Fixed Point Theorem in Fuzzy Metric Space with Implicit relation

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***Abstract*— This paper presents some common fixed point theorem for compatible type (β) mapping in fuzzy metric space. Our result generalizes the result of I.Altun and D.Turkoglu [6].**

***Keywords*— Common Fixed Points, Fuzzy Metric Space, Compatible map, Compatible maps of type (β), Weak compatible maps, Implicit relation. Put your keywords here, keywords are separated by comma.**

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [9], in 1965, as a new way to represent the vagueness in every day life. In mathematical programming problems are expressed as optimizing some goal function given certain constraints, and there are real life problems that consider multiple objectives.Genrally it is very difficult to get a feasible solution that brings us to the optimum of all objective functions. A possible method of resolution, that is quite useful is the one using fuzzy sets [4] .It was developed extensively by many authors and used in various fields to use this concept in topology and analysis several researcher have been defined fuzzy metric space in various ways [3, 7, 8, 10, 11, 14, 17, 19, 21].George and Veeramani [1] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [13] in order to get the Hausdorff topology. Jungck [5] introduced the notion of compatible maps for a pair of self mapping.Mishra et al. [16] obtained common fixed point theorems for compatible maps and asymptocilly commuting maps on fuzzy metric space which generalize,extened and fuzzily several fixed point theorems for contractive-type maps on metric spaces and other spaces.Popa [18] proved theorem for weakly compatible non-continuous mapping using implicit relation..In 2008 I.Altun and D.Turkoglu [6] proved common fixed point theorem on FM space with an implicit relation. In this paper a fixed point theorem has been established using the concept of compatible maps of type (β) which generalized the result of some standard result on fuzzy metric space.

II. PRELIMINARIES

Definition 2.1 *[2]* A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is called a continuous t norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a*b\le c*d$ whenever $a\le c$ and $b\le d$ $a,b,c,d\in[0,1]$.

Example of t-norms are $a*b=ab$ and $a*b=\min\{a,b\}$.

Definition 2.2 *[1]* A Triplet $(X,M,*)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times[0,\infty)$ satisfying the following conditions: for all $x,y,z\in X$ and $t,s>0$,

(i) $M(x,y,0)=1$,

(ii) $M(x,y,t)=M(y,x,t)$,

(iii) $M(x,y,t)=1$ for all t > 0 if and only if $x=y$,

(iv) $M(x,y,\cdot):[0,\infty)\rightarrow[0,1]$ is left continuous,

(v) $M(x,y,t)*\mathrm{M(y,z,s)}\le\mathrm{M(x,z,t+s)}$,

(vi) $\lim_{t\to\infty}M(x,y,t)=1$

Note that $M(\mathrm{x,y,t})$ can be considered as the degree of nearness between x and y with respect to t.We identify $x=y$ with $M(x,y,t)=1$ for all $t>0$.

Definition 2.3 *[2]* Let (X, d) be a metric space. Define $a * b = \min\{\mathrm{a}, \mathrm{b}\}$ and $M(x, y, t) = \dfrac{t}{t + d(x, y)}$ for all $x, y \in X$ and all $t > 0$ .Then $(X, M, *)$ is a fuzzy metric space.It is called the fuzzy metric space induced b the metric d.

Definition 2.4 *[1]* Let (X, M, $*$) be a fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to converges to x in X if for each $\in >0$ and each t > 0, there exists $n_0 \in$ N such that M (x_n, x, t) > 1- $\in$ for all $n \ge n_0$.
(ii) A sequence $\{x_n\}$ in X is said to be Cauchy if for each $\in > 0$ and each t > 0, there exists
$n_0 \in$ N such that M (x_n, x_m, t) > $1 - \in$ for all $n, m \ge n_0$.
(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5 *[18]* Let ϕ be the set of real and continuous function:
ϕ : $(R^{+5}) \longrightarrow$ R, non decreasing in the first, second, third, fourth, fifth argument satisfying the following conditions

$$\phi(u, v, u, v, v) \le 0, \quad u \le v.$$

Example: $\phi(t_1, t_2, t_3, t_4, t_5) = \min(t_2, t_3, t_4, t_5) - t_1$

Definition 2.6 *[15]* Let F and G be maps from a fuzzy metric space $(X, M, *)$ into itself. The maps F and G are said to compatible if

$$\lim_{n\to\infty} M(FGx_n, GFx_n, t) = 1$$

For all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = z \text{ for some } z \in X .$$

