Role of Ψ_μ Operator in Ideal Supra Topological Spaces

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Abstract- In this paper, we shall discuss the ideal on supra topological space and the properties of Ψ operator on topological space. Moreover, We introduce Ψ_μ operator and its properties on supra topological space. In addition with the help of Ψ_μ operator, ( )*μ operator and its properties are also discussed here.

Keywords- Ψ- operator, Ψ -C set, supra topological space, Ψ_μ -operator, Ψ_μ - C set, ( )*μ – operator.

I. Introduction

In 1960, Kuratowski[5] and Vaidyanathswamy[14] were the first to introduce the concept of the ideal in topological space. They also have defined local function in ideal topological space. Further Hamlett and Jankovic in [3] and [4] studied the properties of ideal topological spaces and they have introduced another operator called Ψ operator. They have also obtained a new topology from original ideal topological space. Using the local function, they defined a Kuratowski Closure operator in new topological space.

Further, they showed that interior operator of the new topological space can be obtained by Ψ - operator. In 2007, Modak and Bandyopadhyay[9] have defined generalized open sets using Ψ operator. More recently AI-Omri and Noiri[1] have defined the ideal m-space and introduced two operators as like similar to the local function and Ψ operator. Different types of generalized open sets like semi-open[6], preopen[7], semi-preopen[2], α-open[12] have a common property which is closed under arbitrary union. Mashhour et al[8] put all of the sets in a pocket and defined a generalized space which is supra topological space.

In this space, two type of set operators i.e. μ-local function and Ψ_μ operator are defined under ideal. Further the properties of these two operators are discussed. Finally, μ-codense ideal, μ- compatible ideal and Ψ_μ – C set with the help of Ψ_μ operator and their properties are discussed.

II. Preliminaries

Definition 2.1 : A topology on a set X is a collection $\mathcal{T}$ of subsets of X having the following properties:

1. ϕ and X are in $\mathcal{T}$.
2. The union of the elements of any subcollection of $\mathcal{T}$ is in $\mathcal{T}$.
3. The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$.

A set X for which a topology $\mathcal{T}$ has been specified is called a topological space.

Definition 2.2 : A subfamily μ of the power set $\wp(X)$ of a nonempty set X is called a supra topology on X if μ satisfies the following conditions:

a. μ contains ϕ and X,
b. μ is closed under the arbitrary union.

The member of μ is called supra open set in (X, μ). The pair (X, μ) is called a supra topological space.

Definition 2.3 : A subfamily μ of the power set $\wp(X)$ is said to be supra closed set in (X, μ) if X – μ is supra open set.

Definition 2.4 : A supra topological space (X, μ) with an ideal I on X is called an ideal supra topological space and denoted as (X, μ, I).

Definition 2.5 : A nonempty collection I of subsets of X is called an ideal on X if:

a. A ∈ I and B ⊂ A implies B ∈ I (heredity) ;
b. A ∈ I and B ∈ I implies A ∪ B ∈ I (finite additivity).

For a subset A of X, $A^* = \{x \in X : U \cap A \notin I$, for every $U \in \tau(x)$ where $\tau(x)$ is the collection of all nonempty open sets containing x}. A^* is a closed subset for any $A \subset X$.

Definition 2.6 : If X is a topological space with topology $\mathcal{T}$, then a subset U of X is an open set of X if U belongs to the collection $\mathcal{T}$.

Definition 2.7 : A subset U of a topological space X is said to be closed if $X - U$ is open.

Definition 2.8 : A subset A of a topological space X, the interior of A is defined as the union of all the open sets contained in A.

Definition 2.9 : A subset A of a topological space X, the closure of A is defined as the intersection of all the closed sets containing A.

Definition 2.10 : A neighborhood of x is a subset $W \subset X$ such that there exists an open set A such that $x \in A \subset W$.

Definition 2.11 : Let X and Y be topological spaces. A function $f : X \longrightarrow Y$ is said to be continuous if for each open subset V of Y, the set $f^{-1}(V)$ is an open subset of X.

Definition 2.12 : Let (X, μ) be a supra topological space and $A \subset X$. Then supra interior of A in (X, μ) defined as $\cup\{U : U \subseteq A, U \in \mu\}$ and it is denoted as $Int^\mu(A)$.

Definition 2.13 : Let (X, μ) be a supra topological space and $A \subset X$ .Then supra closure of A in (X, μ) defined as $\cap\{ F : A \subseteq F, X - F \in \mu\}$ and it is denoted as $Cl^{\mu}(A)$.

Definition 2.14 : Let (X, μ) be a supra topological space and $M \subset X$.Then M is said to a supra neighborhood of a point x of X if for some supra open set $U \in \mu$, $x \in U \subset M$.

Definition 2.15 : Let (X, μ, I) be an ideal supra topological space. A subset A of X is called a Baire set with respect to μ and I, denoted $A \in B_r(X, \mu, I)$, if there exists a supra open set $U \in \mu$ such that $A = U[\text{mod } I]$.

Definition 2.16 : A subset A of a topological space (X, τ, I) is called I-dense if every point of X is in the local function of A with respect to I and τ , i.e. if $A^*(I) = X$.

Definition 2.17 : An ideal I in a space (X, μ, I) is called μ-codense ideal if $\mu \cap I = \{\phi\}$.

Definition 2.18 : Let (X, μ, I) be an ideal supra topological space. Then the μ- structure μ is μ - compatible with the ideal I, denoted $\mu \sim_{\pi} I$, if the following holds : For every $A \subseteq X$, if for every $x \in A$ there exists $U \in \mu(x)$ such that $U \cap A \in I$, then $A \in I$.

Definition 2.19 : Let A be the subset of a nonempty set X, Then Ψ-operator is defined as $\Psi(A) = X - (X - A)^*$.

Definition 2.20 : Let (X, τ, I) be a topological space and $A \subset X$, A is said to be a Ψ-C set if $A \subset Cl(\Psi(A))$. The collection of all Ψ-C sets in (X, τ, I) is denoted by Ψ(X, τ).

Definition 2.21 : Let (X, μ, I) be an ideal supra topological space. A set operator $(\ )^{*\mu} : \wp(X) \to \wp(X)$, is called the μ-local function of I on X with respect to μ, is defined as:$(A)^{*\mu}(I, \mu) = \{ x \in X : U \cap A \notin I$,for every $U \in \mu(x)$ },where $\mu(x) = \{U \in \mu : x \in U\}$.This is simply called μ-local function and simply denoted as $(A)^{*\mu}$.

