Total Outer Equitable Connected Domination of a Graph

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ABSTRACT:

Let G = (V, E) be a graph. A set D $\subseteq$ V (G) is equitable dominating set of G if $\forall$ v $\in$ V – D $\exists$ a vertex u $\in$ D such that uv $\in$ E (G) and |d(u) – d(v)| $\leq$ 1. A set D $\subseteq$ V (G) is outer equitable dominating set if D is equitable dominating and <V – D> is connected graph. The outer equitable connected domination number of G is the minimum cardinality of the outer-equitable connected dominating set of G and is denoted by $\gamma_{oec}(G)$. We introduce in this paper the concept of total outer equitable connected domination, exact values for some particular classes of graphs are found, some results on total outer equitable domination number are also established.

Keywords: Graph, total outer equitable connected dominating set, total outer equitable connected domination number.

Mathematics Subject Classification: 05C69

INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A set D $\subseteq$ V(G) is a dominating set of G if for every vertex v $\in$ V(G) – D, there exists a vertex u $\in$ D such that v and u are adjacent. For a survey of results on domination, see[3]. A subset D of V is called an equitable dominating set if for every v $\in$ V – D there exists a vertex u $\in$ D such that uv $\in$ E(G) and |deg(u) – deg(v)| $\leq$ 1, where deg(u) is degree of vertex u and deg(v) is degree of vertex v. The equitable neighborhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V/v \in N(u), |deg(u) - deg(v)| \leq 1\}$ and $u \in I_e \Leftrightarrow N_e(u) = \phi$. The maximum and minimum equitable degree of a point in G are denoted respectively by $\Delta_e(G)$ and $\delta_e(G)$. That is $\Delta_e(G) = \max_{u \in V(G)} |N_e(u)|$, $\delta_e(G) = \min_{u \in V(G)} |N_e(u)|$. An equitable dominating set D is said to be minimal equitable dominating set if no proper subset of D is an equitable dominating set[1].

Let D be a subset of V. D is said to be a total equitable dominating set if for all x $\in$ V, there exists y $\in$ D such that y is adjacent to x and y is degree equitable with x.
If G has no equitable isolated point, then V is a total equitable dominating set. For such graphs, the minimum cardinality of a total equitable dominating set is called the total equitable

domination number and is denoted by $\gamma_{te}(G)$. If G without any equitable isolated H vertices then set D $\subseteq$ V(G) is a total outer connected equitable dominating set if D is total equitable dominating set of G and the induced subgraph <V – D> is connected. The minimum cardinality of total outer connected equitable dominating set in G is the total outer connected equitable domination number of G and is denoted by $\gamma_{toec}(G)$. For any undefined terms we refer Harary[2].

RESULTS

Proposition 1. For any graph G without any equitable isolated vertices.

$$\gamma(G) \leq \gamma_e(G) \leq \gamma_{oec}(G) \leq \gamma_{toec}(G)$$

Proof.

Since every total outer connected equitable dominating set of G is outer connected equitable dominating set and every outer connected equitable dominating set is equitable dominating set and every equitable dominating set is dominating set of G.

Hence, $\gamma(G) \leq \gamma_e(G) \leq \gamma_{oec}(G) \leq \gamma_{toec}(G)$.

In the following results we present the total outer equitable connected domination number of complete graphs, cycles, paths and complete bipartite graphs.

Proposition 2.

i) $\gamma_{toec}(k_p) = 2$ if $p \geq 3$

ii) $\gamma_{toec}(k_{n,m}) = 2$ if $|m - n| \leq 1$

iii) $\gamma_{toec}(W_p) = 2$ if $p \leq 5$

iv) $\gamma_{toec}(C_p) = p - 2$ if $p \geq 4$

v) $\gamma_{toec}(P_p) = \begin{cases} p-1 & \text{if } p = 3, 4, 5 \\ p-2 & \text{if } p \geq 6 \end{cases}$

Proposition 3. For any connected graph G without any equitable isolated vertices.

$$\gamma_{te}(G) \leq \gamma_{toec}(G)$$

Proof.

The proof is straight forward since any total outer equitable connected dominating set of G is also total equitable dominating set of G implies,

$$\gamma_{te}(G) \leq \gamma_{toec}(G).$$

Definition 4. Let G be a graph then the pendent vertex $v \in V(G)$ is called equitable pendent vertex if the equitable degree of v is equal to the degree of v.

$$\text{i.e.,} \quad deg_e(v) = deg(v) = 1.$$

Proposition 5. For any tree T which has atleast one equitable pendent vertex with $p \geq 3$ vertices, then

$$\gamma_{toec}(G) \leq p - 1.$$

Proof.

Let v be the pendent vertex of T.

Thus $V - \{v\}$ is total outer equitable connected dominating set.

Hence, $\gamma_{toec}(G) \leq p - 1$.

Proposition 6. For any connected graph G whose edges are equitable edges and if H is spanning subgraph of G, then

$$\gamma_{toec}(G) \leq \gamma_{toec}(H).$$

Proof.

Let G be any connected graph whose edges are equitable and H be any spanning subgraph of G. Then, $V(H) = V(G)$ and $E(H) \leq E(G)$. Thus every total outer equitable connected dominating of H is also total outer equitable connected dominating of G.

Hence, $\gamma_{toec}(G) \leq \gamma_{toec}(H)$.

Theorem 7. For any connected (p, q) graph

$$\frac{2p - q}{2} \leq \gamma_{toec}(G).$$

Proof.

Let D be γ_{toec} - set of G, since $\langle V - D \rangle$ is connected.

We have $|E| \geq 2|V - D|$.

Thus, $q \geq 2(p - \gamma_{toec}(G))$

This implies, $\frac{2p-q}{2} \leq \gamma_{toec}(G)$.

Theorem 8. For any tree whose edges are equitable, if T has atleast two adjacent cut vertices which are not support vertices then,

$$\gamma_{toec}(T) \leq p - 2.$$

Proof.

Let T be any tree with p vertices. Since T has atleast two adjacent cut-vertices which are not support vertices, then there exists two adjacent cut-vertices u and v such that $deg_e(u)$ and $deg_e(v) \geq 2$.

Let D = V – {u, v}

It is clear that V – {u, v} is total outer equitable dominating set for T and hence the result holds good.

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