Interval Valued Intuitionistic Fuzzy MAGDM Problems with OWA Entropy Weights

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***Abstract*— In this paper, the Multiple Attribute Group Decision Making (MAGDM) problems is based on the Renyi's, Daroczy's and R-norm entropy weights especially when the attribute weights are completely unknown. The interval-valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid averaging (IIFHA) operator are utilized to aggregate the interval-valued intuitionistic fuzzy decision matrices provided by the decision-makers. Correlation coefficient of Interval Valued Intuitionistic Fuzzy Sets (IVIFS) is utilized to rank the alternatives and select the most desirable one. A numerical illustration is presented to demonstrate the proposed approach.**

***Keywords*— MAGDM, Ordered Weighted Averaging, Correlation of Interval valued intuitionistic fuzzy sets, Entropy weights.**

I. Introduction

Atanassov [2, 3, 4] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set. Szmidt & Kacprzyk [18, 19] proposed some solution concepts in group decision making with intuitionistic (individual and social) fuzzy preference relations, such as intuitionistic fuzzy core and consensus winner, etc. Herrera et al. [7] developed an aggregation process for combining numerical, interval valued and linguistic information, and then proposed different extensions of this process to deal with contexts in which can appear other type of information such as IFSs or multi-granular linguistic information. Yager [26] developed the Ordered Weighted Averaging (OWA) operator and applied in decision making problems. Xu & Yager [25] developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information. Xu [22,23] and Xu & Chen [24] also developed some arithmetic aggregation operators, such as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid averaging (IFHA) operator. The interval-valued intuitionistic fuzzy sets (IVIFSs), introduced in [5], which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Wei & Wang [20], developed some geometric aggregation operators for MAGDM problems. Solairaju et al. [16,17] have worked on decision making problems with vague sets. In [10] was presented various decision making techniques to suit the present day necessity in the decision making environment. Amirtharaj & Robinson [15] worked on the MAGDM problems with interval valued vague sets under TOPSIS method.

Using the approach as in [6] we investigate MAGDM problems in which all the information provided by the decision-makers is presented as interval valued intuitionistic fuzzy decision matrices where each of its elements is characterised by Interval Valued Intuitionistic Fuzzy Number (IVIFN). Park et al. [9] proposed an Ordered Weighted Geometric (OWG) model to aggregate all individual Interval Valued Intuitionistic Fuzzy decision matrices provided by the decision makers into the collective Interval Valued Intuitionistic Fuzzy decision matrix. In the proposed model, from the maximal entropy attribute weight information, an optimization model is established to determine the unknown weights. Correlation coefficient is used as a tool to rank alternatives since it preserves the linear relationship between the variables. Robinson & Amirtharaj [11-15] defined correlation coefficient for Interval vague sets and triangular and trapezoidal intuitionistic fuzzy sets and proposed different MAGDM algorithms. Wei et al. [21] and Park et al. [9] have also adopted correlation coefficient as a ranking tool for deciding the best alternatives. In this paper, the correlation coefficient proposed in [9] for IVIFSs is utilized for ranking the alternatives. Correlation coefficient of the overall Interval Valued Intuitionistic Fuzzy values and the ideal Interval Valued Intuitionistic Fuzzy Numbers (IVIFN) value is calculated and the ranking of the most desirable alternatives is done based on the obtained correlation coefficients. A MAGDM model based on the maximal entropy weights [6] is presented for computing the attributes weights, and a numerical illustration is given.

II. INTERVAL-VALUED INTUITIONISTIC FUZZY SET

An interval-valued intuitionistic fuzzy set (IVIFS) A in X, $X \neq \varnothing$ and card(X)=n, is an object having the form: $A=\left\{\left\langle x, \mu_A(x), \gamma_A(x)\right\rangle : x \in X\right\}$, where

$\mu_A : X \to [0,1]$, $\gamma_A : X \to [0,1]$ with the condition $\sup \mu_A(x) + \sup \gamma_A(x) \le 1$ for any $x \in X$. The intervals $\mu_A(x)$ and $\gamma_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to A. We denote by IVIFS(X) the set of all IVIFSs in X. Then for each $x \in X$, $\mu_A(x)$ and $\gamma_A(x)$ are closed intervals and their lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\gamma_{AL}(x)$ and $\gamma_{AU}(x)$, respectively, and thus we can replace with $A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\gamma_{AL}(x), \gamma_{AU}(x)] \rangle : x \in X\}$, where $0 \le \mu_{AU}(x) + \gamma_{AU}(x) \le 1$, $x \in X$.