Definition 2.22 : Let (X, μ, I) be an ideal supra topological space. An operator $\Psi_{\mu} : \wp(X) \to \mu$ is defined as follows for every $A \in \wp(X)$, $\Psi_{\mu}(A) = \{x \in X$: there exists a $U \in \mu(x)$ such that $U - A \in I\}$ and $\Psi_{\mu}(A) = X - (X - A)^{*\mu}$.

Definition 2.23 : Let (X, μ, I) be an ideal supra topological space. A subset A of X is called a Ψ_{μ} - C set if $A \subseteq Cl^{\mu}(\Psi_{\mu}(A))$. The collection of all Ψ_{μ} - C sets in (X, μ, I) is denoted by Ψ_{μ} (X, μ).

Definition 2.24 : A subset A of a space $(X, \mathcal{T})$ is called a generalized closed set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.25 : A map $f : X \to Y$ is called a generalized continuous if $f^{-1}(F)$ is g-closed in X or every closed set F of Y.

Definition 2.26 : A subset $A \subset X$ is semi-open set if $A \subset Cl(Int(A))$. The collection of all semi-open sets in a topological space (X, τ) is denoted by $SO(X, \tau)$.

Definition 2.27 : A subset A of X is said to be a semi-preopen set if $A \subset Cl(Int(Cl(A)))$. The collection of all semi-preopen sets in (X, τ) is denoted by $SPO(X, \tau)$.

Definition 2.28 : A subset $A \subset X$ is semi-closed set if $A \subset Int(Cl(A))$. The collection of all semi-closed sets in a topological space (X, τ) is denoted by $SO(X, \tau)$.

Definition 2.29 : A subset $A \subset X$ is semi-closure if the intersection of all semi-closed sets that contain A and it is denoted by $sCl(A)$

Theorem 2.1 : Let (X, μ) be a supra topological space and $A \subset X$. Then

a) $Int^{\mu}(A) \subseteq A$.
b) $A \in \mu$ if and only if $Int^{\mu}(A) = A$.
c) $Cl^{\mu}(A) \supseteq A$.
d) A is a supra closed set if and only if $Cl^{\mu}(A) = A$.
e) $x \in Cl^{\mu}(A)$ if and only if every supra open set U_x containing x, $U_x \cap A \neq \phi$.

Proof :

a) From the definition of supra interior,
Let (X, μ) be a supra topological space and $A \subset X$.
Then supra interior of A in (X, μ) defined as $\cup\{ U : U \subseteq A, U \in \mu\}$ and it is denoted as $Int^{\mu}(A)$.

$\Longrightarrow Int^{\mu}(A) = U$.

Since $U \subseteq A$,

$\Longrightarrow Int^{\mu}(A) \subseteq A$.

Since arbitrary union of supra open sets is again a supra open set, then proof is obvious.

c) From the definition of supra closure,
Let (X, μ) be a supra topological space and $A \subset X$.
Then supra closure of A in (X, μ) defined as $\cap\{ F : A \subseteq F, X - F \in \mu\}$ and it is denoted as $Cl^{\mu}(A)$.

$\Longrightarrow Cl^{\mu}(A) = F$.

Since $A \subseteq F \Longrightarrow F \supseteq A$

$\Longrightarrow Cl^{\mu}(A) \supseteq A$

d) If A is a supra closed set,then smallest supra closed set containing A is A .Hence $Cl^{\mu}(A) = A$.

e) Let $x \in Cl^{\mu}(A)$. If possible suppose that $U_x \cap A = \phi$, where U_x is a supra open set containing x.
Then $A \subset (X - U_x)$ and $X - U_x$ is a supra closed set containing A.
Therefore $x \in (X - U_x)$, a contradiction.
Conversely,
Suppose that $U_x \cap A \neq \phi$,for every supra open set U_x containing x.
If possible suppose that $x \notin Cl^{\mu}(A)$, then $x \in X - Cl^{\mu}(A)$.
Then there is a $U_x^{/} \in \mu$ such that $U_x^{/} \subset (X - Cl^{\mu}(A))$,

i.e., $U_x^{/} \subset (X - Cl^{\mu}(A)) \subset (X - A)$.

Hence $U_x^{/} \cap A = \phi$, a contradiction. So $x \in Cl^{\mu}(A)$.

Theorem 2.2[5] : Let (X, μ) be a supra topological space and $A \subset X$. Then $Int^{\mu}(A) = X - Cl^{\mu}(X - A)$.

Proof :

Let $x \in Int^{\mu}(A)$.Then there is $U \in \mu$, such that $x \in U \subset A$.
Hence $x \notin X - U$, i.e., $x \notin Cl^{\mu} X - U$,
Since $X - U$ is a supra closed set.

By the definition of supra closure,

$Cl^{\mu}(X - A) \subset Cl^{\mu}(X - U)$

So $x \notin Cl^{\mu}(X - A)$.And hence $x \in X - Cl^{\mu}(X - A)$.

Conversely,

Suppose that $x \in X - Cl^{\mu}(X - A)$.So $x \notin Cl^{\mu}(X - A)$, then there is a supra open set U_x containing x, such that $U_x \cap (X - A) = \phi$.

So $U_x \subset A$. Therefore $x \in Int^{\mu}(A)$.

III. ( )*µ Operator

Theorem 3.1 : Let (X, µ, I) be an ideal supra topological space, and let A, B, A1, A2,----- Ai,---- be subsets of X. Then

a) $\phi^{*\mu} = \phi$.
b) $A \subset B$ implies $A^{*\mu} \subset B^{*\mu}$.
c) for another ideal $J \supseteq I$ on X, $A^{*\mu}(J) \subset A^{*\mu}(I)$.
d) $A^{*\mu} \subset Cl^{\mu}(A)$.
e) $A^{*\mu}$ is a supra closed set.
f) $(A^{*\mu})^{*\mu} \subset A^{*\mu}$.
g) $A^{*\mu} \cup B^{*\mu} \subset (A \cup B)^{*\mu}$.
h) $\cup_i A^{*\mu} \subset (\cup_i A_i)^{*\mu}$.
i) $(A \cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$.
j) for $V \in \mu$, $V \cap (V \cap A)^{*\mu} \subset V \cap A^{*\mu}$.
k) for $I \in I$, $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.

Proof :

a) From the definition of µ-local function,

A set operator $(\ )^{*\mu} : \wp(X) \to \wp(X)$, is called the µ-local function of I on X with respect to µ, is defined as:

$(A)^{*\mu}(I, \mu) = \{ x \in X: U \cap A \notin I$,for every $U \in \mu(x) \}$,

where $\mu(x) = \{U \in \mu: x \in U\}$.