Definition 2.7 *[15]* Let F and G be maps from a fuzzy metric space $(X, M, *)$ into itself. The maps F and G are said to compatible of type (β) if

$$\lim_{n\to\infty} M(FFx_n, GGx_n, t) = 1$$

For all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = z \text{ for some } z \in X .$$

Definition 2.8 *[2]* A pair (F, G) of self –map of a fuzzy metric space is said to be weak compatible if F and G commute at their coincidence points .i.e. For $x \in X$ if $Fx = Gx$ then $FGx = GFx$.

Proposition 2.9 *[15]* Let $(X, M, *)$ be a fuzzy metric space and F and G be continuous maps from X into itself. Then F and G are said to compatible if they are compatible of type (β) .

Proposition 2.10*[15]* Let $(X, M, *)$ be a fuzzy metric space and F and G be maps from X into itself. Then F and G are compatible of type (β) and $Fz = Gz$ for some $z \in X$, then $FGz = GGz = GFz = FFz$.

Proposition 2.11 *[15]* Let $(X, M, *)$ be a fuzzy metric space and F and G be compatible of type (β) from X into itself. Let $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = z \text{ for some } z \in X .$$

Then we have the following:

(i) $\lim_{n\to\infty} GGx_n = Fz$ if F is continuous at z,

(ii) $\lim_{n\to\infty} FFx_n = Gz$ if G is continuous at z,

(iii) $FGz = GFz$ and $Fz = Gz$ if F and G are continuous at z.

Lemma 2.12 *[18]* Let $(X, M, *)$ be a fuzzy metric space. If there exists k > 1 such that for all x, y ∈ X, $M(x, y, kt) \le M(x, y, t)$ ∀ t > 0, then x = y.

Lemma 2.13 *[12]* Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X, M(x, y, \cdot)$ is a non-decreasing function.

Lemma 2.14 *[18]* Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, ∗). If there exists a number k > 1 such that $M(x_n, x_{n+1}, kt) \le M(x_{n+1}, x_{n+2}, t)$ ∀ t > 0 and n ∈ N then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.15 *[12]* The only t-norm ∗ satisfying r ∗ r ≥ r for all r ∈ [0, 1] is the minimum t-norm, that is $a\,b = \min\{a, b\}$ for all a, b ∈ [0, 1].

III. MAIN RESULTS

Theorem 3.1 Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(a) P(X) ⊂ ST(X), Q(X) ⊂ AB(X),

(b) One of P or AB is continuous,

(c) PB = BP, ST = TS, AB = BA, QT = TQ,

(d) The pair (P, AB) is compatible maps of type (β) and the pair (Q, ST) is weak compatible,

(e) There exists k > 1 such that for every x, y ∈ X and t > 0

$$\phi\left[M(ABx, STy, kt), M(Px, Qy, t), M(Px, ABx, kt), M(Qy, STy, t), M(Px, STy, t)\right] \le 0$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let x_0 ∈ X be an arbitrary point in X. construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Px_{2n} = STx_{2n+1} = y_{2n+1}$ and $Qx_{2n+1} = ABx_{2n+2} = y_{2n+2}$; $n = 0, 1, 2, --$

Step 1

Put $x = x_{2n}$ and $y = x_{2n+1}$ in equation (e), we get

$$\phi\begin{bmatrix} M(ABx_{2n}, STx_{2n+1}, kt), M(Px_{2n}, Qx_{2n+1}, t), M(Px_{2n}, ABx_{2n}, kt), \\ M(Qx_{2n+1}, STx_{2n+1}, t), M(Px_{2n}, STx_{2n+1,} t) \end{bmatrix} \le 0$$

Letting $n \to \infty$

$$\phi\begin{bmatrix} M(y_{2n,} y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1,} y_{2n}, kt), \\ M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+1}, t) \end{bmatrix} \le 0$$

$$\phi\left[M(y_{2n,} y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), 1\right] \le 0$$