For each $A \in IVIFS(X)$, $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) = [1 - \mu_{AU}(x) - \gamma_{AU}(x), 1 - \mu_{AL}(x) - \gamma_{AL}(x)]$ is called an intuitionistic fuzzy interval or hesitation degree of x in A. Its lower and upper points are $\pi_{AL}(x) = 1 - \mu_{AU}(x) - \gamma_{AU}(x)$ and $\pi_{AU} = 1 - \mu_{AL}(x) - \gamma_{AL}(x)$, respectively.

The following expressions are defined for $A, B \in IVIFS(X)$:

- $A \le B \Leftrightarrow \mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$
- $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all $x \in X$
- $A^c \Leftrightarrow \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X\}$;
- $A \prec B \Rightarrow \mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \le \gamma_B(x)$ for all $x \in X$.

III. CORRELATION OF INTERVAL-VALUED INTUITIONISTIC FUZZY SET

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the finite universal set and $A, B \in IVIFS(X)$. Now, we utilize the method of calculating the correlation and the correlation coefficient of IVIFSs as follows. For each $A \in IVIFS(X)$, the informational intuitionistic energy of A is defined as follows:

$$E_{IVIFS}(A) = \frac{1}{2} \sum_{I=1}^{N} \left[\mu_{AL}^2(x_i) + \mu_{AU}^2(x_i) + \gamma_{AL}^2(x_i) + \gamma_{AU}^2(x_i) + \pi_{AL}^2(x_i) + \pi_{AU}^2(x_i) \right] \quad (1)$$

The function E satisfies the following conditions:

1) $E_{IVIFS}(A) = E_{IVIFS}(A^c)$ for all $A \in IVIFS(X)$
2) $E_{IVIFS}(A) \le n$ for all $A \in IVIFS(X)$

The correlation of A and B is defined by the formula:

$$C_{IVIFS}(A,B) = \frac{1}{2} \left(\sum_{i=1}^{n} \begin{matrix} [\mu_{AL}(x_i)\mu_{BL}(x_i) + \mu_{AU}(x_i)\mu_{BU}(x_i) + \\ \gamma_{AL}(x_i)\gamma_{BL}(x_i) + \gamma_{AU}(x_i)\gamma_{BU}(x_i) + \\ \pi_{AL}(x_i)\pi_{BL}(x_i) + \pi_{AU}(x_i)\pi_{BU}(x_i)] \end{matrix} \right) \quad (2)$$

For $A, B \in IVIFS(X)$, the correlation has the following properties:

1) $C_{IVIFS}(A, A) = E(A)$.
2) $C_{IVIFS}(A, B) = C_{IVIFS}(B, A)$.

Furthermore, the correlation coefficient of A and B is defined by the formula:

$$K_{IVIFS}(A,B) = \frac{C_{IVIFS}(A,B)}{\sqrt{E_{IVIFS}(A) \cdot E_{IVIFS}(B)}} \quad (3)$$

Theorem 1: (proved in [9])

For all $A, B \in IVIFS(X)$, the correlation coefficient satisfies:

1) $K_{IVIFS}(A,B) = K_{IVIFS}(B,A)$
2) $0 \le K_{IVIFS}(A,B) \le 1$
3) $A = B \Leftrightarrow K_{IVIFS}(A,B) = 1$

IV. MAXIMAL ENTROPY OWA OPERATOR WEIGHTS

An OWA operator of dimension n is a mapping $F : R^n \to R$ that has an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$ of having the properties $w_1 + w_2 + \ldots + w_n = 1, 0 \le w_i \le 1$, i=1,2,...,n and such that

$$F(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \quad (4)$$

where b_j is the jth largest element of the collection of the aggregated objects $\{a_1, a_2, \ldots, a_n\}$.

Yager introduced two characterizing measures associated with the weighting vector W of an OWA operator. The first one, the measure of orness of the aggregation, is defined as:

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \quad (5)$$

and it characterizes the degree to which the aggregation is like an **or** operation. It is clear that $orness(W) \in [0,1]$ holds for any weighting vector.

The second one, the measure of dispersion of the aggregation, is defined as:

$$disp(W) = -\sum_{i=1}^{n} w_i \ln w_i, \quad (6)$$

and it measures the degree to which W takes into account all information in the aggregation.

The classical measure of uncertainty introduced by Shannon in 1948 has been dominating the literature of information theory since its appearance. It is the same as the measure of dispersion up to a positive constant multiplier, i.e.,

$$H_S(W) = -\sum_{i=1}^{n} w_i \log_2 w_i. \quad (7)$$

This is called the Shannon entropy.

