection of soft sets over a universe  $X$  with a fixed set of parameters  $E$ , then  $\tau \subseteq SS(X)_E$  is called a soft topology on  $X$  if

- 1-  $\tilde{X}, \tilde{\phi} \in \tau$ , where  $\tilde{\phi}(e) = \phi$  and  $\tilde{X}(e) = X, \forall e \in E$ ,
  - 2- the union of any number of soft sets in  $\tau$  belongs to  $\tau$ ,
  - 3- the intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .
- The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ .

**Definition 2.13** [8] Let  $(X, \tau, E)$  be a soft topological space. A soft set  $(F, A)$  over  $X$  is said to be closed soft set in  $X$ , if its relative complement  $(F, A)'$  is open soft set.

**Definition 2.14** [8] Let  $(X, \tau, E)$  be a soft topological space. The members of  $\tau$  are said to be open soft sets in  $X$ . We denote the set of all open soft sets over  $X$  by  $OS(X, \tau, E)$ , or when there can be no confusion by  $OS(X)$  and the set of all closed soft sets by  $CS(X, \tau, E)$ , or  $CS(X)$ .

**Definition 2.15** [21] Let  $(X, \tau, E)$  be a soft topological space and  $(F, A) \in SS(X)_E$ . The soft closure of  $(F, A)$ , denoted by  $cl(F, A)$  is the intersection of all closed soft super sets of  $(F, A)$ . Clearly  $cl(F, A)$  is the smallest closed soft set over  $X$  which contains  $(F, A)$  i.e  

$$cl(F, A) = \tilde{\cap}\{(H, C) : (H, C) \text{ is closed soft set and } (F, A) \subseteq (H, C)\}.$$

**Definition 2.16** [26] Let  $(X, \tau, E)$  be a soft topological space and  $(F, A) \in SS(X)_E$ . The soft interior of  $(G, B)$ , denoted by  $int(G, B)$  is the union of all open soft subsets of  $(G, B)$ . Clearly  $int(G, B)$  is the largest open soft set over  $X$  which contained in  $(G, B)$  i.e  

$$int(G, B) = \tilde{\cup}\{(H, C) : (H, C) \text{ is an open soft set and } (H, C) \subseteq (G, B)\}.$$

**Definition 2.17** [26] The soft set  $(F, E) \in SS(X)_E$  is called a soft point in  $X_E$  if there exist  $x \in X$  and  $e \in E$  such that  $F(e) = \{x\}$  and  $F(e') = \phi$  for each  $e' \in E - \{e\}$ , and the soft point  $(F, E)$  is denoted by  $x_e$ .

**Proposition 2.1** [22] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.

**Definition 2.18** [26] The soft point  $x_e$  is said to be belonging to the soft set  $(G, A)$ , denoted by  $x_e \tilde{\in} (G, A)$ , if for the element  $e \in A$ ,  $F(e) \subseteq G(e)$ .

**Definition 2.19** [26] A soft set  $(G, B)$  in a soft topological space  $(X, \tau, E)$  is called a soft neighborhood (briefly: nbd) of the soft point  $x_e \tilde{\in} X_E$  if there exists an open soft set  $(H, C)$  such that  $x_e \tilde{\in} (H, C) \subseteq (G, B)$ .

A soft set  $(G, B)$  in a soft topological space  $(X, \tau, E)$  is called a soft neighborhood of the soft  $(F, A)$  if there exists an open soft set  $(H, C)$  such that  $(F, A) \tilde{\in} (H, C) \subseteq (G, B)$ . The neighborhood system of a soft point  $x_e$ , denoted by  $N_\tau(x_e)$ , is the family of all its neighborhoods.

**Theorem 2.1** [23] Let  $(X, \tau, E)$  be a soft topological space. A soft point  $e_x \tilde{\in} cl(F, A)$  if and only if each soft neighborhood of  $e_x$  intersects  $(F, A)$ .

**Definition 2.20** [19] Let  $(X, \tau, E)$  be a soft topological space and  $(F, E) \in SS(X)_E$ . Define  $\tau_{(F,E)} = \{(G, E) \tilde{\cap} (F, E) : (G, E) \in \tau\}$ , which is soft topology on  $(F, E)$ . This soft topology is called a soft relative topology of  $\tau$  on  $(F, E)$ , and  $[(F, E), \tau_{(F,E)}]$  is called a soft subspace of  $(X, \tau, E)$ .

**Definition 2.21** [?] Let  $SS(X)_A$  and  $SS(Y)_B$  be families of soft sets,  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be mappings. Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a mapping. Then;

If  $(F, A) \in SS(X)_A$ . Then the image of  $(F, A)$  under  $f_{pu}$ , written as  $f_{pu}(F, A) = (f_{pu}(F), p(A))$ , is a soft set in  $SS(Y)_B$  such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{x \in p^{-1}(b) \cap A} u(F(x)), & p^{-1}(b) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise.} \end{cases}$$

for all  $b \in B$ .

If  $(G, B) \in SS(Y)_B$ . Then the inverse image of  $(G, B)$  under  $f_{pu}$ , written as  $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$ , is a soft set in  $SS(X)_A$  such that

$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \emptyset, & \text{otherwise.} \end{cases}$$

for all  $a \in A$ .

The soft function  $f_{pu}$  is called surjective if  $p$  and  $u$  are surjective, also is said to be injective if  $p$  and  $u$  are injective.

**Definition 2.22** [26] Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces and  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then

The function  $f_{pu}$  is called continuous soft (cts-soft) if  $f_{pu}^{-1}(G, B) \in \tau_1 \forall (G, B) \in \tau_2$ .

The function  $f_{pu}$  is called open soft if  $f_{pu}(G, A) \in \tau_2 \forall (G, A) \in \tau_1$ .