ϕ is non-decreasing in fifth argument

$$\phi\left[M(y_{2n,} y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1}, y_{2n+1}, t)\right] \le 0$$

Therefore by the definition of implicit relation (2.5)

$$M(y_{2n,} y_{2n+1}, kt) \le M(y_{2n+1}, y_{2n+2}, t)$$

Similarly we have

$$M\left(y_{2n+1,}y_{2n+2},kt\right)\le M\left(y_{2n+2},y_{2n+3},t\right)$$

Thus we have

$$M\left(y_{n},y_{n+1}kt\right)\le M\left(y_{n+1},y_{n+2},t\right)$$

Therefore lemma (2.14) $\{y_n\}$ is a Cauchy sequence in X.

Since (X, M, $*$) is complete, $\{y_n\}$ converges to some point z $\in$ X. Also their subsequence converges to the same point. i.e. z $\in$ X

i.e. $\{Px_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$ and $\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$

Case 1

Let P is continuous. Since P is continuous, we have

$$PABx_{2n} \to Pz \text{ and } PPx_{2n} \to Pz$$

As (P, AB) is compatible pair of type (β) therefore we have

$$ABABx_{2n} \to Pz$$

Step 2

Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in equation (e)

$$\phi\begin{bmatrix} M\left(ABABx_{2n},STx_{2n+1},kt\right),M\left(PABx_{2n},Qx_{2n+1},t\right), \\ M\left(PABx_{2n},ABABx_{2n},kt\right),M(Qx_{2n+1},STx_{2n+1},t),M\ (PABx_{2n},STx_{2n+1,}t) \end{bmatrix}\le 0$$

Letting $n \to \infty$

$$\phi\left[M\left(Pz,z,kt\right),M\left(Pz,z,t\right),M\left(Pz,Pz,kt\right),M(z,z,t),M\ (Pz,z,t)\right]\le 0$$

$$\phi\left[M\left(Pz,z,kt\right),M\left(Pz,z,t\right),1,1,M\ (Pz,z,t)\right]\le 0$$

ϕ is non-decreasing in third and fourth argument

$$\phi\left[M\left(Pz,z,kt\right),M\left(Pz,z,t\right),M\left(Pz,z,kt\right),M\left(Pz,z,t\right),M\ (Pz,z,t)\right]\le 0$$

Therefore by the definition of implicit relation (2.5)

$$M\left(Pz,z,kt\right)\le M\left(Pz,z,t\right)$$

Therefore using lemma 2.12 we get

$$Pz = z$$

Step 3

As $P(X)\subseteq ST(X)$, there exists $w \in X$ such that

$$z = Pz = STw$$

We put $x = x_{2n}, y = w$ in (e) we get

$$\phi\left[M\left(ABx_{2n},STw,kt\right),M\left(Px_{2n},Qw,t\right),M\left(Px_{2n},ABx_{2n},kt\right),M(Qw,STw,t),M\ (Px_{2n},STw,t)\right]\le 0$$

Letting $n \to \infty$

$$\phi\left[\mathrm{M}\left(z,z,\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{t}\right),\mathrm{M}\left(z,z,\mathrm{kt}\right),\mathrm{M}(Qw,z,t),M\ (\mathrm{z},z,\mathrm{t})\right]\leq 0$$

$$\phi\left[1,\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{t}\right),1,\mathrm{M}(Qw,z,t),1\right]\leq 0$$

ϕ is non-decreasing in first, third and fifth argument

$$\phi\left[\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{t}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{kt}\right),\mathrm{M}(Qw,z,t),\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{t}\right)\right]\leq 0$$

Therefore by the definition of implicit relation (2.5)

$$\mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{kt}\right)\leq \mathrm{M}\left(\mathrm{z},\mathrm{Q}w,\mathrm{t}\right)$$