In the literature there have been described several classes of entropies each including the Shannon entropy as a special case. They include:

Renyi's entropies H_α (also called entropies of degree α) defined for all real numbers α ≠ 1 as follows:

$$H_\alpha(W) = \frac{1}{1-\alpha} \log_2 \sum_{i=1}^{n} w_i^\alpha \quad (8)$$

Entropies of order β, H_β introduced by Daroczy having the following form for all β≠ 1:

$$H_\beta(W)=\frac{1}{2^{1-\beta-1}}\left(\sum_{i=1}^{n}w_i^{\beta}-1\right) \quad (9)$$

R-norm entropies H_R, defined for all $R\neq 1$ by the following formula:

$$H_R(w)=\frac{R}{R-1}\left[1-\left(\sum_{i=1}^{n}w_i^{R}\right)^{1/R}\right] \quad (10)$$

It is well known that $H_S(w)=\lim_{\alpha\to 1}H_\alpha(w)=\lim_{\beta\to 1}H_\beta(w)=\lim_{R\to 1}H_R(w)$.

Hence it is clear that the actual type of aggregation performed by an OWA operator depends upon the form of the weighting vector.

OBTAINING MAXIMAL RENYI'S, DAROCZY'S AND R-NORM ENTROPY WEIGHTS:

In this section the maximal Renyi's and *R*-norm entropy weights are derived when their parameter values equal to 2.

Since $H_\alpha(w)=-\log_2\sum_{i=1}^{n}w_i^2$. if $\alpha=2$,

$H_\beta(w)=-2\left(\sum_{i=1}^{n}w_i^2-1\right)$, if $\beta=2$ and $H_R(w)=2\left(1-\left(\sum_{i=1}^{n}w_i^2\right)^{1/2}\right)$.

If *R*=2, therefore determining a special class of OWA operators having maximal entropy of the OWA weights for a given level of orness is based on the solution of the following mathematical programming problem

$$\text{minimize} \quad \sum_{i=1}^{n}w_i^2 \quad , \quad \text{subject to}$$

$$\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i=\alpha, 0\le\alpha\le 1 \quad (11)$$

$$\sum_{i=1}^{n}w_i=1, 0\le w_i\le 1, \ i=1,2,\ldots,n.$$

The solution for this problem is based on the use of the method Kuhn-Tucker multipliers and is rather complicated. Solving this problem it could be found that the optimal solution is a window-type OWA operator, i.e., there exists $1\le k\le 1\le n$ such that $k\le i\le 1\Leftrightarrow w_i\neq 0$.

As in the previous section, without loss of generality we can assume that $n\ge 3$ and $0<\alpha\le 1/2$. If $\alpha=1/2$ then $w_1=\ldots=w_n=1/n$ is the optimal solution to (9), furthermore this is the global optimal solution of all OWA operators of dimension n. To obtain the optimal solution for arbitrary $\alpha\in\left(0,1/2\right)$, consider the following disjoint union of intervals of $\left(0,1/2\right)$:

$\left(0,1/2\right)=\bigcup_{j=1}^{n-1}I_j$, Where $I_j=\left(\frac{j-1}{3(n-1)},\frac{j}{3(n-1)}\right)$, j=1,2,…,n-2.

$I_{n-1}=\left(\frac{n-2}{3(n-2)},1/2\right)$.

Now, considering α, there uniquely exists $1\le p\le n-1$ such that $\alpha\in I_p$.

Let *r* = *n-p*, then the optimal solution to (9) can be obtained as,

$w_i^*=0$ if $1\le i<r$,

$$w_r^*=\frac{6(n-1)\alpha-2(n-r-1)}{(n-r+1)(n-r+2)},$$

$$w_n^*=\frac{2(2n-2r+1)-6(n-1)\alpha}{(n-r+1)(n-r+2)},$$

$$w_i^*=\frac{n-i}{n-r}w_r^*+\frac{i-r}{n-r}w_n^* \quad \text{if } r<i<n.$$

ILLUSTRATIONS:

Obtaining the maximal Renyi's, Daroczy's and *R*-norm entropy weights it could be found that:

$\left(0,1/2\right)=\bigcup_{j=1}^{4}I_j$, where $I_j=\left[\frac{j-1}{12},\frac{j}{12}\right]$, j=1,2,3. $I_4=\left(1/4,1/2\right)$

Since $\alpha\in I_4$, therefore we have that *r* =1, hence

$w_1^*=\frac{24\times 0.4-6}{30}=0.12$, $w_5^*=\frac{18-24\times 0.4}{30}=0.28$, $w_2^*=\frac{3}{4}w_1^*+$ $w_3^*=\frac{1}{2}w_1^*+\frac{1}{2}w_5^*=0.20$, $w_4^*=\frac{1}{4}w_1^*+\frac{3}{4}w_5^*=0.24$.

and the corresponding Renyi's, Daroczy's and *R*-norm entropies are 2.2109, 1.5680 and 1.0705, respectively.

