Weak Reciprocally Continuity and Common Fixed Point Theorem In Complete Intuitionistic Fuzzy Metric Spaces

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Abstract: In this paper, we define the new concept of reciprocally and weak reciprocally continuity in complete intuitionistic fuzzy metric spaces and prove a common fixed point theorem for self mappings in intuitionistic fuzzy metric spaces under the condition of weak reciprocally continuous. All the results of this paper are new.

Keywords: Intuitionistic fuzzy metric space, continuous t-norm and t-conorm, common fixed point, coincidence point, weak reciprocally continuous maps.

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1. INTRODUCTION

Fixed point theory has wide and significant contribution in modern mathematics. In solving the existence and uniqueness solution of differential equations, integral equations and random differential equations in other related areas. Fixed point theory plays a vital role in solving Eigen value problems and boundary value problems. It is also helpful in mechanics, theory of games and topological dynamics and for the characterization of the completeness of metric space. The theory of fuzzy sets has revolved in many directions after investigation the notion fuzzy set by Zadeh [23] in 1965 and in finding applications and wide variety of fields in which the phenomenon under study are too complex or too ill defined to be analyzed by the conventional techniques. It is a new way to represent vagueness in our everyday life. There is large number of authors who studied applications of fuzzy set theory in pure and applied mathematics. Here we mention some of them Fet'z [4] established results on finite elements method with fuzzy parameters and applications of possibilities and evidence theory in civil engineering. A method utilizing the mathematics of fuzzy sets has been shown to be effective in solving engineering problems such as air craft gas turbine [10], car body structure NVH design [11], multi-objective system optimization [16], preliminary passenger vehicle structure [17], computational tools for preliminary engineering design [22], knowledge base system design [25], intelligent system design support [24], machine flexibility [19] and many others. The concept of fuzzy set corresponds to the degree of nearness between two objects. Deng [3], George and Veeramani [6], Kramosil and Michalek [9] have introduced the concept of fuzzy metric spaces in different ways. Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and non-nearness. Park [15] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [6]. Turkoglu et. al. [20] gave a generalization of Jungck's common fixed point theorem [7] to intuitionistic fuzzy metric spaces. They first formulated the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [13]. Recently, Pant, Bisht and Arora [14] introduced the concept of weakly reciprocally continuous. In this paper, we give some concept in weak reciprocally continuity and prove a common fixed point theorem in intuitionistic fuzzy metric space under the condition of weak reciprocally continuity.

2. PRELIMINARIES

Definition 2.1[5]: A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2[5]: A binary operation $\Diamond: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$;

(iv) $a \lozenge b \geq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3[1]: A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) For all $x, y \in X$, $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous;

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) For all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.1: Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated as

$x \lozenge y = 1 - ((1-x) * (1-y))$ for all x, y∈ X.

Remark 2.2: In intuitionistic fuzzy metric space X, M(x, y, ·) is non-decreasing and N(x, y, ·) is non-increasing for all x, y ∈ X.

Example 2.1[2]: Let (x, d) be a metric space, define t-norm a * b = min {a, b} and t-conorm

a ◊ b = max {a, b} and for all x, y ∈ X and t > 0,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition 2.4[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,
$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x∈ X if, for all t > 0,

$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi) of definition (3), respectively.

Definition 2.5[1]: An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.6[15]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. Then the maps A and B are said to be compatible if, for all t > 0,

$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$
whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some x∈ X.

Definition 2.7[8]: Two self maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weak compatible if they commute at their coincidence points. i.e. Ax = Bx for some x in X, then ABx = BAx.

Definition 2.8[5]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. A and B be self mappings in X. Then a point x in X is called a coincidence point of A and B iff Ax = Bx. In this case y = Ax = Bx is called a point of coincidence of A and B.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.9[18]: Two self maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of A and B at which A and B commute.

Definition 2.10[12]: Two mappings A and B of a intuitionistic fuzzy metric space (X, M, N, *, ◊) will be called reciprocal continuous if, $ABx_n \to Ax$ and $BAx_n \to Bx$ whenever $\{x_n\}$ is a sequence such that $Ax_n, Bx_n \to x$, for some $x \in X$.