**Definition 2.23** [5] Let  $(X, \tau, E)$  be a soft topological space and  $x, y \in X$  such that  $x \neq y$ . Then  $(X, \tau, E)$  is called a soft Hausdorff space or soft  $T_2$  space if there exist open soft sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$ ,  $y \in (G, E)$  and  $(F, E) \tilde{\cap} (G, E) = \tilde{\emptyset}$

### 3 Supra soft topology

**Definition 3.1** Let  $\tau$  be a collection of soft sets over a universe  $X$  with a fixed set of parameters  $E$ , then  $\mu \subseteq SS(X)_E$  is called supra soft topology on  $X$  with a fixed set  $E$  if

1-  $\tilde{X}, \tilde{\emptyset} \in \mu$ ,

2- the union of any number of soft sets in  $\mu$  belongs to  $\mu$ .

The triplet  $(X, \mu, E)$  is called supra soft topological space (or supra soft spaces) over  $X$ .

**Remark 3.1** Every soft topological space is supra soft topological space, but the converse is not true in general as shown in the following example.

**Example 3.1** Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$  where  $(F_1, E), (F_2, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e_1) = \{h_1, h_2\}, \quad F_1(e_2) = \{h_2, h_3\},$$

$$F_2(e_1) = \{h_1, h_3\}, \quad F_2(e_2) = \{h_1, h_2\}.$$

Then  $(X, \mu, E)$  is a supra soft topology, but it is not soft topology.

**Definition 3.2** Let  $(X, \tau, E)$  be a soft topological space and  $(X, \mu, E)$  be a supra soft topological space. We say that  $\mu$  is a supra soft topology associated with  $\tau$  if  $\tau \subset \mu$ .

**Definition 3.3** Let  $(X, \mu, E)$  be a supra soft topological space over  $X$ , then the members of  $\mu$  are said to be supra open soft sets in  $X$ . We denote the set of all supra open soft sets over  $X$  by  $SOS(X, \mu, E)$ , or when there can be no confusion by  $SOS(X)$  and the set of all supra closed soft sets by  $SCS(X, \mu, E)$ , or  $SCS(X)$ .

**Definition 3.4** Let  $(X, \mu, E)$  be a supra soft topological space. A soft set  $(F, A)$  over  $X$  is said to be supra closed soft set in  $X$ , if its relative complement  $(F, A)'$  is supra open soft set.

**Definition 3.5** The soft set  $(F, E) \in SS(X)_E$  is called supra soft point in  $X_E$ , denoted by  $x_e$ , if there exist  $x \in X$  and  $e \in E$  such that  $F(e) = \{x\}$  and  $F(e') = \phi$  for each  $e' \in E - \{e\}$ .

**Definition 3.6** The supra soft point  $x_e$  is said to be belonging to the soft set  $(G, A)$ , denoted by  $x_e \tilde{\in} (G, A)$ , if for the element  $e \in A$ ,  $F(e) \subseteq G(e)$ .

**Definition 3.7** A soft set  $(G, E)$  in a supra soft topological space  $(X, \mu, E)$  is called supra soft neighborhood (briefly: supra nbd) of the supra soft point  $x_e \tilde{\in} X_E$  if there exists a supra open soft set  $(H, A)$  such that  $x_e \tilde{\in} (H, E) \subseteq (G, E)$ . The supra soft neighborhood system of a supra soft point  $x_e$ , denoted by supra  $N_\mu(x_e)$ , is the family of all its supra soft neighborhoods.

**Definition 3.8** Let  $(X, \mu, E)$  be a supra soft topological space over and  $(F, E) \in SS(X)_E$ . Then the supra soft interior of  $(G, E)$ , denoted by  $int^s(G, E)$  is the soft union of all supra open soft subsets of  $(G, E)$ . Clearly  $int^s(G, E)$  is the largest supra open soft set over  $X$  which contained in  $(G, E)$  i.e

$$int^s(G, E) = \tilde{\cup}\{(H, E) : (H, E) \text{ is supra open soft set and } (H, E) \subseteq (G, E)\}.$$

**Definition 3.9** Let  $(X, \mu, E)$  be a supra soft topological space over and  $(F, E) \in SS(X)_E$ . Then the supra soft closure of  $(F, E)$ , denoted by  $cl^s(F, E)$  is the soft intersection of all supra closed supra soft sets of  $(F, E)$ . Clearly  $cl^s(F, E)$  is the smallest supra closed soft set over  $X$  which

contains  $(F, E)$  i.e

$$cl^s(F, E) = \tilde{\cap}\{(H, E) : (H, E) \text{ is supra closed soft set and } (F, E) \cong (H, E)\}.$$

**Definition 3.10** Let  $(X, \mu, E)$  be a supra soft topological space over and  $(G, E) \in SS(X)_E$ . Then  $x_e \in SS(X)_E$  is called supra limit soft point of  $(G, E)$  if  $((G, E) - x_e) \tilde{\cap} (H, E) \neq \tilde{\phi} \forall (H, E) \in SOS(X)$ . The set of all supra limit soft points of  $(F, E)$  is called the supra soft derived of  $(F, E)$  and denoted by  $d^s(F, E)$  or  $F_E^{sd}$ .

In the next theorem, we list the main properties of the operations which give the deviations between these operations and that in soft topological spaces.

**Theorem 3.1** Let  $(X, \mu, E)$  be a supra soft topological space and  $(F, E), (G, E) \in SS(X)_E$ . Then

- 1-  $cl^s(F_E) \tilde{\cup} cl^s(G_E) \cong cl^s((F_E) \tilde{\cup} (G_E))$ .
- 2-  $d^s(F_E) \tilde{\cup} d^s(G_E) \cong d^s((F_E) \tilde{\cup} (G_E))$
- 3-  $int^s((F_E) \tilde{\cap} (G_E)) \cong int^s(F_E) \tilde{\cap} int^s(G_E)$ .