Therefore using lemma 2.12 we get

$$z=Qw$$

Hence $STw=Qw=z$ .As (Q,ST) is weak compatible we have $Qz=STz$

Step 4

Taking $x=x_{2n},y=z$ in (e) we get

$$\phi\left[\mathrm{M}\left(\mathrm{ABx}_{2n},\mathrm{STz},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{Px}_{2n},\mathrm{Q}z,\mathrm{t}\right),\mathrm{M}\left(\mathrm{Px}_{2n},\mathrm{ABx}_{2n},\mathrm{kt}\right),\mathrm{M}(Qz,STz,t),M\ (\mathrm{Px}_{2n},\mathrm{STz},\mathrm{t})\right]\leq 0$$

Letting $n\to\infty$

$$\phi\left[\mathrm{M}\left(\mathrm{z},\mathrm{z},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Q}z,\mathrm{t}\right),\mathrm{M}\left(\mathrm{z,z,kt}\right),\mathrm{M}(Qz,z,t),M\ (\mathrm{z,z,t})\right]\leq 0$$

$$\phi\left[1,\mathrm{M}\left(\mathrm{z},\mathrm{Q}z,\mathrm{t}\right),1,\mathrm{M}(Qz,z,t),1\right]\leq 0$$

ϕ is non-decreasing in first, third and fifth argument

$$\phi\left[\mathrm{M}(Qz,z,\mathrm{kt}),\mathrm{M}\left(\mathrm{Q}z,z,\mathrm{t}\right),\mathrm{M}(Qz,z,\mathrm{kt}),\mathrm{M}(Qz,z,t),\mathrm{M}(Qz,z,t)\right]\leq 0$$

Therefore by the definition of implicit relation (2.5)

$$\mathrm{M}(Qz,z,\mathrm{kt})\leq \mathrm{M}\left(\mathrm{Q}z,z,\mathrm{t}\right)$$

Therefore using lemma 2.12 we get

$$Qz=z$$

Thus $Pz=Qz=STz=z$

Since $Q(X)\subseteq AB(\mathrm{X})$ there exists $v\in X$ such that

$$z=Qz=ABv$$

Step 5

Taking $x = v, y = z$ in (e) we get

$$\phi\left[M(ABv, STz, kt), M(Pv, Qz, t), M(Pv, ABv, kt), M(Qz, STz, t), M\ (Pv, STz, t)\right] \le 0$$

Letting $n \to \infty$

$$\phi\left[M(z, z, kt), M(Pv, z, t), M(Pv, z, kt), M(z, z, t), M\ (Pv, z, t)\right] \le 0$$

$$\phi\left[1, M(Pv, z, t), M(Pv, z, kt), 1, M\ (Pv, z, t)\right] \le 0$$

ϕ is non-decreasing in first and fourth argument

$$\phi\left[M(Pv, z, kt), M(Pv, z, t), M(Pv, z, kt), M(Pv, z, t), M\ (Pv, z, t)\right] \le 0$$

Therefore by the definition of implicit relation (2.5)

$$M(Pv, z, kt) \le M(Pv, z, t)$$

Therefore using lemma 2.12 we get

$$Pv = z$$

Thus $Pv = ABv = z$ .As (P, AB) is compatible type (β) and so it is weak compatible.

We get $Pz = ABz$.

Therefore $Pz = ABz = Qz = STz = z$.

Case 2

Let AB is continuous. Since AB is continuous and (P, AB) is compatible type (β) we get

$$\{ABPx_{2n}\} \to ABz,\ \{AB\}^2 x_{2n} \to ABz \text{ and } \{P\}^2 x_{2n} \to ABz$$

Step 6

Taking $x = Px_{2n}, y = x_{2n+1}$ in (e) we get

$$\phi\begin{bmatrix} M(ABPx_{2n}, STx_{2n+1}, kt), M(PPx_{2n}, Qx_{2n+1}, t), \\ M(PPx_{2n}, ABPx_{2n}, kt), M(Qx_{2n+1}, STx_{2n+1}, t), M\ (PPx_{2n}, STx_{2n+1}, t) \end{bmatrix} \le 0$$

Letting $n \to \infty$

$$\phi\left[M(ABz, z, kt), M(ABz, z, t), M(ABz, ABz, kt), M(z, z, t), M\ (ABz, z, t)\right] \le 0$$

$$\phi\left[M(ABz, z, kt), M(ABz, z, t), 1, 1, M\ (ABz, z, t)\right] \le 0$$

ϕ is non-decreasing in third and fourth argument

$$\phi\left[M(ABz, z, kt), M(ABz, z, t), M(ABz, z, kt), M(ABz, z, t), M\ (ABz, z, t)\right] \le 0$$