$\Rightarrow (\phi)^{*\mu}(I, \mu) = \{ \phi \in X: U \cap \phi \notin I$,for every $U \in \mu(x) \}$,

where $\mu(\phi) = \{U \in \mu: \phi \in U\}$.

$\Rightarrow \phi^{*\mu} = \phi$.

b) Let $x \in A^{*\mu}$. Then for every $U \in \mu(x)$, $U \cap A \notin I$. Since $U \cap A \subset U \cap B$, then $U \cap B \notin I$. This implies that $x \in B^{*\mu}$.

Hence $A^{*\mu} \subset B^{*\mu}$.

c) Let $x \in A^{*\mu}(J)$. Then for every $U \in \mu(x)$, $U \cap A \notin J$. This implies that $U \cap A \notin I$, So $x \in A^{*\mu}(I)$.

Hence $A^{*\mu}(J) \subset A^{*\mu}(I)$.

d) Let $x \in A^{*\mu}$. Then for every $U \in \mu(x)$, $U \cap A \notin I$. This implies that $U \cap A \neq \phi$. Then $x \in Cl^{\mu}(A)$.

Hence $A^{*\mu} \subset Cl^{\mu}(A)$.

e) From definition of supra neighbourhood, each supra neighbourhood M of x contains a $U \in \mu(x)$.

If $A \cap M \in I$ then for $A \cap U \subset A \cap M$, $A \cap U \in I$.

It follows that $X - A^{*\mu}$ is the union of supra open sets.

Since the arbitrary union of supra open sets is a supra open set. So $X - A^{*\mu}$ is a supra open set and hence $A^{*\mu}$ is a supra closed set.

f) From (d), $(A^{*\mu})^{*\mu} \subset Cl^{\mu}(A^{*\mu}) = A^{*\mu}$, since $A^{*\mu}$ is a supra closed set. Hence $(A^{*\mu})^{*\mu} \subset A^{*\mu}$.

g) Since $A \subset (A \cup B)$ and $B \subset (A \cup B)$.

Then from (b), $A^{*\mu} \subset (A \cup B)^{*\mu}$ and $B^{*\mu} \subset (A \cup B)^{*\mu}$.

Hence $A^{*\mu} \cup B^{*\mu} \subset (A \cup B)^{*\mu}$.

h) Proof is by induction method,

Consider i = 1,2; Since A_1 $(A_1 \cup A_2)$ and A_2 $(A_1 \cup A_2)$.

Then from (b),

$A_1{}^{*\mu} \subset (A_1 \cup A_2)^{*\mu}$ and $A_2{}^{*\mu} \subset (A_1 \cup A_2)^{*\mu}$.

$\Rightarrow A_1{}^{*\mu} \cup A_2{}^{*\mu} \subset (A_1 \cup A_2)^{*\mu}$.

Similarly, For i =1,2,3,… $\Rightarrow \cup_i A^{*\mu} \subset (\cup_i A_i)^{*\mu}$.

i) Since $A \cap B \subset A$ and $A \cap B \subset B$, then from (b),

$(A \cap B)^{*\mu} \subset A^{*\mu}$ and $(A \cap B)^{*\mu} \subset B^{*\mu}$.

Then $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.

Hence $(A \cap B)^{*\mu} \subset A^{*\mu} \cap B^{*\mu}$

j) Since $V \cap A \subset A$, then $(V \cap A)^{*\mu} \subset A^{*\mu}$.

So $V \cap (V \cap A)^{*\mu} \subset V \cap A^{*\mu}$.

k) Since $A \subset (A \cup I)$, then

$A^{*\mu} \subset (A \cup I)^{*\mu}$ -----------------(i).

Let $x \in (A \cup I)^{*\mu}$. Then for every $U \in \mu(x)$, $U \cap (A \cup I) \notin I$. This implies that $U \cap A \notin I$.

If possible suppose that $U \cap A \in I$.

$U \cap I \subset I$ implies $U \cap I \in I$ and hence $U \cap (A \cup I) \in I$, a contradiction. Hence $x \in A^{*\mu}$ and

$(A \cup I)^{*\mu} \subset A^{*\mu}$ -----------------(ii).

From (i) and (ii) it implies

$(A \cup I)^{*\mu} = A^{*\mu}$ -----------------(iii).

Since $(A - I) \subset A$, then

$(A - I)^{*\mu} \subset A^{*\mu}$ -------------------(iv).

For reverse inclusion, let $x \in A^{*\mu}$. Claim that $x \in (A - I)^{*\mu}$, if not, then there is $U \in \mu(x)$, $U \cap (A - I) \in I$.

Given that $I \in I$, then $I \cup (U \cap (A - I)) \in I$.

$\Rightarrow I \cup (U \cap A) \in I$. So, $U \cap A \in I$, a contradiction to the fact that $x \in A^{*\mu}$. Hence

$A^{*\mu} \subset (A - I)^{*\mu}$ -------------------(v).

From (iv) and (v) it implies

$A^{*\mu} = (A - I)^{*\mu}$ --------------------(vi).

From (iii) and (vi),

$\Rightarrow (A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.

Example 3.1 : Show that $A^{*\mu} \cup B^{*\mu} = (A \cup B)^{*\mu}$ does not hold in general.

Proof :

Let, X ={a, b, c, d}, I = {ϕ,{c}}.

µ={ϕ, X,{a},{b},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b ,d},{a,c,d},{b,c,d}},

Then,Supra open sets containing'a' are:

X,{a},{a,b}, {a,c},{a,d},{a,b,c},{a,b,d},{a,c,d};

Supra open sets containing 'b' are:

X,{b},{a,b},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d};

Supra open sets containing'c' are:

X,{a,c},{b,c}, {a,b,c}, {a,c,d},{b,c,d};

Supra open sets containing 'd' are:

X,{a,d},{b,d},{a,b,d},{a,c,d},{b,c,d}.

Consider A = {a,c} and B = {b,c}.

Then,

$A^{*\mu} = \{a\}$, $B^{*\mu} = \{b\}$ and

$(A \cup B)^{*\mu} = \{a, b, c\}^{*\mu} = \{a,b,c,d\}$.

Hence $A^{*\mu} \cup B^{*\mu} \neq (A \cup B)^{*\mu}$.

IV. Ψ- Operator And Ψ - C Set

A. *Ψ- Operator*

Theorem A.1 : Let (X, τ, I) be a topological space, then $U \subset \Psi(U)$ for every open set U of (X, τ).

Proof :
Since $\Psi(U) = X - (X-U)^*$.
Then $(X-U)^* \subset Cl(X-U) = X - U$,
Since $X - U$ is closed.
Therefore $X - (X-U)^* \supset X - (X-U) = U$
$\Longrightarrow U \subset \Psi(U)$.