If A and B are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings A and B implies their reciprocal continuity but not conversely.

Definition 2.11[14]: Two mappings A and B of a intuitionistic fuzzy metric space (X, M, N, *, ◊) will be called weakly reciprocal continuous $\lim_{n\to\infty} ABx_n = Az$ or $\lim_{n\to\infty} BAx_n = Bz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X.

Lemma 2.1[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. if there exists a number $k \in (0, 1)$, such that

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t) \text{ and } N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.2[21]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all x, y in X,

$t > 0$ and if there exists a number $k \in (0, 1)$

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t), \text{ then } x = y.$$

3. MAIN RESULTS

Theorem 3.1: Let $(X, M, N, *, \Diamond)$ be an complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \Diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$ and let A, B, S and T be four mappings from X into itself, such that

(3.1) the pairs (A, S) and (B, T) are weak reciprocally continuous;

(3.2) A and S have a coincidence point;

(3.3) B and T have a coincidence point;

(3.4) then there exists a constant $k \in (0,1)$ such that

$$[1 + M(Sx, Ty, kt)] * M(Ax, By, kt) \geq \mathrm{Min}\{ M(Sx, By, kt), M(Ax, Ty, kt), M(Ax, By, 2kt),$$

$$\left(\frac{M(Sx,Ax,t)+M(Ty,By,t)+M(Sx,Ax,t)\cdot M(Ty,By,t)}{3(M(Sx,Ax,t)\cdot M(Ty,By,t))}\right), M(Sx, Ty, 2kt)\} + \mathrm{Min}\{M(Sx, By, t), M(Ax, Ty, t),$$

$$M(Ax, By, t), M(Sx, Ty, t)\}$$

and $[1 + N(Sx, Ty, kt)] \Diamond N(Ax, By, kt) \leq \mathrm{Max}\{N(Sx, By, kt), N(Ax, Ty, kt), N(Ax, By, 2kt),$

$$\left(\frac{N(Sx,Ax,t)+N(Ty,By,t)+N(Sx,Ax,t)\cdot N(Ty,By,t)}{3(N(Sx,Ax,t)\cdot N(Ty,By,t))}\right), N(Sx, Ty, 2kt)\} + \mathrm{Max}\{N(Sx, By, t), N(Ax, Ty, t),$$

$$N(Ax, By, t), N(Sx, Ty, t)\}$$

For all $x, y \in X$, $t > 0$. Then A, B, S and T have a unique common fixed point in X.

Proof: Since the pairs (A, S) and (B, T) are weak reciprocally continuous, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$, $u \in X$, and satisfy $\lim_{n\to\infty} ASx_n = Au$ or $\lim_{n\to\infty} SAx_n = Su$.

and $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = v$, $v \in X$, and satisfy $\lim_{n\to\infty} BTy_n = Bv$ or $\lim_{n\to\infty} TBy_n = Tv$.

Therefore, $Au = Su$ and $Bv = Tv$, that is u is a coincidence point of A and S and v is a coincidence point of B and T.

Now we have to show that $u = v$. Putting $x = x_n$ and $y = y_n$ in (3.4), we get

$[1 + M(Sx_n, Ty_n, kt)]*M(Ax_n, By_n, kt) \geq Min\{M(Sx_n, By_n, kt), M(Ax_n, Ty_n, kt), M(Ax_n, By_n, 2kt),$

$\left(\frac{M(Sx_n,Ax_n,t)+M(Ty_n,By_n,t)+M(Sx_n,Ax_n,t)\cdot M(Ty_n,By_n,t)}{3(M(Sx_n,Ax_n,t)\cdot M(Ty_n,By_n,t))}\right), M(Sx_n, Ty_n, 2kt)\} + Min\{M(Sx_n, By_n, t),$

$M(Ax_n, Ty_n, t), M(Ax_n, By_n, t), M(Sx_n, Ty_n, t)\}$

and $[1 + N(Sx_n, Ty_n, kt)]\Diamond N(Ax_n, By_n, kt) \leq Max\{N(Sx_n, By_n, kt), N(Ax_n, Ty_n, kt),$

