On Soft Čech Closure Spaces

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Abstract

The purpose of the present paper is to introduce the basic notions of Soft Čech closure spaces and investigate some separation axioms in Soft Čech closure spaces.

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I. INTRODUCTION

Čech closure spaces were introduced by Čech [2]. In Čech's approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalisation of a topological space.

Separation axioms in Čech closure spaces have different implications in comparison with the corresponding topological spaces. According to Čech, a Čech closure space is said to be separated [2] if any two distinct points are separated by distinct neighbourhoods. Separation properties in closure spaces have been studied by various authors D.N.Roth and J.W.Carlson studied [10] a number of separation properties in closure spaces. Some separation properties were studied be Chattopadhy and Hazara[4]

In 1999 D.Molodtsov [7] introduced the notion of soft set to deal with problems of incomplete information. Later, he applied this theory to several directions [8] and [9].

In this paper, we introduce and study the concept of soft Čech closure spaces. Also we study the relation between separation properties in soft Čech closure space (F_A, k) and those in the associated soft topological space (F_A, τ).

II. PRELIMINARIES

Definition 2.1: (see[7]). Let X be an initial universe set, P(X) the power set of X and 'A' a set of parameters. A pair (F,A) or F_A , where F is a map from A to P(X) is called a soft set over X.

In other words, the soft set is a parameterized family of subsets of the set X. Every set $F(\varepsilon), \varepsilon \in A$, from this may $F: A \to P(X)$ defined as follows: $F(a) = \cap \{F_i(a): i \in I\}$, for every $a \in A$. Symbolically, we write be considered as the set of ε-elements of the soft set (F, A) or as the set of ε-approximate elements of the soft set. We denote the family of all soft sets (F, A) or F_A over X as SS(X, A).

As an illustration, let us consider the following example.

Example 2.2: (see[7]) A soft set (F, A) describes the attractiveness of the houses which Mr. Y is going to buy.

X – is the set of houses under consideration.

A – is the set of parameters. Each parameter is a word or a sentence.

A = {expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair}.

In this case, a soft set means to point out expensive houses, beautiful houses and so on. It is worth noting that the sets $F(\varepsilon)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

Definition 2.3: (see[7])Let $F_A, G_A \in SS(X, A)$. We say that F_A is a soft subset of G_A if $F(a) \subseteq G(a)$, for every $a \in A$. Symbolically, we write $F_A \subseteq G_A$. Also, we say F_A and G_A are soft equal if $F_A \subseteq G_A$ and $G_A \subseteq F_A$. Symbolically, we write $F_A = G_A$.

Definition 2.4: (see[12]) Let I be an arbitrary index set and $\{(F_i, A): i \in I\} \subseteq$ SS(X,A). The soft union of these soft sets is the soft set (F, A) ∈ SS(X,A), where the map $F: A \to P(X)$ defined as follows: $F(a) = \cup \{F_i(a): i \in I\}$,for every $a \in A$. Symbolically, we write $(F, A) = F_A = \cup \{(F_i, A): i \in I\}$.

Definition 2.5: (see[12]) Let I be an arbitrary index set and $\{(F_i, A): i \in I\} \subseteq$ SS(X,A). The soft intersection of these soft sets is the soft set (F,A) ∈ SS(X,A), where the map

$$(F, A) = F_A = \cap \{(F_i, A): i \in I\}.$$

Definition 2.6: (see [12]) Let $F_A \in SS(X,A)$. The soft complement of F_A is the soft set $H_A \in SS(X,A)$, where the map $H: A \to P(X)$ defined as follows: H(a) = X\F(a), for every $a \in A$. Symbolically, we write $H_A = (F_A)^C$.

Definition 2.7: (see[7])The soft set $F_A \in SS(X,A)$ where $F(a) = \emptyset,\ for\ every\ a \in A$ is called the A-null soft set of SS(X,A) and denoted by $\mathbf{0_A}$. The soft set $F_A \in SS(X,A)$ where $F(a) = X, for\ every\ a \in A$ is called the A-absolute soft set of SS(X,A) and denoted by $\mathbf{1_A}$.