Example A.1 : A set A which is not open but satisfies $A \subset \Psi(A)$.

Proof :
Let $X = \{a,b,c\}, \tau = \{\phi, X, \{a,c\}\}, I = \{\phi, \{c\}\}$.
Then $\Psi(\{a\}) = X - (X - \{a\})^*$
$= X - \{b,c\}^*$
$= X - \{b\}$
$= \{a,c\}$.
Therefore $\{a\} \subset \Psi(\{a\})$, but $\{a\}$ is not open.

Corollary A.1[3] : Let (X, τ, I) be a space, then $Int(A) \subset \Psi(A)$ for any subset A of X.

Proof :
Since $Int(A)$ is open, then by theorem *A.1*,
$Int(A) \subset \Psi(Int(A))$ ------------------ (i)
$\Longrightarrow Int(A) \subset A$,
Therefore,
$\Psi(Int(A)) \subset \Psi(A)$ ------------------ (ii)
From (i) and (ii),
$\Longrightarrow Int(A) \subset \Psi(A)$.

Theorem A.2[3] : Let (X, τ, I) be a space, where I is codense. Then for $A \subset X$, $\Psi(A) \subset A^*$.

Proof :
Suppose $\alpha \in \Psi(A)$ but $\alpha \notin A^*$. Then there exists a nonempty neighborhood U_α of α such that $U_\alpha \cap A \in I$.
Since $\alpha \in \Psi(A)$, therefore $\alpha \in \cup\{M - A \in I\}$,
$\Longrightarrow$ there exist $V \in \tau$ such that $\alpha \in V$ and $V - A \in I$.
$\Longrightarrow U_\alpha \cap V$ is a neighborhood of α.
$\Longrightarrow U_\alpha \cap V \cap A \in I$, by heredity.
$\Longrightarrow U_\alpha \cap V - A \in I$ by heredity.
$U_\alpha \cap V = (U_\alpha \cap V \cap A) \cup (U_\alpha \cap V - A) \in I$
(By finite additivity).
Since $U_\alpha \cap V$ is nonempty open,
A contradiction to I being codense.
Therefore,
$\alpha \in A^*. \Longrightarrow \Psi(A) \subset A^*$.

Corollary A.2 : Let (X, τ, I) be a topological space, where I is codense. Then for $A \subset X$, $\Psi(A) \subset Cl(A)$.

Theorem A.3 : Let (X, τ, I) be a topological space and I be codense. Then

(a) for any $A \subset X$, $\Psi(A) \subset Int(Cl(A))$.
(b) for any closed subset A, $\Psi(A) \subset A$.
(c) for any $A \subset X$, $Int(Cl(A)) = \Psi(Int(Cl(A)))$.
(d) for any regular open subset A, $A = \Psi(A)$.
(e) for any $U \in \tau$, $\Psi(U) \subseteq Int(Cl(U)) \subseteq U^*$.
(f) for $J \in I$, $\Psi(J) = \phi$.

Proof :
(a) From corollary *A.2*,
$\Psi(A) \subset Cl(A)$.
Since $\Psi(A)$ is open, then $\Psi(A) \subset Int(Cl(A))$.
(b) From the definition of Ψ-operator,
For any closed subset A, $\Psi(A) \subset A$.
(c) For any set A,
$\Psi\left(Int(Cl(A))\right) \subset Cl(Int(Cl(A)))$,by corollary *A.2*.
Since $\Psi\left(Int(Cl(A))\right)$ is open,
$\Psi\left(Int(Cl(A))\right) \subset Int(Cl\left(Int(Cl(A))\right))$
$\Longrightarrow \Psi\left(Int(Cl(A))\right) \subset Int(Cl(A))$ ---------- (i)
Since $Int(Cl(A))$ is open, therefore by theorem *A.1*,
$\Longrightarrow Int(Cl(A)) \subset \Psi\left(Int(Cl(A))\right)$ ---------- (ii)
From (i) and (ii),
$\Longrightarrow Int(Cl(A)) = \Psi\left(Int(Cl(A))\right)$
(d) If A is regular open, therefore $A = Int(Cl(A))$.
From (c), $A = \Psi(A)$.
(e) By corollary *A.2*, $\Psi(U) \subset Cl(U)$.
Since $\Psi(A)$ is open, therefore
$\Longrightarrow \Psi(U) \subset Int(Cl(U))$ -------------- (i)
Here I is codense and U is open, therefore $U^* = Cl(U)$
$\Longrightarrow Int(Cl(U)) \subset U^*$ ------------- (ii)
From (i) and (ii),
$\Longrightarrow \Psi(U) \subset Int(Cl(U)) \subset U^*$.
(f) Given that , $J \in I$
From theorem *A.2*,
Let (X, τ, I) be a space, where I is codense.
Then for $A \subset X$, $\Psi(A) \subset A^*$.It follows, $\Psi(J) = \phi$.

Theorem A.4[15] : Let (X, τ, I) be a topological space. Then for each $x \in X$, $X - \{x\}$ is I -dense if and only if $\Psi(\{x\}) = \phi$.

Proof :
From the definition of I-dense set, A subset A of a topological space (X, τ, I) is called I-dense if every point of X is in the local function of A with respect to I and τ.
i.e. if $A^*(I) = X$.
for each $x \in X$, $X - \{x\}$ is I -dense if and only if $\Psi(\{x\}) = \phi$.
Conversely,
$\Psi(\{x\}) = \phi$ if and only $(X - \{x\})^* = X$.
Hence,$\Psi(\{x\}) = \phi$ if and only if for each $x \in X$, $X - \{x\}$ is I -dense

B. Ψ- C Set

Theorem B.1 : Let (X, τ, I) be a topological space. If $A \in \tau$ then $A \in \Psi(X, \tau)$.

Proof :
Since, $\Psi(A) = X - (X-A)^*$.
Then $(X-A)^* \subset Cl(X-A) = X - A$,
Since $X - A$ is closed.
Therefore, $X - (X-A)^* \supset X - (X-A) = A$
$\Longrightarrow A \subset \Psi(A) \Longrightarrow A \in \Psi(X, \tau)$.

Example B.1 : Show that the reverse inclusion of the above theorem is not true.

Proof :
Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{c,d\}\}, I = \{\phi, \{c\}\}$.
Therefore $c(\tau) = \{\phi, X, \{a,b\}\}$.
Then $\Psi(\{a,d\}) = X - \{b,c\}^* = X - \{a,b\} = \{c,d\}$.

Thus $Cl(\Psi(\{a,d\})) = X$. Therefore $\{a,d\} \subset Cl(\Psi(\{a,d\}))$, but $\{a,d\}$ is not open in τ.

