# Solving a Fully Fuzzy Linear Programming Problem by Ranking

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**ABSTRACT**

**In this paper, we propose a new method for solving Fully Fuzzy linear programming Problem (FFLP) using ranking method .In this proposed ranking method, the given FFLPP is converted into a crisp linear programming (CLP) Problem with bound variable constraints and solved by using Robust's ranking technique and the optimal solution to the given FFLP problem is obtained and then compared between our proposed method and the existing method. Numerical examples are used to demonstrate the effectiveness and accuracy of this method.**

**Keywords: Fuzzy linear programming, Triangular fuzzy numbers, Ranking, Optimal solution**

## I. INTRODUCTION

In a fuzzy decision making problem, the concept of maximizing decision was introduced by Bellman and Zadeh[5] (1970).Tanaka et al[ 16] proposed a method for solving fuzzy mathematical programming problems. It is concerned with the optimization of a linear function while satisfying a set of linear equality and/ or inequality constraints or restrictions. In the present practical situations, the available information in the system under consideration are not exact, therefore fuzzy linear programming (FLP) was introduced. Fuzzy set theory has been applied to many disciplines such as control theory, management sciences, mathematical modeling and industrial applications. Campos and Verdegay [8] proposed a method to solve LP problems with fuzzy coefficients in both matrix and right hand side of the constraint. Cadenas and Verdegay [7] solved a LP problem in which all its elements are defined as fuzzy sets. Buckley and Feuring [6] proposed a method to find the solution for a fully fuzzified linear programming problem by changing the objective function into a multi objective LP problem. Maleki et al. [13] solved the LP problems by the comparison of fuzzy numbers in which all decision parameters are fuzzy numbers. Maleki [14] proposed a method for solving LP problems with vagueness in constraints by using ranking function. Ganesan and Veeramani [10] proposed an approach for solving FLP problem involving symmetric trapezoidal fuzzy numbers without converting it into crisp LP problems. Jimenez et al. [12] developed a method using fuzzy ranking method for solving LP problems where all the coefficients are fuzzy numbers . Allahviranloo et al. [1] solved fuzzy integer LP problem by reducing it into two crisp integer Amit Kumar et al. [3, 4] proposed a method for solving the FFLP problems by using fuzzy ranking function in the fuzzy objective function. Jayalakshmi and Pandian[11] proposed a bound and decomposition method to find an optimal fuzzy solution for fully fuzzy linear programming (FFLP) problems . Rangarajan and Solairaju (2010) [15] compute improved fuzzy optimal Hungarian assignment problems with fuzzy numbers by applying Robust's ranking techniques to transform the fuzzy assignment problem to a crisp one.

## II. PRELIMINARIES

### A. *Definition*

Let A be a classical set $\mu_A$ (x) be a real valued function defined from R into [0,1].A fuzzy set $A^*$ with the function $\mu_A$ (x) is defined by $A^*$={ $(x, \mu_A(x)) : x\varepsilon$ A and $\mu_A(x)\varepsilon$ [0,1]}. The function $\mu_A$ (x) is known as the membership function of $A^*$

### B. *Definition*

Given a fuzzy set $A$ defined on X and any number $\alpha \in [0,1]$, the $\alpha -$ cut, $\alpha_A$, is the crisp set $\alpha_A = \{x / A(x) \geq \alpha\}$

### C. *Definition*

Given a fuzzy set $A$ defined on X and any number $\alpha \in [0,1]$, the strong $\alpha -$cut, $\alpha +_A$, is the crisp set $\alpha +_A = \{x / A(x) > \alpha\}$

### D. *Definition*

A fuzzy number $\tilde{A}$ in R is said to be a triangular fuzzy number if its membership function

$\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

### E. *Definition*

Let $(a_1, a_2, a_3)$ and $(b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

i) $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3)$
$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

ii) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ for $k \geq 0$

iii) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ for $k < 0$

iv) $(a_1, a_2, a_3) \otimes (b_1, b_2, b_3) =$

$$\begin{cases} (a_1b_1, a_2b_2, a_3b_3) & \text{for } a_1 \geq 0 \\ (a_1b_3, a_2b_2, a_3b_3) & \text{for } a_1 < 0,\ a_3 \geq 0 \\ (a_1b_3, a_2b_2, a_3b_1) & \text{for } a_3 < 0 \end{cases}$$

### F. *Definition*

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be in F(R), then

i) $\tilde{A} < \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.

ii) $\tilde{A} > \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.

iii) $\tilde{A} = \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

iv) $\tilde{A} - \tilde{B}$ =0 if and only if $R(\tilde{A}) - R(\tilde{B})$=0.

A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ $\in F(R)$ is said to be positive if $R(\tilde{A}) > 0$ and denoted by $\tilde{A} > 0$. Also if $R(\tilde{A}) > 0$ then $\tilde{A} > 0$ and if $R(\tilde{A}) = 0$, then $\tilde{A} = 0$. If $R(\tilde{A}) = R(\tilde{B})$, then the triangular numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and is denoted by $\tilde{A} = \tilde{B}$.