**Proof.** Immediate.

**Remark 3.2** The equality of Theorem 3.1 is not true in general as shown in the following examples.

**Examples 3.1**

1- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e) = \{h_1, h_3\}, F_2(e) = \{h_2, h_4\}, F_3(e) = \{h_1, h_3, h_4\}.$$

Then  $\mu$  defines a supra soft topology on  $X$ . Let  $(G, E)$  and  $(H, E)$  be two soft sets over  $X$  defined by

$$G(e) = \{h_1, h_3\} \text{ and } H(e) = \{h_2\}. \text{ Then } cl^s((G_E) \tilde{\cup} (H_E)) \tilde{\cup} cl^s(G_E) \tilde{\cup} cl^s(H_E).$$

2- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e) = \{h_1, h_3\}, F_2(e) = \{h_2, h_4\}, F_3(e) = \{h_1, h_3, h_4\}.$$

Then  $\mu$  defines a supra soft topology on  $X$ . Let  $(G, E)$  and  $(H, E)$  be two soft sets over  $X$  defined by

$$G(e) = \{h_1, h_3\} \text{ and } H(e) = \{h_2\}. \text{ Then } d^s((G_E) \tilde{\cup} (H_E)) \tilde{\cup} d^s(G_E) \tilde{\cup} d^s(H_E).$$

3- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e) = \{h_1, h_3\}, F_2(e) = \{h_2, h_4\}, F_3(e) = \{h_1, h_3, h_4\}.$$

Then  $\mu$  defines a supra soft topology on  $X$ . Let  $(G, E)$  and  $(H, E)$  be two soft sets over

$X$  defined by

$$G(e) = \{h_1, h_3, h_4\} \quad \text{and} \quad H(e) = \{h_2, h_3, h_4\} \quad . \quad \text{Then}$$

$$int^s(G_E) \tilde{\cap} int^s(H_E) \tilde{\cup} int^s((G_E) \tilde{\cap} (H_E)).$$

#### 4 Subsets of supra soft topological spaces

**Definition 4.1** Let  $(X, \mu, E)$  be a supra soft topological space. A mapping

$\gamma : SS(X)_E \rightarrow SS(X)_E$  is said to be an operation on  $SOS(X)$  if  $F_E \tilde{\subseteq} \gamma(F_E) \forall F_E \in SOS(X)$ .

The collection of all supra  $\gamma$ -open soft sets is denoted by

$SOS(\gamma) = \{F_E : F_E \tilde{\subseteq} \gamma(F_E), F_E \in SS(X)_E\}$ . Also, the complement of supra  $\gamma$ -open soft set is called supra  $\gamma$ -closed soft set, i.e

$SCS(\gamma) = \{F_E : F_E \text{ is a supra } \gamma\text{-open soft set}, F_E \in SS(X)_E\}$  is the family of all supra  $\gamma$ -closed soft sets.

**Definition 4.2** Let  $(X, \mu, E)$  be a supra soft topological space. Different cases of  $\gamma$ -operations on  $SS(X)_E$  are as follows:

1- If  $\gamma = int^s(cl^s)$ , then  $\gamma$  is called supra pre-open soft operator. We denote the set of all supra pre-open soft sets by  $SPOS(X, \mu, E)$ , or when there can be no confusion by  $SPOS(X)$  and the set of all supra pre-closed soft sets by  $SPCS(X, \mu, E)$ , or  $SPCS(X)$ .

2- If  $\gamma = int^s(cl^s(int^s))$ , then  $\gamma$  is called supra  $\alpha$ -open soft operator. We denote the set of all supra  $\alpha$ -open soft sets by  $S\alpha OS(X, \mu, E)$ , or  $S\alpha OS(X)$  and the set of all supra  $\alpha$ -closed soft sets by  $S\alpha CS(X, \mu, E)$ , or  $S\alpha CS(X)$ .

3- If  $\gamma = cl^s(int^s)$ , then  $\gamma$  is called supra semi-open soft operator. We denote the set of all supra semi-open soft sets by  $SSOS(X, \mu, E)$ , or  $SSOS(X)$  and the set of all supra semi-closed soft sets by  $SSCS(X, \mu, E)$ , or  $SSCS(X)$ .

4- If  $\gamma = cl^s(int^s(cl^s))$ , then  $\gamma$  is called supra  $\beta$ -open soft operator. We denote the set of all supra  $\beta$ -open soft sets by  $S\beta OS(X, \mu, E)$ , or  $S\beta OS(X)$  and the set of all supra  $\beta$ -closed soft sets by  $S\beta CS(X, \mu, E)$ , or  $S\beta CS(X)$ .

**Theorem 4.1** Let  $(X, \mu, E)$  be a supra soft topological space and  $\gamma : SS(X)_E \rightarrow SS(X)_E$  be an operation on  $SOS(X)$ .

If  $\gamma \in \{int^s(cl^s), int^s(cl^s(int^s)), cl^s(int^s), cl^s(int^s(cl^s))\}$ . Then

1- Arbitrary soft union of supra  $\gamma$ -open soft sets is supra  $\gamma$ -open soft.

2- Arbitrary soft intersection of supra  $\gamma$ -closed soft sets is supra  $\gamma$ -closed soft.

**Proof.**

1- We give the proof for the case of supra pre-open soft operator i.e  $\gamma = int^s(cl^s)$ . Let



$\{F_{jE} : j \in J\} \subseteq SPOS(X)$ . Then  $\forall j \in J, F_{jE} \cong \text{int}^s(\text{cl}^s(F_{jE}))$ . It follows that  $\bigcup_j F_{jE} \cong \bigcup_j \text{int}^s(\text{cl}^s(F_{jE})) \cong \text{int}^s(\bigcup_j \text{cl}^s(F_{jE})) \cong \text{int}^s(\text{cl}^s(\bigcup_j F_{jE}))$ . Hence  $\bigcup_j F_{jE} \in SPOS(X) \forall j \in J$ . The rest of the proof is similar.

2- Immediate.

**Remark 4.1** The soft intersection of two supra pre-open (resp. supra  $\beta$ -open, supra  $\alpha$ -open, supra semi-open) soft sets need not to be supra pre-open (resp. supra  $\beta$ -open, supra  $\alpha$ -open, supra semi-open), as shown in the following examples.

**Examples 4.1**

1- Let  $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$

where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft sets  $(G, E)$  and  $(H, E)$  which defined as follows:

$$\begin{aligned} G(e_1) &= \{h_1, h_3\}, & G(e_2) &= \{h_1, h_3\}, \\ H(e_1) &= \{h_2, h_3\}, & H(e_2) &= \{h_2, h_3\}, \end{aligned}$$

are supra pre-open soft sets of  $(X, \mu, E)$ , but their soft intersection  $(G, E) \tilde{\cap} (H, E) = (M, E)$ , where  $M(e_1) = \{h_3\}, M(e_2) = \{h_3\}$ , is not supra pre-open soft.