$N(Ax_n, By_n, 2kt), \left(\frac{N(Sx_n,Ax_n,t)+N(Ty_n,By_n,t)+N(Sx_n,Ax_n,t)\cdot N(Ty_n,By_n,t)}{3(N(Sx_n,Ax_n,t)\cdot N(Ty_n,By_n,t))}\right), N(Sx_n, Ty_n, 2kt)\}$

$+Max\{N(Sx_n, By_n, t), N(Ax_n, Ty_n, t), N(Ax_n, By_n, t), N(Sx_n, Ty_n, t)\}.$

Taking $n\to\infty$, we get

$[1 + M(u, v, kt)]*M(u, v, kt) \geq Min\{M(u, v, kt), M(u, v, kt), M(v, v, 2kt),$

$\left(\frac{M(u,u,t)+M(v,v,t)+M(u,u,t)\cdot M(v,v,t)}{3(M(Sx,Ax,t)\cdot M(Ty,By,t))}\right), M(u, v, 2kt)\} + Min\{M(u, v, t), M(u, v, t), M(u, v, t), M(u, v, t)\}$

$M(u, v, kt) + M(u, v, kt)*M(u, v, kt) \geq Min\{M(u, v, kt), M(u, v, kt), 1, \left(\frac{1+1+1\cdot 1}{3(1\cdot 1)}\right), M(u, v, 2kt)\}$

$+M(u, v, t)$

$M(u, v, kt) + M(u, v, kt) \geq Min\{M(u, v, kt), M(u, v, kt), 1, 1, M(u, v, 2kt)\} + M(u, v, t)$

$\geq M(u, v, kt) + M(u, v, t)$

and $[1 + N(u, v, kt)\Diamond N(u, v, kt)] \leq Max\{N(u, v, kt), N(u, v, kt), N(v, v, 2kt),$

$\left(\frac{N(u,u,t)+N(v,v,t)+N(u,u,t)\cdot N(v,v,t)}{3(N(Sx,Ax,t)\cdot N(Ty,By,t))}\right), N(u, v, 2kt)\} + Max\{N(u, v, t), N(u, v, t), N(u, v, t), N(u, v, t)\}$

$N(u, v, kt) + N(u, v, kt)\Diamond N(u, v, kt) \leq Max\{N(u, v, kt), N(u, v, kt), 0, \left(\frac{0+0+0\cdot 0}{3(0\cdot 0)}\right), N(u, v, 2kt)\}$

$+N(u, v, t)$

$N(u, v, kt) + N(u, v, kt) \leq Max\{N(u, v, kt), N(u, v, kt), 0, 0, N(u, v, 2kt)\} + N(u, v, t)$

$\leq N(u, v, kt) + N(u, v, t).$

That is, $M(u, v, kt) \geq M(u, v, t)$ and $N(u, v, kt) \leq N(u, v, t)$,

By lemma (2.2), we get $u = v$.

Now, we claim that $Au = u$. Putting $x = u$ and $y = y_n$ in (3.4), we get

$$[1 + M(Su, Ty_n, kt)]*M(Au, By_n, kt) \geq Min\{M(Su, By_n, kt), M(Au, Ty_n, kt), M(Au, By_n, 2kt), \left(\frac{M(Su,Au,t)+M(Ty_n,By_n,t)+M(Su,Au,t)\cdot M(Ty_n,By_n,t)}{3(M(Su,Au,t)\cdot M(Ty_n,By_n,t))}\right), M(Su, Ty_n, 2kt)\} + Min\{M(Su, By_n, t), M(Au, Ty_n, t), M(Au, By_n, t), M(Su, Ty_n, t)\}$$

and
$$[1 + N(Su, Ty_n, kt)]\Diamond N(Au, By_n, kt) \leq Max\{N(Su, By_n, kt), N(Au, Ty_n, kt), N(Au, By_n, 2kt), \left(\frac{N(Su,Au,t)+N(Ty_n,By_n,t)+N(Su,Au,t)\cdot N(Ty_n,By_n,t)}{3(N(Su,Au,t)\cdot N(Ty_n,By_n,t))}\right), N(Su, Ty_n, 2kt)\} + Max\{N(Su, By_n, t), N(Au, Ty_n, t), N(Au, By_n, t), N(Su, Ty_n, t)\}.$$