G. *Definition*

Minimize (or Minimize) $\tilde{z} = \tilde{c}^T \tilde{x}$

Subject to,

$\tilde{A} \otimes \tilde{x} \ \{\leq, \approx, \geq\} \ \tilde{b}$

$\tilde{x} \geq 0$ and are integers.

Where the cost vector $\tilde{c}^T = (\tilde{c}_j)_{1\times n}$ , $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ and $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{a}_{ij}$ , $\tilde{x}_j$ , $\tilde{b}_i$ , $\tilde{c}_j \in F(R)$ for all $1 \leq j \leq n$ and for all $1 \leq i \leq m$.

## III. Ranking Function

A convenient method for comparing of the fuzzy numbers is by use of ranking functions. A ranking function is a map from *F(R)* into the real line. Since there are many ranking functions for comparing fuzzy numbers, here we have applied Robust's ranking function. Robust's ranking technique satisfies compensation, linearity and additive properties and provides results which are consistent with human intuition. Given a convex fuzzy number $\tilde{a}$ , the Robust's Ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)\, d\alpha$$

Where $(a_\alpha^l, a_\alpha^u)$ is the α- level cut of the fuzzy number $\tilde{a}$.

## IV. Fully Fuzzy Linear Programming Problem (FFLPP)

Consider the fully fuzzy Linear programming Problem (FFLPP)

Maximize( or Minimize) $\tilde{z} = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$

Subject to,

$$\tilde{A} \otimes \tilde{X} \{\leq, \approx, \geq\} \tilde{B}; \quad , \qquad \tilde{X} \geq 0$$

for all $i$ = 1,2,…. $m$ , j=1,2,……..n

Where $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ and $\tilde{B} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{a}_{ij}$ , $\tilde{x}_j$ , $\tilde{b}_i$ , $\tilde{c}_j \in F(R)$ for all $1 \leq j \leq n$ and for all $1 \leq i \leq m$. Let the parameters $\tilde{a}_{ij}$ , $\tilde{x}_j$ , $\tilde{b}_i$ , $\tilde{c}_j$ be the triangular fuzzy number.

In this section ,a new method ,named Roubst's ranking is proposed to find the fuzzy optimal solution of FFLPP.

A. *Algorithm for the Proposed Method:*

**Step1:**

Formulate the chosen problem in to the following fuzzy LPP as

$$\text{Maximize } \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

Subject to,

$$\sum_{j=1}^{n} a_{ij} \tilde{x}_j \leq, \cong, \geq \tilde{b}_i \quad , \text{i= 1, 2,3,……m}$$

$$\tilde{x}_j \geq 0, \quad \text{j=1, 2,……..n}$$

**Step2:**

Substitute the values of $\tilde{x}_j = (x_j, y_j, t_j)$ , $\tilde{c}_j = (p_j, q_j, r_j)$ and $\tilde{b}_i = (b_i, g_i, h_i)$ in the fuzzy LPP obtained in step1,we get

Maximize (or Minimize)

$$\tilde{z} = \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, t_j)$$

Subject to,

$$\sum_{j=1}^{n} a_{ij}\tilde{x}_j \le, \cong, \ge (b_i, g_i, h_i)$$ , i= 1,2,3,……m

$\tilde{x}_j \ge 0,$ j=1,2,……..n

and all decision variables are non-negative.

**Step3**: By using the linearity property of ranking function and the robust's ranking, Convert all the fuzzy constraints and restrictions into the crisp constraints and then the LPP can be written as

Minimize (or minimize)

$$R(\tilde{z}) = \sum_{j=1}^{n} R(p_j, q_j, r_j) \otimes R(x_j, y_j, t_j)$$

Subject to,

$$R(\sum_{j=1}^{n} a_{ij} \otimes (x_j, y_j, t_j)) \le, \cong, \ge R(b_i, g_i, h_i)$$

$$R(x_j, y_j, t_j) \ge 0 \quad \forall j$$

**Step: 4**

Solve the crisp LPP obtained in step3,to find the optimal solution to the given FFLP problem.

## V. NUMERICAL EXAMPLE

Consider the following fully fuzzy linear programming problem

Maximize $\tilde{z} \approx (1,2,3) \otimes \tilde{x}_1 + (2,3,4) \otimes \tilde{x}_2$

subject to,

$$(0,1,2) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 \approx (1,10,27);$$

$$(1,2,3) \otimes \tilde{x}_1 + (0,1,2) \otimes \tilde{x}_2 \approx (2,11,28);$$

$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

### A. *Solution:*

**Step: 1**

Formulate the chosen problem in to the following fuzzy LPP as

Maximize $\tilde{z} = (1,2,3) \otimes \tilde{x}_1 + (2,3,4) \otimes \tilde{x}_2$

Subject to,

$$(0,1,2) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 \le (1,10,27);$$

$$(1,2,3) \otimes \tilde{x}_1 + (0,1,2) \otimes \tilde{x}_2 \le (2,11,28);$$

$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

**Step: 2**

Substitute the values of $\tilde{x}_1 = (x_1, y_1, t_1)$ and $\tilde{x}_2 = (x_2, y_2, t_2)$ ,we get