Definition 2.8: A soft set $U_A \in SS(X,A)$ is called a soft point in F_A ,denoted by e_F , if for the element $e \in \Delta$, $F(e) \neq \emptyset\ and\ F(e') = \emptyset\ for\ all\ e' \in \Delta - \{e\}$.

Definition 2.9: A soft point e_F is said to be in the soft set F_A, denoted by $e_F \in F_A$, if for the element $e \in A \cap C\ and\ \ F(e) \subseteq G(e)$.

Definition 2.10: (see [12])Let X and Y be two initial universe sets, A and B two set of parameters, $f: X \to Y\ and\ e: A \to B$. Then , by φ_{fe} we denote the map from SS(X,A) to SS(Y,B) for which:

(1) If $F_A \in SS(X,A)$, then the image of F_A under φ_{fe} , denoted by $\varphi_{fe}(F_A)$, is the soft set $G_A \in SS(Y,B)$ such that

$$G(ay) = \begin{cases} \cup\{\, f(F(a)) : a \in\ e^{-1}(\{ay\})\}, & if\ e^{-1}(\{ay\}) \neq \emptyset \\ \emptyset\ , & if\ \ e^{-1}(\{ay\}) = \emptyset \end{cases}$$

for every $ay \in B$.

(2) If $G_B \in SS(Y,B)$, then the inverse image of G_B under φ_{fe} , denoted by $\varphi_{fe}^{-1}(G_B)$, is the soft set $F_A \in SS(X,A)$ such that $F(a) = f^{-1}(G(e(a)))$, for every $a \in A$.

III. SOFT ČECH CLOSURE SPACES

In this section we introduce the concept of Soft Čech closure spaces and investigate some of their properties.

Definition 3.1: Let X be an initial universe set, A be a set of parameters. Then the function $k: P(X_{F_A}) \to P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech Closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$
(C2) $F_A \subseteq k(F_A)$
(C3) $k(F_A \cup G_A) = k(F_A) \cup k(G_A)$

The triplet (X, k, A) or (F_A, k) is called a Soft Čech Closure space.

Example 3.2: Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $P(X_{F_A})$ are
$F_{1A} = \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\},\ F_{3A} = \{(x_1, \{u_1, u_2\})\}$,
$F_{4A} = \{(x_2, \{u_1\})\},\ F_{5A} = \{(x_2, \{u_2\})\},\ F_{6A} = \{(x_2, \{u_1, u_2\})\}$,
$F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,
$F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$,
$F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$,
$F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A$.

An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{8A}, k(F_{2A}) = F_{9A}, k(F_{4A}) = F_{7A}, k(F_{5A}) = F_{10A}$,
$k(F_{7A}) = F_{11A}, k(F_{8A}) = F_{14A}, k(F_{9A}) = F_{13A}$,
$k(F_{10A}) = F_{12A}, k(F_{3A}) = k(F_{6A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A,\ k(\emptyset_A) = \emptyset_A$.
The triplet (X, k, A) or (F_A, k) is called a Soft Čech Closure space.

Definition 3.3: A closure operator k on a soft set F_A is called idempotent if $k(U_A) = kk(U_A)$ for all $U_A \subseteq F_A$.

Definition 3.4: A soft subset U_A of a Soft Čech Closure space (F_A, k) is said to be soft closed if $k(U_A) = U_A$.

Definition 3.5: A soft subset U_A of a Soft Čech Closure space (F_A, k) is said to be soft open if $k(U_A{}^C) = U_A{}^C$. i.e., $Int(U_A) = U_A$.

Definition 3.6: A soft set $Int(U_A)$ with respect to the closure operator k is defined as $Int(U_A) = F_A - k(F_A - U_A) = k(U_A{}^C),\ where\ U_A^C = F_A - U_A$.

Definition 3.7: A soft subset U_A in a Soft Čech Closure space (F_A, k) is called Soft neighbourhood of $e_F\ if\ e_F \in Int\ (U_A)$.

Definition 3.8: If (F_A, k) be a Soft Čech Closure space, then the associate soft topology on F_A is $\tau = \{U_A{}^C : k(U_A) = U_A\}$.

Definition 3.9: Let (F_A, k) be a Soft Čech Closure space. A Soft Čech Closure space (G_A, k^*) is called a soft subspace of (F_A, k) if $G_A \subseteq F_A$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq G_A$.