2- Let  $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$

where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft sets  $(G, E)$  and  $(H, E)$  which defined as follows:

$$\begin{aligned} G(e_1) &= \{h_1, h_3\}, & G(e_2) &= \{h_1, h_3\}, \\ H(e_1) &= \{h_2, h_3\}, & H(e_2) &= \{h_2, h_3\}, \end{aligned}$$

are supra  $\beta$ -open soft sets of  $(X, \mu, E)$ , but their soft intersection  $(G, E) \tilde{\cap} (H, E) = (M, E)$  where  $M(e_1) = \{h_3\}, M(e_2) = \{h_3\}$  is not supra  $\beta$ -open soft.

3- Let  $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$  where

$(F_1, E), (F_2, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e_1) = \{h_1, h_2\}, \quad F_1(e_2) = \{h_2\},$$

$$F_2(e_1) = \{h_1, h_3\}, \quad F_2(e_2) = \{h_2, h_3\}.$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft sets  $(F_1, E), (F_2, E)$  are supra  $\alpha$ -open soft sets of  $(X, \mu, E)$ , but their soft intersection  $(F_1, E) \tilde{\cap} (F_2, E) = (M, E)$  where  $M(e_1) = \{h_1\}, M(e_2) = \{h_2\}$ , is not supra  $\alpha$ -open soft set.

4- Let  $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$  where  $(F_1, E), (F_2, E)$  are soft sets over  $X$  defined as follows:

$$F_1(e_1) = \{h_1, h_2\}, \quad F_1(e_2) = \{h_2\},$$

$$F_2(e_1) = \{h_1, h_3\}, \quad F_2(e_2) = \{h_2, h_3\}.$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft sets  $(G, E), (H, E)$  where

$$G(e_1) = \{h_1, h_2\}, \quad G(e_2) = \{h_2, h_3\},$$

$$H(e_1) = \{h_1, h_3\}, \quad H(e_2) = \{h_2, h_3\}.$$

are supra semi-open soft sets of  $(X, \mu, E)$ , but their soft intersection  $(G, E) \tilde{\cap} (H, E) = (M, E)$  where  $M(e_1) = \{h_1\}, M(e_2) = \{h_2\}$ , is not supra semi-open soft set.

## 5 Relations between subsets of supra soft topological spaces

In this section we introduce the relations between some special subsets of a supra soft topological space  $(X, \mu, E)$ .

**Theorem 5.1** *In a supra soft topological space  $(X, \mu, E)$ , the following statements hold,*

- 1- every supra open (resp. closed) soft set is supra pre-open (resp. pre-closed) soft.
- 2- every supra open (resp. closed) soft set is supra semi-open (resp. semi-closed) soft.
- 3- every supra open (resp. closed) soft set is supra  $\alpha$ -open (resp.  $\alpha$ -closed) soft.
- 4- every supra open (resp. closed) soft set is supra  $\beta$ -open (resp.  $\beta$ -closed) soft.

**Proof.** We prove the assertion in the case of supra open soft set, the other case is similar.

1- Let  $(F, E) \in SOS(X)$ . Then  $int^s(F, E) = (F, E)$ . Since  $(F, E) \tilde{\subseteq} cl^s(F, E)$ , then  $(F, E) \tilde{\subseteq} int^s(cl^s(F, E))$ . Therefore,  $(F, E) \in SPOS(X)$ .

2- Let  $(F, E) \in SOS(X)$ . Then  $int^s(F, E) = (F, E)$ . Since  $(F, E) \tilde{\subseteq} cl^s(F, E)$ , then  $(F, E) \tilde{\subseteq} cl^s(int^s(F, E))$ . Thus,  $(F, E) \in SSOS(X)$ .

3- Let  $(F, E) \in SOS(X)$ . Then  $int^s(F, E) = (F, E)$ . Since  $(F, E) \tilde{\subseteq} cl^s(F, E)$ , then  $(F, E) \tilde{\subseteq} int^s(cl^s(F, E)) = int^s(cl^s(int^s(F, E)))$ . Hence  $(F, E) \in S\alpha OS(X)$ .

4- Let  $(F, E) \in SOS(X)$ . Then  $int^s(F, E) = (F, E)$ . Since  $(F, E) \tilde{\subseteq} cl^s(F, E)$ , then  $(F, E) \tilde{\subseteq} int^s(cl^s(F, E))$ . Hence  $(F, E) \tilde{\subseteq} cl^s(F, E) \tilde{\subseteq} cl^s(int^s(cl^s(F, E)))$ . Therefore,  $(F, E) \in S\beta OS(X)$ .

**Remark 5.1** The converse of Theorem 5.1 is not true in general as shown in the following examples.

**Examples 5.1**

1- Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$ , which defined by  $G(e_1) = \{h_1, h_3\}$ ,  $G(e_2) = \{h_1, h_3\}$ , is a supra pre-open soft set of  $(X, \mu, E)$ , but it is not supra open soft.

2- Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$  where  $(F_1, E), (F_2, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1, h_2\}, & F_1(e_2) &= \{h_2\}, \\ F_2(e_1) &= \{h_1, h_3\}, & F_2(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  where  $G(e_1) = \{h_1, h_2\}$ ,  $G(e_2) = \{h_2, h_3\}$ , is a supra semi-open soft set of  $(X, \mu, E)$ , but it is not supra open soft.

3- Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$  where  $(F_1, E), (F_2, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1, h_2\}, & F_1(e_2) &= \{h_2\}, \\ F_2(e_1) &= \{h_1, h_3\}, & F_2(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  where  $G(e_1) = \{h_1, h_2\}$ ,  $G(e_2) = \{h_2, h_3\}$ , is a supra  $\alpha$ -open soft set of  $(X, \mu, E)$ , but it is not supra open soft.

4- Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$ , which defined by  $G(e_1) = \{h_1, h_3\}$ ,  $G(e_2) = \{h_1, h_3\}$ , is a supra  $\beta$ -open soft set of  $(X, \mu, E)$ , but it is not supra open soft.