V. MAGDM PROBLEM WITH MAXIMAL ENTROPY OWA WEIGHTS

Let $R^{(k)}=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}^{(k)}=\left\langle\left[a_{ij}^{(k)},b_{ij}^{(k)}\right],\left[c_{ij}^{(k)},d_{ij}^{(k)}\right]\right\rangle$ is an IVIFN, provided by the decision-maker $d_k\in D$ for the alternative O_j with respect to the attribute $u_i\in U$, $\left[a_{ij}^{(k)},b_{ij}^{(k)}\right]$ indicates the degree that the alternative $O_j\in O$ satisfy the attribute u_i, expressed by the decision-maker d_k, while $\left[c_{ij}^{(k)},d_{ij}^{(k)}\right]$ indicates the degree that the alternative $O_j\in O$ does not satisfy the attribute u_i, expressed by the decision-maker d_k, and

$\left[a_{ij}^{(k)},b_{ij}^{(k)}\right]\subset[0,1]$, $\left[c_{ij}^{(k)},d_{ij}^{(k)}\right]\subset[0,1]$, $b_{ij}^{(k)}+d_{ij}^{(k)}\le 1$, $i=1,2,...,m$, $j=1,2,...,n$. To make a final decision in the process of group decision making, we need to fuse all individual decision opinion into group opinion. To do this, we use the IIFHA operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ $(k = 1,2,3,4)$ into the collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{m\times n}$.

Definition: Interval Valued Intuitionistic Fuzzy Ordered Weighted Average Operator

$$r_{ij} = IIFOWA_{\alpha,\lambda}(r_{ij}^{(1)}, r_{ij}^{(2)},..., r_{ij}^{(l)})$$
$$= \left\langle \begin{array}{l} \left[1-\prod_{k=1}^{n}\left(1-\dot{a}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, 1-\prod_{k=1}^{n}\left(1-\dot{b}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right], \\ \left[\prod_{k=1}^{n}\left(\dot{c}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, \prod_{k=1}^{n}\left(\dot{d}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right] \end{array} \right\rangle.$$

where $\alpha = (\alpha_1, \alpha_2,..., \alpha_l)^T$ is weight vector of IIFOWA operator with $\alpha_k > 0,\ (k = 1,2,...,l)$ and $\sum_{k=1}^{l} \alpha_k = 1$, and $\dot{r}_{ij} = \left\langle [\dot{a}_{ij}, \dot{b}_{ij}], [\dot{c}_{ij}, \dot{d}_{ij}] \right\rangle = \left\langle [\dot{a}_{ij}^{\sigma((k))}, \dot{b}_{ij}^{\sigma((k))}], [\dot{c}_{ij}^{\sigma((k))}, \dot{d}_{ij}^{\sigma((k))}] \right\rangle$, is the k^th^ largest of the weighted IVIFNs $\dot{r}_{ij}^{(k)} = (r_{ij}^{(k)})^{l\lambda_k},\ i = 1,2,...,m,\ j = 1,2,...,n$.

Definition: Interval Valued Intuitionistic Fuzzy Hybrid Average Operator

$$r_j = IIFHA\ (r_{1j}, r_{2j},..., r_{mj})$$
$$= \left\langle \left[1-\prod_{i=1}^{m}\left(1-\dot{a}_{ij}\right)^{w_i}, 1-\prod_{i=1}^{m}\left(1-\dot{b}_{ij}\right)^{w_i}\right], \left[\prod_{i=1}^{m}\dot{c}_{ij}^{w_i}, \prod_{i=1}^{m}\dot{d}_{ij}^{w_i}\right] \right\rangle$$

where the weights $w = (w_1, w_2,..., w_m)^T$ of the attributes can be completely determined in advance.

For the ranking order of the alternatives in accordance with the decision making problem, we give the largest IVIFN $r^* = \langle [1,1],[0,0] \rangle$ as the value of the ideal alternative.

ALGORITHM:

Step: 1 Utilize the IIFOWA operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)}$ into a collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{mxn}$.

Step: 2 Derive the weights by the Renyi's, Daroczy's and *R*-norm entropy weight by using

$$w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)}, \qquad w_n^* = \frac{2(2n-2r+1) - 6(n-1)\alpha}{(n-r+1)(n-r+2)} ,$$

and $w_i^* = \frac{n-i}{n-r} w_r^* + \frac{i-r}{n-r} w_n^*$, if $r < i < n$.

Step: 3 Use the IIFHA operator to get the overall values r_j of the alternatives O_j (*j*=1, 2,…,n).