Definition 3.10: Let (F_A, k) and (G_B, k^*) be two Soft Čech Closure spaces over X and Y respectively. For $x \in X\ and\ e: A \to B$, a map $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous if $\Phi_{fe}(k(F,A)) \subseteq k^*\Phi_{fe}(F,A)$, for every soft subset $(F,A) \subseteq SS(X,A)$.

On the other hand a map $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous if and only if $k\Phi_{fe}^{-1}(G,B) \subseteq \Phi_{fe}^{-1}(k^*(G,B))$, for every soft subset $(G,B) \subseteq SS(Y,B)$. Clearly, if $f: (F_A, k) \to (G_B, k^*)$ is said to be soft e-continuous then $\Phi_{fe}^{-1}(U_B)$ is a soft closed subset of (F_A, k) for every soft closed subset U_B of (G_B, k^*).

Example 3.11: Let us consider the soft subsets of F_A that are given in *example 3.2.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$k(F_{1A}) = k(F_{2A}) = k(F_{3A}) = F_{3A}, k(F_{4A}) = k(F_{6A}) = F_{6A}$, $k(F_{5A}) = F_{5A}$, $k(F_{8A}) = k(F_{10A}) = k(F_{14A}) = F_{14A}$, $k(F_{7A}) = k(F_{9A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_A) = F_A$, $k(\emptyset_A) = \emptyset_A$. Therefore, (F_A, k) is a Soft Čech Closure space.

Let the initial universe set $Y = \{v_1, v_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $B = \{x_1, x_2\} \subseteq E$ and $G_B = \{(x_1, \{v_1, v_2\}), (x_2, \{v_1, v_2\})\}$. Then $P(Y_{G_B})$ are $G_{1B} = \{(x_1, \{v_1\})\}$, $G_{2B} = \{(x_1, \{v_2\})\}, G_{3B} = \{(x_1, \{v_1, v_2\})\}$, $G_{4B} = \{(x_2, \{v_1\}, G_{5B} = \{(x_2, \{v_2\})\}$, $G_{6B} = \{(x_2, \{v_1, v_2\})\}$, $G_{7B} = \{(x_1, \{v_1\}), (x_2, \{v_1\})\}, G_{8B} = \{(x_1, \{v_1\}), (x_2, \{v_2\})\}$, $G_{9B} = \{(x_1, \{v_2\}), (x_2, \{v_1\})\}, G_{10B} = \{(x_1, \{v_2\}), (x_2, \{v_2\})\}$, $G_{11B} = \{(x_1, \{v_1\}), (x_2, \{v_1, v_2\})\}$, $G_{12B} = \{(x_1, \{v_2\}), (x_2, \{v_1, v_2\})\}$, $G_{13B} = \{(x_1, \{v_1, v_2\}), (x_2, \{v_1\})\}$, $G_{14B} = \{(x_1, \{v_1, v_2\}), (x_2, \{v_2\})\}, G_{15B} = G_B, G_{16B} = \emptyset_B$.

An operator $k^*: P(Y_{G_B}) \to P(Y_{G_B})$ is defined from soft power set $P(Y_{G_B})$ to itself over Y as follows.
$k^*(G_{1B}) = G_{1B}, k^*(G_{2B}) = k^*(G_{3B}) = G_{3B}, k^*(G_{4B}) = G_{4B}$, $k^*(G_{7B}) = G_{7B}$, $k^*(G_{5B}) = k^*(G_{6B}) = G_{6B}$, $k^*(G_{8B}) = k^*(G_{11B}) = G_{11B}, k^*(G_{9B}) = k^*(G_{13B}) = G_{13B}$, $k^*(G_{10B}) = k^*(G_{12B}) = k^*(G_{14B}) = k^*(G_B) = G_B$, $k^*(\emptyset_B) = \emptyset_B$. The triplet (Y, k^*, B) or (G_B, k^*) is also a Soft Čech Closure space.

Let $f: (F_A, k) \to (G_B, k^*)$ be the map such that $f(u_1) = f(u_2) = v_1$ and $e: A \to B$ the map such that $e(x_1) = x_2$ *and* $e(x_2) = x_1$. Then the map f is soft e-continuous.