Maximize(or Minimize)

$$\tilde{z} = (1,2,3) \otimes (x_1, y_1, t_1) + (2,3,4) \otimes (x_2, y_2, t_2)$$

Subject to,

$$(0,1,2) \otimes (x_1, y_1, t_1) + (1,2,3) \otimes (x_2, y_2, t_2) \le (1,10,27)$$

$$(1,2,3) \otimes (x_1, y_1, t_1) + (0,1,2) \otimes (x_2, y_2, t_2) \le (2,11,28)$$

$x_1, y_1, t_1, x_2, y_2, t_2 \ge 0$

**Step3**: By using the linearity property of ranking function and the Robust's ranking technique, convert all the fuzzy constraints and restrictions into the crisp constraints and the LPP can be written as

Minimize (or Minimize) $R(\tilde{z}) =$

$$R(1,2,3) \otimes R(x_1, y_1, t_1) + R(2,3,4) \otimes R(x_2, y_2, t_2)$$

Subject to,

$$R(0,1,2) \otimes R(x_1, y_1, t_1) + R(1,2,3) \otimes R(x_2, y_2, t_2) \le R(1,10,27)$$

$$R(1,2,3) \otimes R(x_1, y_1, t_1) + R(0,1,2) \otimes R(x_2, y_2, t_2) \le R(2,11,28)$$

$x_1, y_1, t_1, x_2, y_2, t_2 \ge 0$

Now we calculate R(0,1,2) by applying Robust's ranking method. The membership function of the triangular fuzzy number (0,1,2) is

$$\mu(x) = \begin{cases} \dfrac{x-0}{1} & 0 \le x \le 1 \\ 1 & x = 1 \\ \dfrac{2-x}{1} & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

The $\alpha -$ Cut of the fuzzy number (0, 1, 2) is $(a_\alpha^l, a_\alpha^u) = (\alpha, 2-\alpha)$ for which

$$R(a_{11}) = R(0,\ 1,\ 2) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)d\alpha$$

$$= \int_0^1 0.5(\alpha,\ 2-\alpha)d\alpha$$

$$= 0.5\int_0^1 (2)d\alpha = 1$$

$$\Rightarrow \quad R(a_{11}) = R(0,\ 1,\ 2) = 1$$

Similarly, the Robust's ranking indices for the fuzzy costs $\tilde{C}_{ij}$ , $\tilde{a}_{ij}$ and the fuzzy numbers $\tilde{B}$ are

$$R(c_{11}) = R(1,2,3) = 2, \quad R(c_{12}) = R(2,3,4) = 3$$

$$R(a_{12}) = R(1,\ 2,\ 3) = 2 \quad ,$$

$$R(a_{21}) = R(1,\ 2,\ 3) = 2$$

$$R(a_{22}) = R(0,\ 1,\ 2) = 1$$

$$R(b_1) = R(1,\ 10,\ 27) = 12 \quad ,$$

$$R(b_2) = R(2,\ 11,\ 28) = 11$$

Now the given FLPP becomes

Maximize $z = 2x_1 + 3x_2$

Subject to

$$x_1 + 2x_2 \le 12$$
$$2x_1 + x_2 \le 11$$
$$x_1, x_2 \ge 0$$

**Step 4**:By solving the above equation by integer linear programming, we obtain the optimal solution max $z$ =19, $x_1$ =2, $x_2$ =5

B. *Comparison of Our proposed method with Pandian and Jayalakshmi method:[11]*

In the proposed method we obtain the optimal solution for the above integer linear programming problem is , max $z$ =19, $x_1$ =2, $x_2$ =5

By the definition of ranking parameters, the ranking value of triangular fuzzy number using Robust's ranking for Pandian method the optimal solution for the above integer linear programming problem is , max $z$ =19 , $x_1$ =4, $x_2$ =3

Table 5.1

| Method | $x_1$ | $x_2$ | Z | ranking |
|---|---|---|---|---|
| Pandian method | 4 | 3 | 19 | 1 |
| proposed method | 2 | 5 | 19 | 1 |

## VI. CONCLUSION

In this paper a new method for solving fully fuzzy linear programming problem (FFLPP) are discussed and these methods are illustrated with suitable numerical example. We have also given the comparison between the two methods. In this paper the general fuzzy numbers and the decision variables are considered as triangular fuzzy numbers. The effectiveness of our proposed method is demonstrated by using the example given by Pandian and Jayalakshmi[11] and the results are tabulated. It is obvious from the results shown in table 5.1 that by using both the existing and the proposed method the ranking value and the optimum value are same. Moreover we have also obtained a crisp value rather than the fuzzy values. This is possible only by using ranking where a decision maker can take a decision which will be optimum and the proposed method is very easy to understand and can be applied for fully fuzzy linear programming problem occurring in real life situation as compared to the existing method.

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