**Theorem 5.2** Let  $(X, \mu, E)$  be a supra soft topological space, then the following statements are hold,

- 1- Every supra  $\alpha$ -open (resp.  $\alpha$ -closed) soft set is supra semi-open (resp. semi-closed) soft.
- 2- Every supra semi-open (resp. semi-closed) soft set is supra  $\beta$ -open (resp.  $\beta$ -closed) soft.
- 3- Every supra pre-open (resp. pre-closed) soft set is supra  $\beta$ -open (resp.  $\beta$ -closed) soft.
- 4- Every supra  $\alpha$ -open (resp. supra  $\alpha$ -closed) soft set is supra pre-open (resp. supra pre-closed) soft.

**Proof.** We prove the assertion in the case of open soft set, the other case is similar.

1- Let  $(F, E) \in S\alpha OS(X)$ . Then

$$(F, E) \cong \text{int}^s(\text{cl}^s(\text{int}^s(F, E))) \cong \text{cl}^s(\text{int}^s(F, E)). \text{ Hence } (F, E) \in SSOS(X).$$

2- Let  $(F, E) \in SSOS(X)$ . Then  $(F, E) \cong \text{cl}^s(\text{int}^s(F, E))$ . Since

$$(F, E) \cong \text{cl}^s(F, E), \text{ then } (F, E) \cong \text{cl}^s(\text{int}^s(F, E)) \cong \text{cl}^s(\text{int}^s(\text{cl}^s(F, E))). \text{ Thus, } (F, E) \in S\beta OS(X).$$

3- Let  $(F, E) \in SPOS(X)$ . Then

$$(F, E) \cong \text{int}^s(\text{cl}^s(F, E)) \cong \text{cl}^s(\text{int}^s(\text{cl}^s(F, E))). \text{ Hence } (F, E) \in S\beta OS(X).$$

4- Let  $(F, E) \in S\alpha OS(X)$ . Since  $\text{int}^s(F, E) \cong \text{cl}^s(F, E)$ . Then

$$\text{cl}^s(\text{int}^s(F, E)) \cong \text{cl}^s(F, E). \text{ Hence } (F, E) \cong \text{int}^s(\text{cl}^s(\text{int}^s(F, E))) \cong \text{int}^s(\text{cl}^s(F, E)). \text{ Thus, } (F, E) \cong \text{int}^s(\text{cl}^s(F, E)). \text{ It follows that } (F, E) \in SPOS(X).$$

The converse of Theorem 5.2 is not true in general as shown by the following examples.

**Examples 5.2**

1- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e_1, e_2\}$  and

$$\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\} \text{ where}$$

$(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$  are supra soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_2\}, & F_2(e_2) &= \{h_2\}, \\ F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_1, h_2\}, \\ F_4(e_1) &= \{h_1, h_4\}, & F_4(e_2) &= \{h_1, h_4\}, \\ F_5(e_1) &= \{h_1, h_2, h_4\}, & F_5(e_2) &= \{h_1, h_2, h_4\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  which defined as follows:

$$G(e_1) = \{h_2, h_3\}, \quad G(e_2) = \{h_2, h_3\}.$$

is a supra semi-open soft set of  $(X, \mu, E)$ , but it is not supra  $\alpha$ -open soft.

2- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e_1, e_2\}$  and

$$\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\} \text{ where}$$

$(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$  are supra soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}, \\ F_4(e_1) &= \{h_1, h_2, h_3\}, & F_4(e_2) &= \{h_1, h_2, h_3\}, \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  which defined as follows:

$$G(e_1) = \{h_2\}, \quad G(e_2) = \{h_2\}.$$

is a supra  $\beta$ -open soft set of  $(X, \mu, E)$ , but it is not supra semi-open soft.

3- Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $E = \{e_1, e_2\}$  and

$\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$  where

$(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$  are supra soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_2\}, & F_2(e_2) &= \{h_2\}, \\ F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_1, h_2\}, \\ F_4(e_1) &= \{h_1, h_4\}, & F_4(e_2) &= \{h_1, h_4\}, \\ F_5(e_1) &= \{h_1, h_2, h_4\}, & F_5(e_2) &= \{h_1, h_2, h_4\}. \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  which defined as follows:

$$G(e_1) = \{h_1, h_3\}, \quad G(e_2) = \{h_1, h_3\}.$$

is a supra  $\beta$ -open soft set of  $(X, \mu, E)$ , but it is not supra pre-open soft.

4- Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$  and  $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$

where  $(F_1, E), (F_2, E), (F_3, E)$  are soft sets over  $X$  defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= \{h_1, h_2\}, \\ F_3(e_1) &= \{h_2, h_3\}, & F_3(e_2) &= \{h_2, h_3\}, \end{aligned}$$

Then  $\mu$  defines a supra soft topology on  $X$ . Hence the soft set  $(G, E)$  which defined as follows:

$$G(e_1) = \{h_1, h_3\}, \quad G(e_2) = \{h_1, h_3\}.$$

is a supra pre-open soft set of  $(X, \mu, E)$ , but it is not supra  $\alpha$ -open soft.

**Remark 5.2** The following implications hold from Theorem 5.1 and Theorem 5.2 for a soft topological space  $(X, \mu, E)$ . These implications are not reversible.

$$\begin{array}{ccccc} SOS(X) & \rightarrow & S\alpha OS(X) & \rightarrow & SSOS(X) \\ & & \downarrow & \swarrow & \downarrow \\ & & SPOS(X) & \rightarrow & S\beta OS(X) \end{array}$$

**Theorem 5.3** Let  $(X, \tau, E)$  be a soft topological space,  $\gamma : SS(X)_E \rightarrow SS(X)_E$  be one of the operations in Definition 4.2 and  $F_E \in SS(X)_E$ . Then the following hold:

$$1- \gamma(int^s(F'_E)) = \tilde{X} - \gamma(cl^s(F_E)).$$

$$2- \gamma(cl^s(F'_E)) = \tilde{X} - \gamma(int^s(F_E)).$$

**Proof.** We give the proof for the case of supra pre-open soft operator i.e  $\gamma = int^s(cl^s)$ , the other cases is similar.