Taking $n \to \infty$ and $Su = Au$, we get

$$[1 + M(Au, v, kt)]*M(Au, v, kt) \geq Min\{M(Au, v, kt), M(Au, v, kt), M(Au, v, 2kt), \left(\frac{M(Au,Au,t)+M(v,v,t)+M(Au,Au,t)\cdot M(v,v,t)}{3(M(Au,Au,t)\cdot M(v,v,t))}\right), M(Au, v, 2kt)\} + Min\{M(Au, v, t), M(Au, v, t), M(Au, v, t), M(Au, v, t)\}$$

$$M(Au, v, kt + M(Au, v, kt)*M(Au, v, kt) \geq Min\{M(Au, v, kt), M(Au, v, kt), M(Au, v, 2kt), \left(\frac{1+1+1\cdot 1}{3(1\cdot 1)}\right), M(Au, v, 2kt)\} + M(Au, v, t)$$

$$M(Au, v, kt + M(Au, v, kt) \geq Min\{M(Au, v, kt), M(Au, v, kt), M(Au, v, 2kt), 1, M(Au, v, 2kt)\} + M(Au, v, t)$$

$$\geq M(Au, v, kt) + M(Au, v, t)$$

and
$$[1 + N(Au, v, kt)]\Diamond N(Au, v, kt) \leq Max\{N(Au, v, kt), N(Au, v, kt), N(Au, v, 2kt), \left(\frac{N(Au,Au,t)+N(v,v,t)+N(Au,Au,t)\cdot N(v,v,t)}{3(N(Au,Au,t)\cdot N(v,v,t))}\right), N(Au, v, 2kt)\} + Max\{N(Au, v, t), N(Au, v, t),$$

N(Au, v, t),N(Au, v, t)}

$N(Au, v, kt) + N(Au, v, kt)\lozenge N(Au, v, kt) \leq Max\{N(Au, v, kt), N(Au, v, kt), N(Au, v, 2kt),$

$\left(\frac{0+0+0\cdot 0}{3(0\cdot 0)}\right), N(Au, v, 2kt)\} + N(Au, v, t)$

$N(Au, v, kt) + N(Au, v, kt) \leq Max\{N(Au, v, kt), N(Au, v, kt), N(Au, v, 2kt), 0, N(Au, v, 2kt)\}$

$+ N(Au, v, t)$

$\leq N(Au, v, kt) + N(Au, v, t)$

That is, $M(Au, v, kt) \geq M(Au, v, t)$ and $N(Au, v, kt) \leq N(Au, v, t)$

By lemma (2.2), we get Au = v = u.

Again we claim that Bu = u. putting x = u and y = u in (3.4), we get

$[1 + M(Su, Tu, kt)] * M(Au, Bu, kt) \geq Min\{M(Su, Bu, kt), M(Au, Tu, kt), M(Au, Bu, 2kt),$

$\left(\frac{M(Su,Au,t)+M(Tu,Bu,t)+M(Su,Au,t)\cdot M(Tu,Bu,t)}{3(M(Su,Au,t)\cdot M(Tu,Bu,t))}\right), M(Su, Tu, 2kt)\} + Min\{M(Su, Bu, t), M(Au, Tu, t),$

$M(Au, Bu, t), M(Su, Tu, t)\}$

and $[1 + N(Su, Tu, kt)]\lozenge N(Au, Bu, kt) \leq Max\{N(Su, Bu, kt), N(Au, Tu, kt), N(Au, Bu, 2kt),$

$\left(\frac{N(Su,Au,t)+N(Tu,Bu,t)+N(Su,Au,t)\cdot N(Tu,Bu,t)}{3(N(Su,Au,t)\cdot N(Tu,Bu,t))}\right), N(Su, Tu, 2kt)\} + Max\{N(Su, Bu, t), N(Au, Tu, t),$

$N(Au, Bu, t), N(Su, Tu, t)\}.$

For Su = u and Tu = Bu, we get

$[1 + M(u, Bu, kt)] * M(u, Bu, kt) \geq Min\{M(u, Bu, kt), M(u, Bu, kt), M(u, Bu, 2kt),$