Example B.2 : Show that any closed set in (X, τ, I) may not be a Ψ-C set.

Proof :

Let $X = \{a,b,c\}, \tau = \{\phi, X, \{b\}, \{a,b\}, \{b,c\}\}, I = \{\phi, \{a\}\}$.

$C(\tau) = \{\phi, X, \{a,c\}, \{c\}, \{a\}\}$.

Then $\Psi(\{a\}) = X - \{b,c\}^* = X - \{a,b,c\} = \phi$.

Therefore $\{a\}$ is closed in (X, τ) but $\{a\} \not\subset Cl(\Psi(\{a\}))$.

Theorem B.2 : Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of nonempty Ψ - C sets in a topological space (X, τ, I), then $\cup_\alpha A_\alpha \in \Psi_\mu(X,\tau)$.

Proof :

For each α,

$$A_\alpha \subset Cl(\Psi(A_\alpha)) \subset Cl(\Psi(\cup_{\alpha\in\Delta} A_\alpha)$$
$$\Rightarrow \quad \cup_\alpha A_\alpha \subset Cl(\Psi(\cup_\alpha A_\alpha)$$

Thus $\cup_{\alpha\in\Delta} A_\alpha \in \Psi_\mu(X,\tau)$.

Example B.3 : Show that intersection of two Ψ- C sets in (X, τ, I) may not be a Ψ-C set.

Proof :

Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}, I = \{\phi, \{c\}\}$.

$C(\tau) = \{\phi, X, \{b,c,d\}, \{a,d\}, \{d\}\}$.

Then $\Psi(\{a,d\}) = X - \{b,c\}^* = X - \{b,c,d\} = \{a\}$.

Therefore $Cl(\Psi(\{a,d\})) = \{a,d\} \Rightarrow \{a,d\} \subset Cl(\Psi(\{a,d\}))$.

Then $\Psi(\{b,c,d\}) = X - \{a\}^* = X - \{a,d\} = \{b,c\}$,

$\Rightarrow Cl(\Psi(\{b,c,d\}) = \{b,c,d\}$.

Therefore $\{b,c,d\} \subset Cl(\Psi(\{b,c,d\}))$.

Then $\{b,c,d\} \cap \{a,d\} = \{d\}$ and

$\Psi(\{d\}) = X - \{a,b,c\}^* = X - \{a,b,c,d\} = \phi$.

Therefore $\{d\} \not\subset Cl(\Psi(\{d\}))$.

Theorem B.3 : Let (X, τ, I) be a topological space then $SO(X,\tau) \subset \Psi(X,\tau)$.

Proof :

Let $A \in SO(X,\tau)$, therefore $A \subset Cl(Int(A))$.

By *Corollary A.1*, $Int(A) \subset \Psi(A)$.

Therefore $Cl(Int(A) \subset Cl(\Psi(A))$.

Thus $A \subset Cl(Int(A)) \subset Cl(\Psi(A))$.

Theorem B.4 : Let A be a Ψ - C set in a topological space (X,τ,I), where I is codense. Then $A \in SPO(X,\tau)$

Proof :

From the *Theorem A.3.*(a),

for any $A \subset X$, $\Psi(A) \subset Int(Cl(A))$.

Since $\Psi(A) \subset Cl(A)$

$$\Rightarrow A \subset Cl(Int(Cl(A)))$$

Hence $A \in SPO(X,\tau)$.

Example B.4 : Show that the converse of the above theorem does not hold.

Proof :

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a,b\}\}, I = \{\phi, \{a\}\}$.

$C(\tau) = \{\phi, X, \{c\}\}$.

Then $\Psi(\{a\}) = X - \{b,c\}^* = X - \{a,b,c\} = \{a\} = \phi$.

Therefore $\{a\} \not\subset Cl(\Psi(\{a\})$,i.e., $\{a\}$ is not a Ψ - C set.

But $\{a\} \subset Cl(Int(Cl(\{a\})))$, therefore $\{a\}$ is a semi-preopen set.

Corollary B.1 : $SO(X,\tau) \subset \Psi(X,\tau) \subset SPO(X,\tau)$, when I is a codense ideal.

Proof :

From *Theorem B.3*,

$$SO(X,\tau) \subset \Psi(X,\tau) \quad \text{------------------ (i)}$$

From *Theorem B.4*,

$$\Psi(X,\tau) \subset SPO(X,\tau) \quad \text{------------------ (ii)}$$

From (i) and (ii),

$$\Rightarrow SO(X,\tau) \subset \Psi(X,\tau) \subset SPO(X,\tau).$$

Theorem B.5 : Let (X,τ,I) be a topological space and $A \in \Psi(X,\tau)$.If $U \in \tau^\alpha$, then $U \cap A \in \Psi(X,\tau)$.

Proof :

Consider, if G is open,

for any $A \subset X$, $G \cap Cl(A) \subset Cl(G \cap A)$,

As well as that, $\Psi(A \cap B) = \Psi(A) \cap \Psi(B)$.

Hence if $U \in \tau^\alpha$ and $A \in \Psi(X,\tau)$.

Therefore,

$$U \cap A \subset Int\Big(Cl\big(Int(U)\big)\Big) \cap Cl\big(\Psi(A)\big)$$
$$\subset Int\Big(Cl\big(\Psi(U)\big)\Big) \cap Cl\big(\Psi(A)\big)$$
$$\subset Cl\Big(Int\Big(Cl\big(\Psi(U)\big)\Big) \cap \Psi(A)\Big)$$
$$= Cl\Big(Int\Big(Cl\big(\Psi(U) \cap \Psi(A)\big)\Big)\Big)$$
$$= Cl\big(\Psi(U) \cap \Psi(A)\big) = Cl(\Psi(U \cap A)) \text{ and}$$

Hence $U \cap A \in \Psi(X,\tau)$.

Corollary B.2 : Let (X,τ,I) be a topological space and $A \in \Psi(X,\tau)$. If $U \in \tau$, then $U \cap A \in \Psi(X,\tau)$.

Example B.5 : Show that the converse of the above corollary does not hold.

Proof :

Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}$,

$I = \{\phi, \{c\}\}$.

$C(\tau) = \{\phi, X, \{b,c,d\}, \{a,d\}, \{d\}\}$.

Then $\Psi(\{a,c\}) = X - \{b,d\}^* = X - \{b,c,d\} = \{a\}$.

Therefore $Cl(\Psi(\{a,c\})) = \{a,d\}$.

Thus$\{a,c\} \notin \Psi(X,\tau)$,where as $\{a,c\} \notin I$.