Step: 4 Use equation (1) to calculate the informational intuitionistic energies of the obtained values r_j (*j*=1,2,...,n). use equation (2), to calculate the correlation between the value r^* of the ideal alternative O^* and the value r_j (*j*=1, 2,..., n).

Step: 5 Utilize equation (3) to calculate the correlation coefficients $K_{IVIFS}(r^*, r_j)$ (*j*=1, 2, ….,n) between the values r^* and r_j (*j*=1, 2, …., n).

Step: 6 Utilize the obtained correlation coefficients $K_{IVIFS}(r^*, r_j)$, (*j*=1, 2,..., n) to rank the alternatives O_j (*j*=1, 2,…, n), and then select the most desirable one(s).

NUMERICAL ILLUSTRATION:

A problem concerning with a manufacturing company is discussed, searching the best global supplier for one of its most critical parts used in assembling process. The attributes which are considered here in selection of four potential global suppliers O_j (*j*=1, 2, 3, 4) are:

U_1: Overall cost of the product; U_2: Quality of the product; U_3: Service performance of supplier;
U_4: Supplier's profile; U_5: Risk factor.

An expert group is formed which consists of four experts $d_k\ (k = 1,2,3,4)$ (whose weight vector is $\lambda = (0.3, 0.2, 0.3, 0.2)^T$) from each strategic decision area. The experts $d_k (k = 1,2,3,4)$ represent, respectively, the characteristics of the potential global suppliers $O_j\ (j = 1,2,3,4)$ by the IVIFSs $r_{ij}^{(k)}$ $(i = 1,2,3,4,5;\ j = 1,2,3,4)$ with respect to the attributes $u_i\ (i = 1,2,3,4,5)$,

$$R^{(1)} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{cccc} O_1 & O_2 & O_3 & O_4 \\ \langle[0.5,0.6],[0.2,0.3]\rangle & \langle[0.3,0.4],[0.4,0.6]\rangle & \langle[0.4,0.5],[0.3,0.5]\rangle & \langle[0.3,0.5],[0.4,0.5]\rangle \\ \langle[0.3,0.5],[0.4,0.5]\rangle & \langle[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.7,0.8]\rangle \\ \langle[0.6,0.7],[0.2,0.3]\rangle & \langle[0.3,0.4],[0.4,0.5]\rangle & \langle[0.5,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.5,0.8]\rangle \\ \langle[0.5,0.7],[0.1,0.2]\rangle & \langle[0.2,0.4],[0.5,0.6]\rangle & \langle[0.4,0.6],[0.2,0.3]\rangle & \langle[0.2,0.3],[0.4,0.6]\rangle \\ \langle[0.1,0.4],[0.3,0.5]\rangle & \langle[0.7,0.8],[0.1,0.2]\rangle & \langle[0.5,0.6],[0.2,0.3]\rangle & \langle[0.2,0.3],[0.5,0.6]\rangle \end{array}$$

$$R^{(2)}=\begin{matrix} & O_1 & O_2 & O_3 & O_4 \\ u_1 & \langle[0.4,0.5],[0.2,0.4]\rangle & \langle[0.3,0.5],[0.4,0.5]\rangle & \langle[0.4,0.6],[0.3,0.4]\rangle & \langle[0.3,0.4],[0.4,0.6]\rangle \\ u_2 & \langle[0.3,0.4],[0.4,0.6]\rangle & \langle[0.1,0.3],[0.3,0.7]\rangle & \langle[0.6,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.6,0.8]\rangle \\ u_3 & \langle[0.6,0.7],[0.1,0.2]\rangle & \langle[0.3,0.4],[0.4,0.5]\rangle & \langle[0.7,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.7,0.8]\rangle \\ u_4 & \langle[0.5,0.6],[0.1,0.3]\rangle & \langle[0.2,0.3],[0.6,0.7]\rangle & \langle[0.4,0.6],[0.3,0.4]\rangle & \langle[0.3,0.4],[0.4,0.6]\rangle \\ u_5 & \langle[0.1,0.3],[0.3,0.5]\rangle & \langle[0.6,0.8],[0.1,0.2]\rangle & \langle[0.5,0.6],[0.2,0.4]\rangle & \langle[0.2,0.4],[0.5,0.6]\rangle \end{matrix}$$