$\left(\frac{M(u,u,t)+M(Bu,Bu,t)+M(u,u,t)\cdot M(Bu,Bu,t)}{3(M(u,u,t)\cdot M(Bu,Bu,t))}\right), M(u, Bu, 2kt)\} + Min\{M(u, Bu, t), M(u, Bu, t),$

$M(u, Bu, t), M(u, Bu, t)\}$

$M(u, Bu, kt) + M(u, Bu, kt) * M(u, Bu, kt) \geq Min\{M(u, Bu, kt), M(u, Bu, kt), M(u, Bu, 2kt),$

$\left(\frac{1+1+1\cdot1}{3(1\cdot1)}\right)$,M(u, Bu, 2kt)}+M(u, Bu, t)

M(u, Bu, kt) + M(u, Bu, kt)≥Min{M(u, Bu, kt),M(u, Bu, kt),M(u, Bu, 2kt),1,M(u, Bu, 2kt)}

+M(u, Bu, t)

≥M(u, Bu, t) + M(u, Bu, t)

and [1 + N(u, Bu, kt)]◊N(u, Bu, kt)≤Max{N(u, Bu, kt),N(u, Bu, kt),N(u, Bu, 2kt),

$\left(\frac{N(u,u,t)+N(Bu,Bu,t)+N(u,u,t)\cdot N(Bu,Bu,t)}{3(N(u,u,t)\cdot N(Bu,Bu,t))}\right)$,N(u, Bu, 2kt)}+Max{N(u, Bu, t), N(u, Bu, t),

N(u, Bu, t),N(u, Bu, t)}

N(u, Bu, kt) + N(u, Bu, kt)◊N(u, Bu, kt)≤Max{N(u, Bu, kt),N(u, Bu, kt),N(u, Bu, 2kt),$\left(\frac{0+0+0\cdot0}{3(0\cdot0)}\right)$,

N(u, Bu, 2kt)}+N(u, Bu, t)

N(u, Bu, kt) + N(u, Bu, kt)◊≤Max{N(u, Bu, kt),N(u, Bu, kt),N(u, Bu, 2kt),0,N(u, Bu, 2kt)}

+N(u, Bu, t)

≤N(u, Bu, kt) + N(u, Bu, t)

That is, M(u, Bu, kt)≥M(u, Bu, k) and N(u, Bu, kt)≤N(u, Bu, t)

By lemma (2.2), we get Bu = u = Tu. Therefore, u = Au = Bu = Su = Tu.

That is, u is a common fixed point of A, B, S and T.

For uniqueness, let w be another fixed point of A, B, S and T. Then we have Aw = Bw = Sw = Tw = w. Putting x = u and y = w in (3.4), we get

[1 + M(Su, Tw, kt)]∗M(Au, Bw, kt)≥Min{M(Su, Bw, kt),M(Au, Tw, kt),M(Au, Bw, 2kt),

$\left(\frac{M(Su,Au,t)+M(Tw,Bw,t)+M(Su,Au,t)\cdot M(Tw,Bw,t)}{3(M(Su,Au,t)\cdot M(Tw,Bw,t))}\right)$,M(Su, Tw, 2kt)}+Min{M(Su, Bw, t),M(Au, Tw, t),

M(Au, Bw, t),M(Su, Tw, t)}

$[1 + M(u, w, kt)]*M(u, w, kt) \geq \text{Min}\{M(u, w, kt), M(u, w, kt), M(u, w, 2kt),$

$\left(\frac{M(u,u,t)+M(w,w,t)+M(u,u,t)\cdot M(w,w,t)}{3(M(u,u,t)\cdot M(w,w,t))}\right), M(u, w, 2kt)\} + \text{Min}\{M(u, w, t), M(u, w, t) M(u, w, t),$

$M(u, w, t)\}$

$M(u, w, kt) + M(u, w, kt)*M(u, w, kt) \geq \text{Min}\{M(u, w, kt), M(u, w, kt), M(u, w, 2kt), \left(\frac{1+1+1\cdot 1}{3(1\cdot 1)}\right),$