Also that $\Psi(A) = X - (X - A)^*$, from the definition of I - dense set

$$\Rightarrow \Psi(A) = \phi \text{ if and only if } (X - A) \text{ is I - dense.}$$

Therefore for a topological space (X,τ,I) if I is codense $A \neq \phi, A \notin \Psi(X,\tau)$ if $A \in I$ or $(X - A)$ is I- dense.

Theorem B.6 : A set $A \notin \Psi(X,\tau)$ if and only if there exists an element $x \in A$ such that there is a neighborhood V_x of x for which $X - A$ is relatively I -dense in V_x.

Proof :

Let $A \notin \Psi(X,\tau)$.

Since $A \not\subset Cl(\Psi(A))$, there exists $x \in X$ such that $x \in A$ but $x \notin Cl(\Psi(A))$.

Hence there exists a neighborhood V_x of x such that $V_x \cap \Psi(A) = \phi$.

$$\Rightarrow V_x \cap (X - (X - A)^*) = \phi,$$

therefore $V_x \subset (X - A)^*$.

Conversely,

Let U be any nonempty open set in V_x.

Since $V_x \subset (X - A)^*$,

therefore $U \cap (X - A) \notin I$.
$\Longrightarrow X - A$ is relatively I-dense in V_x.

V. Ψ_μ- OPERATOR AND Ψ_μ - C SET

The behaviors of the operator Ψ_μ has been discussed in the following theorem:

A. Ψ_μ- Operator

Theorem A.1 : Let (X, μ, I) be an ideal supra topological space.

a) If $A \subseteq X$, then $\Psi_\mu(A) \supset Int^\mu(A)$.
b) If $A \subseteq X$, then $\Psi_\mu(A)$ is supra open.
c) If $A \subseteq B$, then $\Psi_\mu(A) \subseteq \Psi_\mu(B)$.
d) If $A, B \in \wp(X)$, then $\Psi_\mu(A) \cup \Psi_\mu(B) \subset \Psi_\mu(A \cup B)$.
e) If $A, B \in \wp(X)$, then $\Psi_\mu(A \cap B) \subset \Psi_\mu(A) \cap \Psi_\mu(B)$.
f) If $U \in \mu$, then $U \subset \Psi_\mu(U)$.
g) If $A \subseteq X$, then $\Psi_\mu(A) \subset \Psi_\mu(\Psi_\mu(A))$.
h) If $A \subseteq X$, then $\Psi_\mu(A) = \Psi_\mu(\Psi_\mu(A))$ if and only if $(X - A)^{*\mu} = ((X - A)^{*\mu})^{*\mu}$
i) If $A \in I$, then $\Psi_\mu(A) = X - X^{*\mu}$.
j) If $A \subseteq X$, $I \in I$, then $\Psi_\mu(A - I) = \Psi_\mu(A)$.
k) If $A \subseteq X$, $I \in I$, then $\Psi_\mu(A \cup I) = \Psi_\mu(A)$.
l) If $(A - B) \cup (B - A) \in I$, then $\Psi_\mu(A) = \Psi_\mu(B)$.

Proof :

a) From definition of Ψ_μ operator, $\Psi_\mu(A) = X - (X - A)^{*\mu}$.
Then $\Psi_\mu(A) = X - (X - A)^{*\mu} \supset X - Cl^\mu(X - A)$, from the theorem 3.1.(d),
$A^{*\mu} \subset Cl^\mu(A)$.
By using the theorem 2.2, Let (X, μ) be a supra topological space and $A \subset X$. Then $Int^\mu(A) = X - Cl^\mu(X - A)$.
Hence $\Psi_\mu(A) \supset Int^\mu(A)$.

b) Since $(X - A)^{*\mu}$ is a supra closed set.
From the theorem 3.1.(e),
$A^{*\mu}$ is a supra closed set.
Then $X - (X - A)^{*\mu}$ is a supra open set. Hence $\Psi_\mu(A)$ is supra open.

c) Given that $A \subseteq B$, then $(X - A) \supseteq (X - B)$.
Then from the theorem 3.1.(b),
$A \subset B$ implies $A^{*\mu} \subset B^{*\mu}$, $(X - A)^{*\mu} \supseteq (X - B)^{*\mu}$ and hence $\Psi_\mu(A) \subseteq \Psi_\mu(B)$.

d) Given that $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$, then $(X - A) \supseteq (X - (A \cup B)$ and $(X - B) \supseteq (X - (A \cup B)$.
Then from the result, $A \subset B$ implies $A^{*\mu} \subset B^{*\mu}$, $(X - A)^{*\mu} \supseteq (X - (A \cup B))^{*\mu}$ and $(X - B)^{*\mu} \supseteq (X - (A \cup B))^{*\mu}$.
Hence $\Psi_\mu(A) \subseteq \Psi_\mu(A \cup B)$ and $\Psi_\mu(B) \subseteq \Psi_\mu(A \cup B)$.
$\Longrightarrow \Psi_\mu(A) \cup \Psi_\mu(B) \subset \Psi_\mu(A \cup B)$

e) Since $A \cap B \subset A$ and $A \cap B \subset B$,
From (c), If $A \subseteq B$, then $\Psi_\mu(A) \subseteq \Psi_\mu(B)$.
$\Longrightarrow \Psi_\mu(A \cap B) \subset \Psi_\mu(A) \cap \Psi_\mu(B)$.

f) Let $U \in \mu$ .Then $(X - U)$ is a supra closed set and hence $Cl^\mu(X - U) = (X - U)$.
$\Longrightarrow (X - U)^{*\mu} \subset Cl^\mu(X - U) = (X - U)$.
Hence $U \subset X - (X - U)^{*\mu}$, so $U \subset \Psi_\mu(U)$.

g) From (b), If $A \subseteq X$, then $\Psi_\mu(A)$ is supra open, $\Psi_\mu(A) \in \mu$.
From (f), If $U \in \mu$, then $U \subset \Psi_\mu(U)$.
$\Longrightarrow \Psi_\mu(A) \subset \Psi_\mu(\Psi_\mu(A))$.

h) Let $\Psi_\mu(A) = \Psi_\mu(\Psi_\mu(A))$. Then,
$X - (X - A)^{*\mu} = \Psi_\mu(X - (X - A)^{*\mu}) = X - (X - (X - (X - A)^{*\mu})^{*\mu}) = X - ((X - A)^{*\mu})^{*\mu}$.
$\Longrightarrow (X - A)^{*\mu} = ((X - A)^{*\mu})^{*\mu}$.
Conversely,
Suppose that $(X - A)^{*\mu} = ((X - A)^{*\mu})^{*\mu}$ hold.
Then $X - (X - A)^{*\mu} = X - ((X - A)^{*\mu})^{*\mu}$
$X - (X - A)^{*\mu} = X - (X - (X - (X - A)^{*\mu}))^{*\mu} = X - (X - \Psi_\mu(A))^{*\mu}$.
Hence $\Psi_\mu(A) = \Psi_\mu(\Psi_\mu(A))$.