$$R^{(3)}=\begin{matrix} & O_1 & O_2 & O_3 & O_4 \\ u_1 & \langle[0.4,0.7],[0.1,0.2]\rangle & \langle[0.4,0.5],[0.2,0.4]\rangle & \langle[0.2,0.4],[0.3,0.4]\rangle & \langle[0.3,0.4],[0.2,0.4]\rangle \\ u_2 & \langle[0.3,0.5],[0.3,0.4]\rangle & \langle[0.2,0.4],[0.4,0.5]\rangle & \langle[0.6,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.6,0.8]\rangle \\ u_3 & \langle[0.6,0.7],[0.1,0.2]\rangle & \langle[0.4,0.5],[0.3,0.4]\rangle & \langle[0.5,0.7],[0.1,0.3]\rangle & \langle[0.1,0.3],[0.5,0.7]\rangle \\ u_4 & \langle[0.5,0.6],[0.1,0.3]\rangle & \langle[0.1,0.2],[0.7,0.8]\rangle & \langle[0.5,0.7],[0.2,0.3]\rangle & \langle[0.2,0.3],[0.5,0.7]\rangle \\ u_5 & \langle[0.3,0.5],[0.4,0.5]\rangle & \langle[0.6,0.7],[0.2,0.3]\rangle & \langle[0.6,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.6,0.8]\rangle \end{matrix}$$

$$R^{(4)}=\begin{matrix} & O_1 & O_2 & O_3 & O_4 \\ u_1 & \langle[0.6,0.7],[0.2,0.3]\rangle & \langle[0.4,0.5],[0.4,0.5]\rangle & \langle[0.4,0.5],[0.3,0.4]\rangle & \langle[0.3,0.4],[0.4,0.5]\rangle \\ u_2 & \langle[0.3,0.4],[0.3,0.4]\rangle & \langle[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.3]\rangle & \langle[0.1,0.3],[0.6,0.7]\rangle \\ u_3 & \langle[0.7,0.8],[0.1,0.2]\rangle & \langle[0.3,0.4],[0.5,0.6]\rangle & \langle[0.5,0.8],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.5,0.8]\rangle \\ u_4 & \langle[0.5,0.6],[0.1,0.3]\rangle & \langle[0.2,0.3],[0.4,0.6]\rangle & \langle[0.4,0.5],[0.2,0.3]\rangle & \langle[0.2,0.3],[0.4,0.5]\rangle \\ u_5 & \langle[0.1,0.2],[0.5,0.7]\rangle & \langle[0.6,0.7],[0.1,0.2]\rangle & \langle[0.5,0.6],[0.3,0.4]\rangle & \langle[0.3,0.4],[0.5,0.6]\rangle \end{matrix}$$

Step: 1 Utilize the IIFOWA operator (let $\alpha=(0.155,0.345,0.345,0.155)^T$ be its weight vector derived by the normal distribution based method) to aggregate the individual interval-valued intuitionistic fuzzy decision matrices into the collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{mxn}$.

$$R=\begin{matrix} u_1 & \langle[0.4930,0.6605],[0.1575,0.2727]\rangle & \langle[0.3707,0.4858],[0.3148,0.4761]\rangle & \langle[0.3375,0.4857],[0.2999,0.4139]\rangle & \langle[0.3001,0.4168],[0.3149,0.4762]\rangle \\ u_2 & \langle[0.3001,0.4168],[0.3149,0.4777]\rangle & \langle[0.1358,0.3051],[0.2705,0.4266]\rangle & \langle[0.6175,0.7700],[0.1000,0.2300]\rangle & \langle[0.1001,0.2361],[0.6145,0.7369]\rangle \\ u_3 & \langle[0.6378,0.7871],[0.1114,0.2129]\rangle & \langle[0.3368,0.4366],[0.3912,0.4929]\rangle & \langle[0.5381,0.7700],[0.1000,0.2300]\rangle & \langle[0.1001,0.2361],[0.5368,0.7639]\rangle \\ u_4 & \langle[0.5001,0.6175],[0.1000,0.2818]\rangle & \langle[0.1668,0.2843],[0.5348,0.6786]\rangle & \langle[0.4366,0.6089],[0.2129,0.3137]\rangle & \langle[0.2164,0.3165],[0.4319,0.5942]\rangle \\ u_5 & \langle[0.1748,0.3627],[0.3951,0.5615]\rangle & \langle[0.6175,0.7354],[0.1270,0.2300]\rangle & \langle[0.5371,0.6851],[0.1811,0.3012]\rangle & \langle[0.2044,0.3214],[0.5324,0.6626]\rangle \end{matrix}$$

STEP: 2 To derive a weight vector w by using Renyi's, Daroczy's and R-norm entropy weights.

$$w_i^*=0;\ if\quad 1\le i<r;\qquad w_r^*=\frac{6(n-1)\alpha-2(n-r-1)}{(n-r+1)(n-r+2)},$$

$$w_n^*=\frac{2(2n-2r+1)-6(n-1)\alpha}{(n-r+1)(n-r+2)},$$

$$w_i^*=\frac{n-i}{n-r}w_r^*+\frac{i-r}{n-r}w_n^*\quad \text{if}\quad r<i<n.$$

$n=5;\quad r=1;\quad \alpha=0.4$

Hence $w_1^*=0.12$,

$w_2^*=0.16,\ w_3^*=0.20,\ w_4^*=0.24,\ w_5^*=0.28.$

STEP:3 Using IIFHA operator to obtain the overall value $r_j\,(j=1,2,3,4,5)$ of the alternative.