Proposition 3.12: Let (F_A, k) be a Soft Čech Closure space and let $U_A \subseteq F_A$, then
(i) U_A is soft open if and only if $U_A = F_A - k(F_A - U_A)$.
(ii)If V_A is soft open and $V_A \subseteq U_A$, then $V_A \subseteq F_A - k(F_A - U_A)$.

Proof:(i) Assume that U_A is soft open. Then $F_A - U_A$ is soft closed. This implies, $k(F_A - U_A) = F_A - U_A$.
So, $F_A - k(F_A - U_A) = F_A - (F_A - U_A)$.
Therefore, $U_A = F_A - k(F_A - U_A)$.

Conversely, let V_A be a soft open subset of (F_A, k) such that $F_A - U_A \subseteq V_A$. Then, $F_A - V_A \subseteq U_A$. Since, $F_A - V_A$ is soft closed subset of (F_A, k).
We have, $F_A - V_A \subseteq F_A - k(F_A - U_A)$.

Consequently, $k(F_A - U_A) \subseteq V_A$. Hence, $F_A - U_A$ is soft closed and so U_A is soft open.

(ii)Let V_A is soft open and $V_A \subseteq U_A$, then by (i) , we get $V_A \subseteq F_A - k(F_A - U_A)$.

Proposition 3.13: Let (F_A, k) be a Soft Čech closure space and let (G_A, k^*) be a soft closed subspace of (F_A, k). If U_A is a soft closed subset of (G_A, k^*), then U_A is a soft closed subset of (F_A, k).

Proof: Let U_A is soft closed subset of (G_A, k^*). Then $k^*(U_A) = U_A$. Since G_A is soft closed subset of (F_A, k). This implies $k(U_A) = U_A$. Therefore, U_A is a soft closed subset of (F_A, k).

Proposition 3.14: Let (F_A, k) be a Soft Čech closure space, if U_A and V_A are soft closed sets then $(U_A \, U \, V_A)$ also soft closed.

Proof: Let (F_A, k) be a soft Čech closure space. Let U_A and V_A be two soft Čech closed sets. $k(U_A) = U_A$ and $k(V_A) = V_A$. Since, by additivity,
$k(U_A \cup V_A) = k(U_A) U \, k(V_A) = U_A \, U \, V_A$. Hence, $U_A \, U \, V_A$ is also soft Čech closed.

Proposition 3.15: Let (F_A, k) be a Soft Čech closure space and let $U_A \subseteq F_A$. If U_A is Soft closed set then $k(U_A) - U_A$ contains no non-empty soft closed sets.

Proof: Let, (F_A, k) be a soft Čech closure space. Let V_A be a soft closed subset of (F_A, k) such that $V_A \subseteq k(U_A) - U_A$. Then $U_A \subseteq F_A - V_A$. Since, U_A is soft closed and $F_A - V_A$ is a soft open subset of (F_A, k). Then, $k(U_A) \subseteq F_A - V_A$. This implies, $V_A \subseteq F_A - k(U_A)$ and we get $V_A \subseteq (F_A - k(U_A)) \cap k(U_A) = \emptyset_A$. Therefore $V_A = \emptyset_A$. Hence, $k(U_A) - U_A$ contains no non-empty soft closed sets.

Proposition 3.16: Let (F_A, k) be a Soft Čech closure space. If $U_A \subseteq F_A$ is soft closed, then $k(U_A) - U_A$ is soft open.

Proof: Let (F_A, k) be a Soft Čech closure space. Suppose that $U_A \subseteq F_A$ is soft closed and let V_A be a soft closed subset of (F_A, k) such that $V_A \subseteq k(U_A) - U_A$. *By proposition 3.15* $V_A = \emptyset_A$ and hence $V_A \subseteq F_A - k(F_A - (k(U_A) - U_A))$.
By proposition 3.12(ii) , $k(U_A) - U_A$ is soft open.

Proposition 3.17: Let (F_A, k) be a Soft Čech closure space. If U_A *and* V_A be two soft open sets, then $U_A \cap V_A$ also soft open.