1- Let  $x_e \notin Pcl^s(F_E)$ . Then  $\exists G_E \in SPO(\tilde{X}, x_e)$  such that  $G_E \tilde{\cap} F_E = \tilde{\phi}$ , hence  $x_e \in G_E \tilde{\subseteq} F'_E$ . Thus,  $x_e \in Pint^s(F'_E)$ . This means that,  $\tilde{X} - Pcl^s(F_E) \tilde{\subseteq} Pint^s(F'_E)$ .

2- Let  $x_e \in Pint^s(F'_E)$ . Since  $Pint^s(F'_E) \tilde{\cap} F_E = \tilde{\phi}$ , so  $x_e \notin Pcl^s(F_E)$ . It follows that  $x_e \in \tilde{X} - Pcl^s(F_E)$ . Therefore,  $Pint^s(F'_E) \tilde{\subseteq} \tilde{X} - Pcl^s(F_E)$ .

Let  $x_e \in Pint^s(F_E)$ . Then  $\forall G_E \in SPO(\tilde{X}, x_e)$ ,  $x_e \in G_E \tilde{\subseteq} F_E$ , hence  $G_E \tilde{\cap} F'_E = \tilde{\phi}$ . Thus,  $x_e \notin Pint^s(F'_E)$ . This means that,  $\tilde{X} - Pint^s(F_E) \tilde{\subseteq} Pcl^s(F'_E)$ .

Let  $x_e \notin Pcl^s(F'_E)$ . Then  $\exists G_E \in SPO(\tilde{X}, x_e)$  such that  $G_E \tilde{\cap} F'_E = \tilde{\phi}$ , hence  $x_e \in G_E \tilde{\subseteq} F_E$ . It follows that  $x_e \in Pint^s(F_E)$ . This means that,  $Pcl^s(F'_E) \tilde{\subseteq} \tilde{X} - Pint^s(F_E)$ . This completes the proof.

**Theorem 5.4** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then

$$1- F_E \in SSOS(X) \text{ if and only if } cl^s(F_E) = cl^s(int^s(F_E)).$$

$$2- \text{ If } G_E \in SOS(X), \text{ then } G_E \tilde{\cap} cl^s(F_E) \tilde{\subseteq} cl^s(G_E \tilde{\cap} F_E).$$

**Proof.** Immediate.

**Theorem 5.5** Let  $(X, \tau, E)$  be a soft topological space and  $F_E, G_E \in SS(X)_E$ . Then

$$1- F_E \in S\alpha OS(X) \text{ if and only if } \exists H_E \in SOS(X) \text{ such that } H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E)).$$

$$2- \text{ If } F_E \in S\alpha OS(X) \text{ and } F_E \tilde{\subseteq} G_E \tilde{\subseteq} int^s(cl^s(F_E)), \text{ then } G_E \in S\alpha OS(X).$$

**Proof.**

$$1- \Rightarrow: \text{ Suppose that } int^s(F_E) = H_E \in SOS(X). \text{ Then } H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E)).$$

$$\Leftarrow: \text{ Let } H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E)), \quad H_E \in SOS(X). \text{ Then } int^s(H_E) = H_E \tilde{\subseteq} int^s(F_E). \text{ It follows that } F_E \tilde{\subseteq} int^s(cl^s(int^s(H_E))) \tilde{\subseteq} int^s(cl^s(int^s(F_E))). \text{ Thus, } F_E \in S\alpha OS(X).$$

2- Let  $F_E \in S\alpha OS(X)$ , then  $F_E \cong int^s(cl^s(int^s(F_E)))$ . Hence  $F_E \cong G_E \cong int^s(cl^s(int^s(cl^s(int^s(F_E)))) \cong int^s(cl^s(int^s(F_E))) \cong int^s(cl^s(int^s(G_E)))$ . Thus,  $G_E \in S\alpha OS(X)$ .

**Theorem 5.6** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then

1-  $F_E \in S\alpha OS(X)$  if and only if  $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$ .

2-  $F_E \in S\alpha CS(X)$  if and only if  $F_E \in SPCS(X) \tilde{\cap} SSCS(X)$ .

**Proof.**

1-  $\Rightarrow$ : Let  $F_E \in S\alpha OS(X)$ , Then  $F_E \cong int^s(cl^s(int^s(F_E)))$ . Hence  $F_E \cong cl^s(int^s(F_E))$  and  $F_E \cong int^s(cl^s(F_E))$ . Thus,  $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$ .

$\Leftarrow$ : Let  $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$ . Then  $F_E \cong cl^s(int^s(F_E))$  and  $F_E \cong int^s(cl^s(F_E))$ . Thus,  $F_E \cong int^s(cl^s(int^s(F_E))) = int^s(cl^s(int^s(F_E)))$ . It follows that  $F_E \in S\alpha OS(X)$ .

2- By a similar way.

**Theorem 5.7** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then  $F_E \in SPCS(X)$  if and only if  $cl^s(int^s(F_E)) \cong F_E$ .

**Proof.** Let  $F_E \in SPCS(X)$ , then  $F'_E$  is a supra pre-open soft set, This means that,  $F'_E \cong int^s(cl^s(\tilde{X} - F_E)) = \tilde{X} - (cl^s(int^s(F_E)))$ . Therefore,  $cl^s(int^s(F_E)) \cong F_E$ . Conversely, let  $cl^s(int^s(F_E)) \cong F_E$ . Then  $\tilde{X} - F_E \cong int^s(cl^s(\tilde{X} - F_E))$ , hence  $\tilde{X} - F_E$  is a supra pre-open soft set. Therefore,  $F_E$  is a supra pre-closed soft set.

**Theorem 5.8** Let  $(X, \mu, E)$  be a supra soft topological space. If  $F_E \in S\alpha OS(X)$  and  $F'_E \in SPOS(X)$ . Then  $F_E \in SOS(X)$ .