i) Since $\Psi_\mu(A) = X - (X - A)^{*\mu} = X - X^{*\mu}$ from the theorem 3.1.(k),
For $I \in I$, $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.

j) Since $X - \left(X - (A - I)\right)^{*\mu} = X - \left((X - A) \cup I\right)^{*\mu}$
$= X - (X - A)^{*\mu}$
From the theorem 3.1.(k),
For $I \in I$, $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.
So $\Psi_\mu(A - I) = \Psi_\mu(A)$.

k) Since $X - (X - A \cup I)^{*\mu} = X - \left((X - A) - I\right)^{*\mu}$
$= X - (X - A)^{*\mu}$
By using the theorem 3.1.(k),
For $I \in I$, $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.
Thus $\Psi_\mu(A \cup I) = \Psi_\mu(A)$.

l) Given that $(A - B) \cup (B - A) \in I$, and let $A - B = I_1$, $B - A = I_2$.
Then I_1 and $I_2 \in I$ (by heredity).
And also that $B = (A - I_1) \cup I_2$.
Thus $\Psi_\mu(A) = \Psi_\mu(A - I_1) = \Psi_\mu\left((A - I_1) \cup I_2\right) = \Psi_\mu(B)$.

Example A.1 : A set A which is not supra open set but satisfies $A \subseteq \Psi_\mu(A)$.

Proof :

Let X = {a, b, c, d}, μ = { ϕ, X,{a},{a,c,d},{b,c,d}},
I = {ϕ, {c}}.
Then for $A = \{a,b,d\}$, $\Psi_\mu(A) = X - \{c\}^{*\mu} = X - \phi = X$.
Here $A \subseteq \Psi_\mu(A)$, but A is not a supra open set.

Example A.2 : Show that $\Psi_\mu(A \cap B) = \Psi_\mu(A) \cap \Psi_\mu(B)$ does not hold in general.

Proof :

Let, X ={a, b, c, d}, I = {ϕ,{c}}.
μ={ϕ, X,{a},{b},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},
Then, Supra open sets containing ‘a’ are:
X,{a},{a,b}, {a,c}, {a,d}, {a,b,c},{a,b,d},{a,c,d};
Supra open sets containing ‘b’ are:
X,{b},{a,b},{b,c},{b,d},{a,b,c} ,{a,b,d},{b,c,d};
Supra open sets containing ‘c’ are:
X,{a,c},{b,c}, {a,b,c}, {a,c,d},{b,c,d};
Supra open sets containing ‘d’ are:
X,{a,d},{b,d}, {a,b,d},{a,c,d},{b,c,d}.
Here,

Consider $A = \{b,d\}$ and $B = \{a,d\}$,
then $\Psi_\mu(A) = X - \{a,c\}^{*\mu} = X - \{a\} = \{b,c,d\}$
and $\Psi_\mu(B) = X - \{b,c\}^{*\mu} = X - \{b\} = \{a,c,d\}$.
Then $\Psi_\mu(A \cap B) = \Psi_\mu(\{d\}) = X - \{a,b,c\}^{*\mu}$
$= X - \{a,b,c,d\} = \phi$.
Hence $\Psi_\mu(A \cap B) \neq \Psi_\mu(A) \cap \Psi_\mu(B)$.

B. Ψ_μ- C Set

Theorem B.1 : Let (X, μ, I) be an ideal supra topological space. If $A \in \mu$, then $A \in \Psi_\mu(X, \mu)$.
Proof :
From the previous theorem (f),
If $U \in \mu$, then $U \subset \Psi_\mu(U)$.
It follows that $\mu \subseteq \Psi_\mu(X, \mu)$.
Similarly,
If $A \in \mu$, then $A \in \Psi_\mu(X, \mu)$.
Hence the proof.
Example B.1 : Show that the reverse inclusion of the above theorem is not true.
Proof :
Consider $X = \{a, b, c, d\}$, $\mu = \{\phi, X, \{a\}, \{a,c,d\}, \{b,c,d\}\}$,
$I = \{\phi, \{c\}\}$ and
Then for $A = \{a,b,d\}$, $\Psi_\mu(A) = X - \{c\}^{*\mu} = X - \phi = X$.
Here $A \subseteq \Psi_\mu(A)$,
Hence $A \in \Psi_\mu(X, \mu)$ but $A \notin \mu$.
Which shows that any supra closed in (X, μ, I) may not be a Ψ_μ - C set.
Example B.2 : Let C(μ) be the family of all supra closed sets in (X, μ). Show that any supra closed in (X, μ, I) may not be a Ψ_μ - c set.
Proof :
Consider, $X = \{a, b, c, d\}$, $I = \{\phi, \{c\}\}$.
$\mu = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.
Here $A = \{d\} \in C(\mu)$. Then $\Psi_\mu(A) = X - \{a,b,c\}^{*\mu}$
$= X - X = \phi$.
Therefore $A \in C(\mu)$ but $A \notin \Psi_\mu(X, \mu)$.
Theorem B.2 : Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of nonempty Ψ_μ - C sets in an ideal supra topological space (X, μ, I), then $U_\alpha \in \Delta A_\alpha \in \Psi_\mu(X, \mu)$.
Proof :
For each $\alpha \in \Delta$, $A_\alpha \subseteq Cl^\mu(\Psi_\mu(A_\alpha)) \subseteq Cl^\mu(\Psi_\mu(U_\alpha \in \Delta A_\alpha))$.
$\Rightarrow U_\alpha \in \Delta A_\alpha \subseteq Cl^\mu(\Psi_\mu(U_\alpha \in \Delta A_\alpha))$. Thus $U_\alpha \in \Delta A_\alpha \in \Psi_\mu(X, \mu)$.
Example B.3 : Show that the intersection of two Ψ_μ - C sets in (X, μ, I) may not be a Ψ_μ – C.
Proof :
Consider, $X = \{a, b, c, d\}$, $I = \{\phi, \{c\}\}$.
$\mu = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.
Here, Consider $A = \{b,d\}$ and $B = \{a,d\}$, then
$\Psi_\mu(A) = X - \{a,c\}^{*\mu} = X - \{a\} = \{b,c,d\}$ and
$\Psi_\mu(A) = X - \{b,c\}^{*\mu} = X - \{b\} = \{a,c,d\}$.
So $A, B \in \Psi_\mu(X, \mu)$, but $\Psi_\mu(\{d\}) = X - \{a,b,c\}^{*\mu}$
$= X - \{a,b,c,d\} = \phi$, and $\{d\} \notin \Psi_\mu(X, \mu)$.