Proof: Let (F_A, k) be a Soft Čech closure space. Let U_A *and* V_A be two soft open sets, then U_A^C and V_A^C are soft closed sets. This implies $k(U_A^C) = U_A^C$ and $k(V_A^C) = V_A^C$. Since, by additivity, $k(U_A^C \cup V_A^C) = k(U_A^C) \cup k(V_A^C) = U_A^C \cup V_A^C$. Therefore, $U_A^C \cup V_A^C$ is soft closed . That is, $(U_A \cap V_A)^C$ is soft closed. This implies, $U_A \cap V_A$ is soft open.

IV. SEPARATION AXIOMS ON SOFT ČECH CLOSURE SPACES

In this section we discuss some properties of separation axioms on soft Čech closure spaces. Also we study the relation between soft Čech closure space (F_A, k) and those in the associated soft topological space (F_A, τ).

Definition 4.1: A Soft Čech closure space (F_A, k) is said to be T_0-space if and only if for every distinct points $x \neq y$ and for each $a \in A$ *either* $x \notin_a k(\{y\})$ *or* $y \notin_a k(\{x\})$.

Definition 4.2: A Soft Čech closure space (F_A, k) is said to be $T_{1/2}$-space if each singleton soft subset of (F_A, k) is soft closed or soft open.

Definition 4.3: A Soft Čech closure space (F_A, k) is said to be T_1-space if and only if for every distinct points $x \neq y$ and for each $a \in A$ *there exist* $x \notin_a k(\{y\})$ *and* $y \notin_a k(\{x\})$.

Theorem 4.4: A Soft Čech closure subspace of a T_0-space is T_0.

Proof: Let (F_A, k) be a soft Čech closure T_0-space and (G_A, k^*) be the soft subspace of (F_A, k). Let x *and* y are two distinct points and for each $a \in A$ *in* G_A. Since, $(G_A, k^*) \subseteq (F_A, k)$, then either $x \notin_a k(\{y\})$ or $y \notin_a k(\{x\})$. This implies either $x \notin_a k(\{y\}) \cap G_A$ or $y \notin_a k(\{x\}) \cap G_A$.
Hence, (G_A, k^*) is a T_0-space.

Result 4.5: Let (F_A, k) be a Soft Čech closure space, then $k(U_A) \subset \tau\_cl(U_A)$, where $\tau\_cl(U_A)$ is a soft topological closure with respect to k, $\forall\, U_A \subset F_A$.

Proof: Let (F_A, k) be a Soft Čech closure space.
We have, $U_A \subset \tau\_cl(U_A)$
Therefore, $k(U_A) \subset k(\tau\_cl(U_A))$ ....(1)
Since, $\tau\_cl(U_A)$ is closed
Therefore, $k(\tau\_cl(U_A)) = \tau\_cl(U_A)$ ....(2)
From (1) & (2), we have, $k(U_A) \subset \tau\_cl(U_A), \forall\, U_A \subset F_A$.

Theorem 4.6: If (F_A, τ) is T_0-space, then (F_A, k) is also T_0-space.

Proof: Let (F_A, τ) be a soft topological space. Assume (F_A, τ) be T_0-space. Let $x \neq y$, *and for each* $a \in A$ *either*
$x \notin_a \tau - cl(\{y\})$ *or* $y \notin_a \tau - cl(\{x\})$.
We have, $k(U_A) \subset \tau\_cl(U_A), \forall\, U_A \subset F_A$.
So, $x \notin_a \tau - cl(\{y\})$ implies $x \notin_a k(\{y\})$ *or*
$y \notin_a \tau - cl(\{x\})$ implies $y \notin_a k(\{x\})$.
Therefore, $x \notin_a k(\{y\})$ *or* $y \notin_a k(\{x\})$. Hence, soft Čech closure space (F_A, k) is T_0-space.

Corrolary 4.7: The converse of the above theorem is not true.

Proof: In *example 3.2.* The associate soft topology on F_A is $\tau = \{\emptyset_A, F_A\}$. Therefore, (F_A, τ) is not a T_0-space.

Theorem 4.8: A Soft Čech closure subspace of a T_1-space is T_1.