Now the collective interval-valued intuitionistic fuzzy decision matrix R, is as follows:

$$R=\begin{matrix} & O_1 & O_2 & O_3 & O_4 \\ u_1 & \langle[0.4279,0.6079],[0.1088,0.2103]\rangle & \langle[0.4521,0.5613],[0.3967,0.5523]\rangle & \langle[0.2716,0.4204],[0.2357,0.3469]\rangle & \langle[0.3818,0.4965],[0.3968,0.5524]\rangle \\ u_2 & \langle[0.2359,0.3499],[0.2499,0.4121]\rangle & \langle[0.2025,0.3869],[0.3513,0.5058]\rangle & \langle[0.5607,0.7308],[0.0631,0.1714]\rangle & \langle[0.1586,0.3151],[0.6774,0.8062]\rangle \\ u_3 & \langle[0.5829,0.7503],[0.0718,0.1562]\rangle & \langle[0.4182,0.5153],[0.4719,0.5678]\rangle & \langle[0.4753,0.7308],[0.0631,0.1714]\rangle & \langle[0.1586,0.3151],[0.6079,0.8062]\rangle \\ u_4 & \langle[0.4354,0.5607],[0.0631,0.2187]\rangle & \langle[0.2386,0.3656],[0.6061,0.7333]\rangle & \langle[0.3699,0.5514],[0.1562,0.2488]\rangle & \langle[0.2939,0.3984],[0.5109,0.6594]\rangle \\ u_5 & \langle[0.1233,0.2961],[0.3281,0.5003]\rangle & \langle[0.6800,0.7820],[0.1919,0.3086]\rangle & \langle[0.4743,0.6352],[0.1287,0.2369]\rangle & \langle[0.2808,0.4033],[0.6039,0.7195]\rangle \end{matrix}$$

$$r_1=\left[(0.3681,0.5298),(0.0404,0.2840)\right],$$

$$r_2=\left[(0.4519,0.5758),(0.3639,0.4979)\right]$$

$$r_3=\left[(0.4453,0.6368),(0.1121,0.2234)\right],$$

$$r_4=\left[(0.2561,0.3844),(0.5625,0.7111)\right]$$

Step: 4 To calculate the informational intuitionistic energy $E_{IVIFS}(r_j)$ of the obtained valued r_j (j=1, 2, 3, 4) and to calculate the correlation $C_{IVIFS}(r^*,r_j)$ between the value r^* of the ideal alternative O^* and the value r_j (j=1,2,3,4).

$$C_{IVIFS}(r^*,r_1) = 0.2081, \quad C_{IVIFS}(r^*,r_2) = 0.5139,$$

$$C_{IVIFS}(r^*,r_3) = 0.5411, \quad C_{IVIFS}(r^*,r_4) = 0.3203.$$

Step: 5 To calculate the correlation coefficient $K_{IVIFS}(r^*,r_j)$ between the values r^* and r_j .

$$K_{IVIFS}(r^*,r_1) = 0.3133, \quad K_{IVIFS}(r^*,r_2) = 0.7435,$$

$$K_{IVIFS}(r^*,r_3) = 0.8149, \quad K_{IVIFS}(r^*,r_4) = 0.4364.$$

Step: 6 Utilize the obtained correlation coefficients $K_{IVIFS}(r^*, r_j)$, (j=1, 2, 3, 4) to rank the alternatives O_j (j=1, 2, 3, 4).

$$O_3 > O_2 > O_1 > O_4$$

Hence the most desirable global supplier is O_3.

VI. CONCLUSION

We have investigated the MAGDM problems under interval-valued intuitionistic fuzzy environment, and proposed an approach to handling the situations where the attribute values are characterized by IVIFNs, and the information about attribute weights completely unknown. The proposed approach first fuses all individual interval-valued intuitionistic fuzzy decision matrices into the collective interval-valued intuitionistic fuzzy decision matrix by using the IIFOWA operator. Then we have used the obtained attribute weights and the IIFHA operator to get the overall interval-valued intuitionistic fuzzy values of alternatives and have used the proposed method for calculating correlation coefficients between IVIFNs to rank the alternatives and then to select the most desirable one. The proposed approach in this work not only can comfort the influence of unjust arguments on the decision results, but also avoid losing or distorting the original decision information in the process of aggregation. Thus, the proposed approach provides us an effective and practical way to deal with multi-person multi-attribute decision making problems, where the attribute values are characterized by IVIFNs and the information about attribute weights is partially known.