$M(u, w, 2kt)\} + M(u, w, t)$

$M(u, w, kt) + M(u, w, kt) \geq \text{Min}\{M(u, w, kt), M(u, w, kt), M(u, w, 2kt), 1, M(u, w, 2kt)\} + M(u, w, t)$

$\geq M(u, w, kt) + M(u, w, t)$

and $[1 + N(Su, Tw, kt)]\Diamond N(Au, Bw, kt) \leq \text{Max}\{N(Su, Bw, kt), N(Au, Tw, kt), N(Au, Bw, 2kt),$

$\left(\frac{N(Su,Au,t)+N(Tw,Bw,t)+N(Su,Au,t)\cdot N(Tw,Bw,t)}{3(N(Su,Au,t)\cdot N(Tw,Bw,t))}\right), N(Su, Tw, 2kt)\} + \text{Max}\{N(Su, Bw, t), N(Au, Tw, t),$

$N(Au, Bw, t), N(Su, Tw, t)\}$

$[1 + N(u, w, kt)]\Diamond N(u, w, kt) \leq \text{Max}\{N(u, w, kt), N(u, w, kt), N(u, w, 2kt),$

$\left(\frac{N(u,u,t)+N(w,w,t)+N(u,u,t)\cdot N(w,w,t)}{3(N(u,u,t)\cdot N(w,w,t))}\right), N(u, w, 2kt)\} + \text{Max}\{N(u, w, t), N(u, w, t), N(u, w, t), N(u, w, t)\}$

$N(u, w, kt) + N(u, w, kt)\Diamond N(u, w, kt) \leq \text{Max}\{N(u, w, kt), N(u, w, kt), N(u, w, 2kt), \left(\frac{0+0+0\cdot 0}{3(0\cdot 0)}\right),$

$N(u, w, 2kt)\} + N(u, w, t)$

$N(u, w, kt) + N(u, w, kt) \leq \text{Max}\{N(u, w, kt), N(u, w, kt), N(u, w, 2kt), 0, N(u, w, 2kt)\} + N(u, w, t)$

$\leq N(u, w, kt) + N(u, w, t)$

That is, $M(u, w, kt) \geq M(u, w, t)$ and $N(u, w, kt) \leq N(u, w, t)$

By lemma (2.2), we get u = w. Hence u is a unique common fixed point of A, B, S and T.

If we take T = S in theorem 3.1 we get the following result:

Corollary 3.2: Let (X, M, N, $*$, $\Diamond$) be an complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \Diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$ and let A, B and S be three mappings from X into itself, such that

(3.1) the pairs (A, S) and (B, S) are weakly reciprocally continuous;

(3.2) A and S have a coincidence point;

(3.3) B and S have a coincidence point;

(3.4) then there exists a constant $k \in (0,1)$ such that

$$[1 + M(Sx, Sy, kt)] * M(Ax, By, kt) \geq \mathrm{Min}\{M(Sx, By, kt), M(Ax, Sy, kt), M(Ax, By, 2kt), \left(\frac{M(Sx,Ax,t)+M(Sy,By,t)+M(Sx,Ax,t)\cdot M(Sy,By,t)}{3(M(Sx,Ax,t)\cdot M(Sy,By,t))}\right), M(Sx, Sy, 2kt)\} + \mathrm{Min}\{M(Sx, By, t), M(Ax, Sy, t), M(Ax, By, t), M(Sx, Sy, t)\}$$

and
$$[1 + N(Sx, Sy, kt)] \Diamond N(Ax, By, kt) \leq \mathrm{Max}\{N(Sx, By, kt), N(Ax, Sy, kt), N(Ax, By, 2kt), \left(\frac{N(Sx,Ax,t)+N(Sy,By,t)+N(Sx,Ax,t)\cdot N(Sy,By,t)}{3(N(Sx,Ax,t)\cdot N(Sy,By,t))}\right), N(Sx, Sy, 2kt)\} + \mathrm{Max}\{N(Sx, By, t), N(Ax, Sy, t), N(Ax, By, t), N(Sx, Sy, t)\}$$

for all $x, y \in X$, $t > 0$. Then A, B and S have a unique common fixed point in X.

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