Proof: Let (F_A, k) be a soft Čech closure T_1-space and (G_A, k^*) be the soft subspace of (F_A, k). Let x *and* y are two distinct points and for each $a \in A$ *in* G_A. Since, $(G_A, k^*) \subseteq (F_A, k)$, then there exist $x \notin_a k(\{y\})$ and $y \notin_a k(\{x\})$. This implies $x \notin_a k(\{y\}) \cap G_A$ and $y \notin_a k(\{x\}) \cap G_A$.
Hence, (G_A, k^*) is a T_1-space.

Remark 4.9: Let (F_A, k) be a soft Čech closure space, then for every T_1-space is also a T_0-space. But the converse is not true.

Example 4.10: Let us consider the soft subsets of F_A that are given in *example 3.2.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A},\ k(F_{2A}) = k(F_{3A}) = F_{3A},\ k(F_{4A}) = F_{4A},$
$k(F_{7A}) = F_{7A}, k(F_{5A}) = k(F_{6A}) = F_{6A},$
$k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{9A}) = k(F_{13A}) = F_{13A},$
$k(F_{10A}) = k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.$
Here, (F_A, k) is a soft Čech closure space, which is T_0-space but not T_1.

Theorem 4.11: For a Soft Čech closure space (F_A, k) the following are equivalent.
(1) The soft Čech closure space (F_A, k) is T_1.
(2) For every $x \in X$ *and for every* $a \in A$, the singleton soft set $\{(a, x)\}$ is soft closed with respect to k.
(3) Every finite soft subset of F_A is closed with respect to k.

Proof: (1) → (2)
Let the soft Čech closure space (F_A, k) is T_1. Let $x \neq y$ and for every $a \in A$. Suppose the singleton soft set $\{(a, x)\}$ is not closed with respect to k. Then $k(\{(a, x)\}) \neq \{(a, x)\}, \forall\, a \in A$. Therefore there exists $y \neq x$ such that $y \in_a k(\{x\})$. Which is a contradiction. Therefore, the singleton soft set $\{(a, x)\}$ is soft closed.
(2) → (3) Let for every $(a, x) \in F_A$, the singleton soft set $\{(a, x)\}$ is soft closed with respect to k. Since, finite union of soft closed set is closed, therefore every finite soft subset of F_A is closed with respect to k.
(3) → (2) Let every finite soft subset of F_A is closed with respect to k. Since, the singleton soft set $\{(a, x)\}$ is finite. Therefore, $\{(a, x)\}$ is soft closed with respect to k.
(2) → (1) Let $x \neq y$ and for every $a \in A$, assume that every singleton soft sets are soft closed with respect to k. Therefore, $k(\{(a, x)\}) = \{(a, x)\}$ and $k(\{(a, y)\}) = \{(a, y)\}$. This implies $y \notin_a k(\{x\})$ *and* $x \notin_a k(\{y\})$. Hence (F_A, k) is T_1-space.

Definition 4.12: A Soft Čech closure space (F_A, k) is said to be Hausdorff or T_2-space if for every distinct points $x \neq y$ and for each $a \in A$, there exists disjoint soft open sets G_A, H_A such that $x \in_a G_A$ *and* $y \in_a H_a$.

Example 4.13: Let us consider the soft subsets of F_A that are given in *example 3.2.* An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = F_{2A},\ k(F_{3A}) = F_{3A},\ k(F_{4A}) = F_{4A},$
$k(F_{5A}) = F_{5A}, k(F_{6A}) = F_{6A}, k(F_{7A}) = F_{7A}, k(F_{8A}) = F_{8A},$
$k(F_{9A}) = F_{9A}, k(F_{10A}) = F_{10A,} k(F_{11A}) = F_{11A},$
$k(F_{12A}) = F_{12A}, k(F_{13A}) = F_{13A}, k(F_{14A}) = F_{14A,}$

$k(F_A) = F_A, k(\emptyset_A) = \emptyset_A$. Here, $(F_{A,} k)$ is a is T_2-space

Lemma 4.14: Let $(F_{A,} k)$ be a Soft Čech closure space and let (G_A, k^*) be a soft closed subspace of $(F_{A,} k)$. If U_A is a soft open subset of $(F_{A,} k)$, then $U_A \cap G_A$ is also a soft open subset of (G_A, k^*).