REFERENCES

[1] Amirtharaj, E.C.H., & Robinson, J.P., (2013). Attributes weight determination for interval valued intuitionistic fuzzy multiple attribute group decision making problems. *International Journal of Computing Algorithm.* 2, 339-353.

[2] Atanassov .K, (1986). Intuitionistic Fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.

[3] Atanassov .K, (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 33, 37-46.

[4] Atanassov .K, (1994). Operators over interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 64, 159-174.

[5] Atanassov .K, Gargov .G, (1989). Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 31, 343-349.

[6] Fuller, R & Majlender, P, (2001). An analytic approach for obtaining maximal entropy OWA operator weights, Fuzzy Sets and Systems, 124, 53-57.

[7] Herrera, F., Martinez, L., & Sanchez,P.J. (1999). Managing non-homogenous information in group decision making. *European Journal of Operational Research*, 116, 115-132.

[8] O'Hagan, M. (1988). Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic, in: *Proc. 22nd Annual IEEE Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, 81-689.

[9] Park .D.G., Kwun .Y.C., Park .J.H., Park .I.Y., (2009). Correlation coefficient of Interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. *Mathematical and Computer Modelling*, 50, 1279-1293.

[10] Power, D.J., (2013). Engineering effective decision support technologies: New models and applications. Hershey: IGI Global.

[11] Robinson, J.P., Amirtharaj, E.C.H, (2011a). A short primer on the Correlation coefficient of Vague sets, *International Journal of Fuzzy System Applications*, 1(2), 55-69 .

[12] Robinson, J.P., Amirtharaj, E.C.H, (2011b). Extended TOPSIS with correlation coefficient of Triangular Intuitionistic fuzzy sets for Multiple Attribute Group Decision Making, *International Journal of Decision Support System Technology*, 3(3), 15-40.

[13] Robinson, J.P., Amirtharaj, E.C.H, (2012a). Vague Correlation coefficient of Interval Vague sets, *International Journal of Fuzzy System Applications*, 2(1), 18-34.

[14] Robinson, J.P., Amirtharaj, E.C.H, (2012b). A Search for the Correlation coefficient of Triangular and Trapezoidal intuitionistic Fuzzy sets for Multiple Attribute Group Decision Making, *Communications in Computer and Information Science - 283, Springer-Verlag*, 333-342.

[15] Robinson, J.P.,& Amirtharaj, E.C.H., (2013). A Strategic TOPSIS algorithm with correlation coefficient of interval vague sets. *International Journal of Computing Algorithm.* 2, 314-331.

[16] Solairaju, A., Robinson, J.P., & Lenin, T., (2013). Applications of Transforming Vague Sets into Fuzzy Sets for Knowledge Management. *International Journal of Computing Algorithm.* 2(2), 430-439.

[17] Solairaju, A., Robinson, J.P., & Lenin, T., (2014). A New Horizon for Multiple Attribute Group Decision Making Problems with Vague sets. *International Journal of Applied Mathematics and Statistical Sciences.* 3(2), 1-10.

[18] Szmidt .E, Kacprzyk .J, (2002). Using intuitionistic fuzzy sets in group decision- making. *Control and Cybernetics*, 31, 1037-1053.

[19] Szmidt, E., & Kacprzyk, J, (2003). A consensus-reaching process under intuitionistic fuzzy preference relations. *International Journal of Intelligent Systems*, 18, 837-852.

[20] Wei .G, Wang .X.R., (2007). Some geometric aggregation operators interval-valued intuitionistic fuzzy sets and their application to group decision making. *in:Proc.2007, ICCIS-2007*, 495-499.

[21] Wei,G, Wang, H-J, Lin, R., (2011). Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. *Knowledge and Information Systems*, 26(2),337-349.

[22] Xu Z.S., (2005). An Overview of Methods for Determining OWA Weights, *International Journal of Intelligent Systems*, 20, 843-865.

[23] Xu .Z.S., (2007). Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control and Decision*, 22, 215-219.

[24] Xu .Z.S., Chen .J, (2007). An approach to group decision making based on interval-valued intuitionistic judgment matrices. *Systems Engineering-Theory and Practice*, 27, 126-133.

[25] Xu .Z.S., Yager .R.R., (2006). Some geometric aggregation operators based on intuitionistic fuzzy sets. *International Journal of General Systems*, 35, 417-433.

[26] Yager .R.R., (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Trans. Systems, Man, Cybernetics.*, 18, 183-190.