Therefore by the definition of implicit relation (2.5)

$$M(ABz, z, kt) \le M(ABz, z, t)$$

Therefore using lemma 2.12 we get

$$ABz = z$$

As in step 5 we get $Pz = z$

Hence $Pz = ABz = z$

Hence by step 3 and step 4 it follows that $Pz = STz = Qz = z$ .Thus in both the case we have $Pz = Qz = ABz = STz = z$.

Step 7

Taking $x = z, y = Tz$ in (e) we get

$$\phi\left[\mathrm{M}\left(\mathrm{ABz},\mathrm{ST(Tz)},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{Pz},\mathrm{QTz},\mathrm{t}\right),\mathrm{M}\left(\mathrm{Pz},\mathrm{AB(Tz)},\mathrm{kt}\right),\mathrm{M}(Q(Tz),ST(Tz),t),M\ (\mathrm{Pz},\mathrm{ST(Tz)},\mathrm{t})\right]\leq 0$$

As $QT = TQ$ and $ST = TS$

We have $ST(Tz) = Tz$ and $Q(Tz) = Tz$

Letting $n \to \infty$

$$\phi\left[\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{t}\right),\mathrm{M}\left(\mathrm{z},\mathrm{z},\mathrm{kt}\right),\mathrm{M}(\mathrm{Tz},\mathrm{Tz},t),M\ (\mathrm{z},\mathrm{Tz},\mathrm{t})\right]\leq 0$$

$$\phi\left[\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{t}\right),1,1,M\ (\mathrm{z},\mathrm{Tz},\mathrm{t})\right]\leq 0$$

ϕ is non-decreasing in third and fourth argument

$$\phi\left[\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{t}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{t}\right),M\ (\mathrm{z},\mathrm{Tz},\mathrm{t})\right]\leq 0$$

Therefore by the definition of implicit relation (2.5)

$$\mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{kt}\right)\leq \mathrm{M}\left(\mathrm{z},\mathrm{Tz},\mathrm{t}\right)$$

Therefore using lemma 2.12 we get

$$z = Tz$$

Now $STz = z$ and $Tz = z$ gives $Sz = z$

Step 8

Uniqueness: Let u be another common fixed point of A, B, S, T, P, and Q then $Au = Bu = Pu = Qu = Su = Tu = u$

Taking $x = v, y = u$ in (e) we get

$$\phi\left[\mathrm{M}\left(\mathrm{ABv},\mathrm{STu},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{Pv},\mathrm{Q}u,\mathrm{t}\right),\mathrm{M}\left(\mathrm{Pv},\mathrm{ABv},\mathrm{kt}\right),\mathrm{M}(Qu,STu,t),M\ (\mathrm{Pv},\mathrm{STu},\mathrm{t})\right]\leq 0$$

Letting $n \to \infty$

$$\phi\left[\mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{v},u,\mathrm{t}\right),\mathrm{M}\left(\mathrm{v},\mathrm{v},\mathrm{kt}\right),\mathrm{M}(u,u,t),M\ (\mathrm{v},\mathrm{u},\mathrm{t})\right]\leq 0$$

$$\phi\left[\mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{v},u,\mathrm{t}\right),1,1,M\ (\mathrm{v},\mathrm{u},\mathrm{t})\right]\leq 0$$

ϕ is non decreasing in third and fourth argument

$$\phi\left[\mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{v},u,\mathrm{t}\right),\mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{kt}\right),\mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{t}\right),M\ (\mathrm{v},\mathrm{u},\mathrm{t})\right]\leq 0$$

Therefore by the definition of implicit relation (2.5)

$$\mathrm{M}\left(\mathrm{v},u,\mathrm{kt}\right)\leq \mathrm{M}\left(\mathrm{v},\mathrm{u},\mathrm{t}\right)$$

Therefore using lemma 2.12 we get

$$v = u$$

Thus v is the unique common fixed point of self maps A, B, S, T, P and Q.

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