Proof: Let U_A be a soft open subset of $(F_{A,} k)$. Then $F_A - U_A$ is a soft closed subset of $(F_{A,} k)$. Since, (G_A, k^*) be a soft closed subset of $(F_{A,} k)$, then $(F_A - U_A) \cap G_A$ is a soft closed subset of $(F_{A,} k)$.

But, $(F_A - U_A) \cap G_A = G_A - (U_A \cap G_A)$. Therefore, $G_A - (U_A \cap G_A)$ is a soft closed subset of (G_A, k^*). Hence, $(U_A \cap G_A)$ is a soft open subset of (G_A, k^*).

Proposition 4.15: Let $(F_{A,} k)$ be a Soft Čech closure space and let (G_A, k^*) be a soft closed subspace of $(F_{A,} k)$. If $(F_{A,} k)$ is a Hausdorff space, then (G_A, k^*) is also Hausdorff Soft Čech closure space.

Proof: Let $x \neq y$ be any two distinct points and for each $a \in A$ of (G_A, k^*). Then x and y are distinct points of $(F_{A,} k)$. Since, $(F_{A,} k)$ is a Hausdorff soft Čech closure space, there exists a disjoint soft open subsets U_A *and* V_A such that $x \in U_A,\ y \in V_A$ *and* $U_A \cap V_A = \emptyset_A$.
Consequently, $x \in U_A \cap G_A$, $y \in V_A \cap G_A$ and $(U_A \cap G_A) \cap (V_A \cap G_A) = \emptyset_A$. *By lemma 4.14,* $U_A \cap G_A$ and $V_A \cap G_A$ are soft open subset of (G_A, k^*). Hence (G_A, k^*) is a Hausdorff soft Čech closure space.

Theorem 4.16: If the Soft topological space $(F_{A,} \tau)$ is Hausdorff, then the Soft Čech closure space $(F_{A,} k)$ is also Hausdorff.

Proof: Let $(F_{A,} \tau)$ be a Hausdorff soft topological space. Then for any two points $x \neq y$ and for each $a \in A$, there exists soft τ-open subsets U_A *and* V_A of x and y respectively such that $U_A \cap V_A = \emptyset_A$.

Since, each soft τ-neighbourhood in $(F_{A,} \tau)$ is also soft k-neighbourhood in $(F_{A,} k)$. Therefore, for each $a \in A$ there exists U_A *and* V_A are soft k-neighbourhood of x and y and respectively in $(F_{A,} k)$ such that $U_A \cap V_A = \emptyset_A$.

Definition 4.17: A Soft Čech closure space $(F_{A,} k)$ is said to be Semi Hausdorff if for every $x \neq y$ and for each $a \in A$, either there exists soft open set U_A such that $x \in_a U_A$ and $y \notin_a k(U_A)$ or there exists soft open set V_A such that $y \in_a V_A$ and $x \notin_a k(V_A)$.

Definition 4.18: A Soft Čech closure space $(F_{A,} k)$ is said to be Pseudo Hausdorff if for every $x \neq y$ and for each $a \in A$, either there exists soft open set U_A such that $x \in_a U_A$ and $y \notin_a k(U_A)$ and there exists soft open set V_A such that $y \in_a V_A$ and $x \notin_a k(V_A)$.

Theorem 4.19: If Soft topological space $(F_{A,} \tau)$ is Hausdorff, then $(F_{A,} k)$ is Pseudo Hausdorff.

Proof: Let the soft topological space $(F_{A,} \tau)$ is Hausdorff. Let $x \neq y$ and for each $a \in A$, there exists a soft τ-open set U_A such that $x \in_a U_A$ *and* $y \notin_a \tau - cl(U_A)$ and there exists a soft open set V_A such that $y \in_a V_A$ *and* $x \notin_a \tau - cl(V_A)$.
Since, $k(W_A) \subset \tau - cl(W_A), \forall\, W_A \subset F_A$.
Therefore, $k(U_A) \subset \tau - cl(U_A)$ *and* $k(V_A) \subset \tau - cl(V_A)$.
This implies, $y \notin_a \tau - cl(U_A) = y \notin_a k(U_A)$ and
$x \notin_a \tau - cl(V_A) = x \notin_a k(V_A)$. Hence, the soft Čech closure space $(F_{A,} k)$ is Pseudo Hausdorff.