**Proof.** Let  $F_E \in S\alpha OS(X)$  and  $F'_E \in SPOS(X)$ . Then  $F_E \in SPCS(X)$ . Hence  $cl^s(int^s(F_E)) \cong F_E \cong int^s(cl^s(int^s(F_E))) \cong cl^s(int^s(F_E))$ . This means that,  $cl^s(int^s(F_E)) = F_E$ . Thus,  $F_E \cong int^s(cl^s(int^s(F_E))) = int^s(F_E)$ . Therefore,  $F_E \in SOS(X)$ .

**Theorem 5.9** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then  $F_E \in S\alpha CS(X)$  if and only if  $cl^s(int^s(cl^s(F_E))) \cong F_E$ .

**Proof.** Let  $F_E \in S\alpha CS(X)$ , then  $F'_E$  is supra  $\alpha$ -open soft set, This means that,  $F'_E \cong int^s(cl^s(int^s(\tilde{X} - F_E))) = \tilde{X} - (cl^s(int^s(cl^s(F_E))))$ . Therefore,  $cl^s(int^s(cl^s(F_E))) \cong F_E$ . Conversely, let  $cl^s(int^s(cl^s(F_E))) \cong F_E$ . Then  $\tilde{X} - F_E \cong int^s(cl^s(int^s(\tilde{X} - F_E)))$ , hence  $\tilde{X} - F_E$  is a supra  $\alpha$ -open soft set. Therefore,  $F_E$  is supra  $\alpha$ -closed soft set.

**Theorem 5.10** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then

$F_E \in SSCS(X)$  if and only if  $int^s(cl^s(F_E)) \cong F_E$ .

**Proof.** Let  $F_E \in SSCS(X)$ , then  $F'_E$  is supra semi-open soft set, This means that,  $F'_E \cong cl^s(int^s(\tilde{X} - F_E)) = \tilde{X} - (int^s(cl^s(F_E)))$ . Therefore,  $int^s(cl^s(F_E)) \cong F_E$ . Conversely, let  $int^s(cl^s(F_E)) \cong F_E$ . Then  $\tilde{X} - F_E \cong cl^s(int^s(\tilde{X} - F_E))$ , hence  $\tilde{X} - F_E$  is a supra semi-open soft set. Therefore,  $F_E$  is a supra semi-closed soft set.

**Corollary 5.1** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then  $F_E \in SSCS(X)$  if and only if  $F_E = F_E \tilde{\cap} int^s(cl^s(F_E))$ .

**Proof.** It is obvious from Theorem 5.10.

**Theorem 5.11** Let  $(X, \mu, E)$  be a supra soft topological space and  $F_E \in SS(X)_E$ . Then  $F_E \in S\beta C(X)$  if and only if  $int^s(cl^s(int^s(F_E))) \cong F_E$ .

**Proof.** Let  $F_E \in S\beta C(X)$ , then  $F'_E$  is a supra  $\beta$ -open soft set, This means that,  $F'_E \cong cl^s(int^s(cl^s(\tilde{X} - F_E))) = \tilde{X} - int^s(cl^s(int^s(F_E)))$ . Therefore,  $int^s(cl^s(int^s(F_E))) \cong F_E$ . Conversely, let  $int^s(cl^s(int^s(F_E))) \cong F_E$ . Then  $\tilde{X} - F_E \cong cl^s(int^s(cl^s(\tilde{X} - F_E)))$ , hence  $\tilde{X} - F_E$  is a supra  $\beta$ -open soft set. Therefore,  $F_E$  is a supra  $\beta$ -closed soft set.

## 6 Decompositions of some forms of supra soft continuity

**Definition 6.1** Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces. Let  $\mu_1$  be an associated supra soft topology with  $\tau_1$ . Let  $u: X \rightarrow Y$  and  $p: A \rightarrow B$  be a mappings. Let  $f_{pu}: SS(X)_A \rightarrow SS(Y)_B$  be a function. Then, the function

1-  $f_{pu}$  is called supra continuous soft function (supra cts soft) if

$$f_{pu}^{-1}(G, B) \in SOS(X, \mu_1, E) \forall (G, B) \in OS(Y).$$

2-  $f_{pu}$  is called supra pre-continuous soft function (supra pre-cts soft) if

$$f_{pu}^{-1}(G, B) \in SPOS(X, \mu_1, E) \forall (G, B) \in OS(Y).$$

3-  $f_{pu}$  is called supra semi-continuous soft function (supra semi-cts soft) if

$$f_{pu}^{-1}(G, B) \in SSOS(X, \mu_1, E) \forall (G, B) \in OS(Y).$$

4-  $f_{pu}$  is called supra  $\alpha$ -continuous soft function (supra  $\alpha$ -cts soft soft) if

$$f_{pu}^{-1}(G, B) \in S\alpha OS(X, \mu_1, E) \forall (G, B) \in OS(Y).$$

5-  $f_{pu}$  is called supra  $\beta$ -continuous soft function (supra  $\beta$ -cts soft soft) if

$$f_{pu}^{-1}(G, B) \in S\beta OS(X, \mu_1, E) \forall (G, B) \in OS(Y).$$



**Theorem 6.1** Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces. Let  $\mu_1$  be an associated supra soft topology with  $\tau_1$ . Let  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be mappings. Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then for the classes, supra pre-continuous (resp. supra  $\alpha$ -continuous soft, supra semi-continuous soft and supra  $\beta$ -continuous soft) functions the following are equivalent (we give an example for the the class of supra pre-continuous soft functions).

- 1-  $f_{pu}$  is supra pre-continuous soft function.
- 2-  $f_{pu}^{-1}(H, B) \in SPCS(X, \mu_1, E) \forall (H, B) \in CS(Y)$ .
- 3-  $f_{pu}(Pcl^s(G, A) \subseteq cl_{\tau_2}(f_{pu}(G, A)) \forall (G, A) \in SS(X)_A$ .
- 4-  $Pcl^s(f_{pu}^{-1}(H, B)) \subseteq f_{pu}^{-1}(cl_{\tau_2}(H, B)) \forall (H, B) \in SS(Y)_B$ .
- 5-  $f_{pu}^{-1}(int_{\tau_2}(H, B)) \subseteq Pint^s(f_{pu}^{-1}(H, B)) \forall (H, B) \in SS(Y)_B$ .

**Proof.**

(1  $\rightarrow$  2) Let  $(H, B)$  be a closed soft set over  $Y$ . Then  $(H, B)' \in OS(Y)$  and  $f_{pu}^{-1}(H, B)' \in SPOS(X, \mu_1, E)$  from Definition 6.1. Since  $f_{pu}^{-1}(H, B)' = (f_{pu}^{-1}(H, B))'$  from [[26], Theorem 3.14]. Thus,  $f_{pu}^{-1}(H, B) \in SPCS(X, \mu_1, E)$ .

(2  $\rightarrow$  3) Let  $(G, A) \in SS(X)_A$ . Since  $(G, A) \cong f_{pu}^{-1}(f_{pu}(G, A)) \cong f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A))) \in SPCS(X, \mu_1, E)$  from (2) and [[26], Theorem 3.14]. Then  $(G, A) \cong Pcl^s(G, A) \cong f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A)))$ . Hence  $f_{pu}(Pcl^s(G, A)) \cong f_{pu}(f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A)))) \cong cl_{\tau_2}(f_{pu}(G, A))$  from [[26], Theorem 3.14]. Thus,  $f_{pu}(Pcl^s(G, A)) \cong cl_{\tau_2}(f_{pu}(G, A))$ .

(3  $\rightarrow$  4) Let  $(H, B) \in SS(Y)_B$  and  $(G, A) = f_{pu}^{-1}(H, B)$ . Then  $f_{pu}(Pcl^s f_{pu}^{-1}(H, B)) \cong cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H, B)))$  From (3). Hence  $Pcl^s(f_{pu}^{-1}(H, B)) \cong f_{pu}^{-1}(f_{pu}(Pcl^s(f_{pu}^{-1}(H, B)))) \cong f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H, B)))) \cong f_{pu}^{-1}(cl_{\tau_2}(H, B))$  from [[26], Theorem 3.14]. Thus,  $Pcl^s(f_{pu}^{-1}(H, B)) \cong f_{pu}^{-1}(cl_{\tau_2}(H, B))$ .

(4  $\rightarrow$  2) Let  $(H, B)$  be a closed soft set over  $Y$ . Then  $Pcl^s(f_{pu}^{-1}(H, B)) \cong f_{pu}^{-1}(cl_{\tau_2}(H, B)) = f_{pu}^{-1}(H, B) = f_{pu}^{-1}(H, B) \forall (H, B) \in SS(Y)_B$  from (4), but clearly  $f_{pu}^{-1}(H, B) \cong Pcl^s(f_{pu}^{-1}(H, B))$ . This means that,  $f_{pu}^{-1}(H, B) = Pcl^s(f_{pu}^{-1}(H, B)) \in SPCS(X, \mu_1, E)$ .

(1  $\rightarrow$  5) Let  $(H, B) \in SS(Y)_B$ . Then  $f_{pu}^{-1}(int_{\tau_2}(H, B)) \in SPOS(X, \mu_1, E)$  from (1).

Hence  $f_{pu}^{-1}(int_{\tau_2}(H, B)) = Pint^s(f_{pu}^{-1}int_{\tau_2}(H, B)) \cong Pint^s(f_{pu}^{-1}(H, B))$ . Thus,

$$f_{pu}^{-1}(int_{\tau_2}(H, B)) \cong Pint^s(f_{pu}^{-1}(H, B)).$$

(5  $\rightarrow$  1) Let  $(H, B)$  be an open soft set over  $Y$ . Then  $int_{\tau_2}(H, B) = (H, B)$  and

$$f_{pu}^{-1}(int_{\tau_2}(H, B)) = f_{pu}^{-1}((H, B)) \cong Pint^s(f_{pu}^{-1}(H, B)) \text{ from (5). But we have}$$

$$Pint^s(f_{pu}^{-1}(H, B)) \cong f_{pu}^{-1}(H, B). \text{ This means that,}$$

$Pint^s(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B) \in SPOS(X, \mu_1, E)$ . Thus,  $f_{pu}$  is supra pre-continuous soft function.

**Theorem 6.2** Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces. Let  $\mu_1$  be an associated supra soft topology with  $\tau_1$ . Let  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be a mappings. Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then

- 1- every supra continuous soft function is supra pre-continuous soft function.
- 2- every supra continuous soft function is supra semi-continuous soft function.
- 3- every supra continuous soft function is supra  $\alpha$ -continuous soft function.
- 4- every supra continuous soft function is supra  $\beta$ -continuous soft function.

**Proof.** It is obvious from Theorem 5.1.

**Theorem 6.3** Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces. Let  $\mu_1$  be an associated supra soft topology with  $\tau_1$ . Let  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be a mappings. Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then

- 1- Every supra  $\alpha$ -continuous soft function is supra semi-continuous soft function.
- 2- Every supra semi-continuous soft function is supra  $\beta$ -continuous soft function.
- 3- Every supra pre-continuous soft function is supra  $\beta$ -continuous soft function.
- 4- Every supra  $\alpha$ -continuous soft function is supra pre-continuous soft function.

**Proof.** It is obvious from Theorem 5.2.

**Theorem 6.4** Let  $(X, \tau_1, A)$  and  $(Y, \tau_2, B)$  be soft topological spaces. Let  $\mu_1$  be an associated supra soft topology with  $\tau_1$ . Let  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be a mappings. Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then  $f_{pu}$  is supra  $\alpha$ -continuous soft function if and only if it is a supra pre-continuous and supra semi-continuous soft function .

**Proof.**

It is obvious from Theorem 5.6.

On accounting of Theorem 6.2 and Theorem 6.3 we have the following corollary.

**Corollary 6.1** *For a soft topological space  $(X, \tau, E)$  and its associated supra soft topology  $\mu$  we have the following implications.*

$$\begin{array}{ccccc}
 \text{supra cts soft} & \rightarrow & \text{supra } \alpha\text{-cts soft} & \rightarrow & \text{supra semi-cts soft} \\
 \downarrow & & \swarrow & & \downarrow \\
 \text{supra pre-cts soft} & & & \rightarrow & \text{supra } \beta\text{-cts soft}
 \end{array}$$

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