Theorem 4.20: Let Soft Čech closure space $(F_{A,} k)$ is Pseudo Hausdorff, then every subspace (G_A, k^*) of $(F_{A,} k)$ is also Pseudo Hausdorff.

Proof: Let Soft Čech closure space $(F_{A,} k)$ is Pseudo Hausdorff. Let (G_A, k^*) be the subspace of (F_A, k). Since, (F_A, k) is Pseudo Hausdorff, then $x \neq y$ and for each $a \in A$, there exists soft open sets U_A *and* V_A such that $x \in_a U_A$, $y \notin_a k(U_A)$ *and* $y \in_a V_A$, $x \notin_a k(V_A)$. Then $U_A \cap G_A$ and $V_A \cap G_A$ are soft open sets respectively in G_A such that $x \in_a U_A \cap G_A$ *and* $y \notin_a k(U_A \cap G_A)$
also $y \in_a V_A \cap G_A$ *and* $x \notin_a k(V_A \cap G_A)$. Therefore, (G_A, k^*) is Pseudo Hausdorff.

Theorem 4.21: Let Soft Čech closure space $(F_{A,} k)$ is Pseudo Hausdorff, then $(F_{A,} k)$ is T_1.

Proof: Let $\neq y$. Since, $(F_{A,} k)$ is Pseudo Hausdorff. For each $a \in A$, there exists soft open sets U_A *and* V_A such that $x \in_a U_A$, $y \notin_a k(U_A)$ and $y \in_a V_A$, $x \notin_a k(V_A)$.

This implies, $x \in_a U_A \Rightarrow k(x) \subset k(U_A)$ and $y \in_a V_A \Rightarrow k(y) \subset k(V_A)$. Therefore, $x \notin_a k(y)$ *and* $y \notin_a k(x)$. Hence, $(F_{A,} k)$ is T_1.

Definition 4.22: A Soft Čech closure space $(F_{A,} k)$ is said to be Uryshon space if given $x \neq y$ and for each $a \in A$, there exists soft open sets U_A and V_A such that $x \in_a U_A$,$y \in_a V_A$ *and* $k(U_A) \cap k(V_A) = \emptyset_A$.

Theorem 4.23: If the soft topological space (F_A, τ) is Uryshon space, then the Soft Čech closure space (F_A, k) is also Uryshon space.

Proof: The proof is similar to the proof of *theorem 4.16.*

Theorem 4.24: If a soft Čech closure space (F_A, k) is Uryshon, then every soft subspace (G_A, k^*) of (F_A, k) is Uryshon.

Proof: Let a soft Čech closure space (F_A, k) is Uryshon and let (G_A, k^*) be a soft subspace of (F_A, k). Since, (F_A, k) is Uryshon. Given, $x \neq y$ and for each $a \in A$, there exists soft open sets $U_A \text{ and } V_A$ Such that $x \in_a U_A$, $y \in_a V_A \text{ and } k(U_A) \cap k(V_A) = \emptyset_A$. Now , $U_A \cap G_A$ and $V_A \cap G_A$ are soft open sets respectively in (G_A, k^*) such that $x \in_a U_A \cap G_A$, $y \in_a V_A \cap G_A$.

Consider, $k^*(U_A \cap G_A) \cap k^*(V_A \cap G_A)$

$$= [k^*(U_A \cap G_A)] \cap [k^*(V_A) \cap G_A]$$
$$= [k^*(U_A) \cap k^*(V_A)] \cap G_A$$
$$= [k(U_A) \cap k(V_A)] \cap G_A$$
$$= \emptyset_A \cap G_A = \emptyset_A.$$

Therefore, (G_A, k^*) is Uryshon space.

V. CONCLUSION

Čech closure spaces were introduced by Čech [2]. Molodtsov [7] initiated the mathematical tool of soft set theory. In this paper, we introduced and studied the basic notions of soft Čech closure spaces. Also we discussed the relation between separation properties in soft Čech closure space (F_A, k) and those in the associated soft topological space (F_A, τ).

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