VI. μ- Codense Ideal And μ - Compatible Ideal

A. μ - Codense Ideal

Theorem A.1 : Let (X, μ, I) be an ideal supra topological space and I is μ-codense with μ. Then $X = X^{*\mu}$.
Proof :
Given,
Let (X, μ, I) be an ideal supra topological space and I is μ-codense with μ. Then $X^{*\mu} \subseteq X$ is obvious.
Conversely,
Suppose $x \in X$ but $x \in X^{*\mu}$. Then there exists $U_x \in \mu(x)$ such that $U_x \cap X \in I$.
That is $U_x \in I$, a contradiction to the fact that $\mu \cap I = \{\phi\}$.
Hence $X = X^{*\mu}$.
Theorem A.2 : Let (X, μ, I) be an ideal supra topological space. Then following conditions are equivalent:
i) $\mu \cap I = \{\phi\}$.
ii) $\Psi_\mu(\phi) = \phi$.
iii) if $I \in I$, then $\Psi_\mu(I) = \phi$.
Proof :
(i) ⇒(ii)
Given that $\mu \cap I = \{\phi\}$,
then $\Psi_\mu(\phi) = X - (X - \phi)^{*\mu}$ By previous theorem,
$= X - X^{*\mu} = \phi$.
(ii) ⇒(iii)
$\Psi_\mu(I) = X - (X - I)^{*\mu} = X - X^{*\mu}$
By the result, for $I \in I$, $(A \cup I)^{*\mu} = A^{*\mu} = (A - I)^{*\mu}$.
From the previous theorem,
$\Rightarrow X - X^{*\mu} = \phi$
(iii) ⇒(i)
Suppose that $A \in \mu \cap I$, then $A \in I$ and by (iii), $\Psi_\mu(A) = \phi$.
If $A \in \mu$, then by the theorem *V.A.1*(f),
If $U \in \mu$, then $U \subset \Psi_\mu(U)$
$\Rightarrow A \subseteq \Psi_\mu(A) = \phi$.
Hence $\mu \cap I = \{\phi\}$.

B. μ- Compatible Ideal

In this section we shall discuss a special type of ideal and its various properties.
Theorem B.1 : Let (X, μ, I) be an ideal supra topological space. Then $\mu \sim_\pi I$ if and only if $\Psi_\mu(A) - A \in I$ for every $A \subseteq X$.
Proof :
Suppose $\mu \sim_\pi I$.
Then, $x \in \Psi_\mu(A) - A$ if and only if $x \notin X$ and $x \notin (X - A)^{*\mu}$ if and only if $x \notin A$ and there exists $U_x \in \mu(x)$ such that $U_x - A \in I$ if and only if there exists $U_x \in \mu(x)$ such that $x \in U_x - A \in I$.
Then,
For each $x \in \Psi_\mu(A) - A$ and $U_x \in \mu(x)$,
$U_x \cap (\Psi_\mu(A) - A) \in I$ by heredity and since $\mu \sim_\pi I$,

hence $\Psi_\mu(A) - A \in I$.
Conversely,
Suppose that the condition holds. Let $A \subseteq X$ and assume that for each $x \in A$ there exists $U_x \in \mu(x)$ such that $U_x \cap A \in I$.
Then, $\Psi_\mu(X - A) - (X - A) = A - A^{*\mu}$.
Thus, $A \subseteq \Psi_\mu(X - A) - (X - A) \in I$ and hence $A \in I$ by heredity of I.

Corollary B.1 : Let (X, μ, I) be an ideal supra topological space with $\mu \sim_\pi I$. Then $\Psi_\mu(\Psi_\mu(A)) = \Psi_\mu(A)$ for every $A \subseteq X$.

Proof :
Since $\Psi_\mu(A) \subseteq \Psi_\mu(\Psi_\mu(A))$.
From the previous theorem, $\Psi_\mu(A) \subseteq A \cup I$ for some $I \in I$ and By the result,
If $A \subseteq X$, $I \in I$, then $\Psi_\mu(A \cup I) = \Psi_\mu(A)$, $\Rightarrow \Psi_\mu(\Psi_\mu(A)) = \Psi_\mu(A)$.

Theorem B.2[11] : Let (X, μ, I) be an ideal supra topological space with $\mu \sim_\pi I$. If $U, V \in \mu$ and $\Psi_\mu(U) = \Psi_\mu(V)$, then $U = V$[mod I].

Proof :
Since $U \in \mu$, $\Rightarrow U \subseteq \Psi_\mu(U)$ and
Hence $U - V \subseteq \Psi_\mu(U) - V = \Psi_\mu(V) - V \in I$ by the previous theorem.
Similarly,

$V - U \in I$.

Then $(U - V) \cup (V - U) \in I$ by additivity.
Hence $U = V$[mod I].

Theorem B.3 : Let (X, μ, I) be an ideal supra topological space with $\mu \sim_\pi I$. If $A, B \in B_r(X, \mu, I)$, and $\Psi_\mu(A) = \Psi_\mu(B)$, then $A = B$[mod I].

Proof :
Let $U, V \in \mu$ such that $A = U$[mod I] and $B = V$[mod I].
$\Psi_\mu(A) = \Psi_\mu(B)$ and $\Psi_\mu(B) = \Psi_\mu(V)$ by the result,
If $(A - B) \cup (B - A) \in I$, then $\Psi_\mu(A) = \Psi_\mu(B)$.
Since $\Psi_\mu(A) = \Psi_\mu(U)$ implies that $\Psi_\mu(U) = \Psi_\mu(V)$,
hence $U = V$[mod I] by the previous theorem.
Hence $A = B$[mod I] by transitivity.

Theorem B.4 : Let (X, μ, I) be an ideal supra topological space.

a) If $B \in B_r(X, \mu, I) - I$, then there exists $A \in \mu - \{ \phi \}$ such that $B = A$[mod I].

b) Let $\mu \cap I = \{ \phi \}$, then $B \in B_r(X, \mu, I) - I$ if and only if there exist $A \in \mu - \{ \phi \}$ such that $B = A$[mod I].

Proof :
a) Let $B \in B_r(X, \mu, I) - I$. Then $B \in B_r(X, \mu, I)$.
Suppose,
if there does not exist $A \in \mu - \{ \phi \}$ such that
$B = A$[mod I], $\Rightarrow B = \phi$[mod I].
$\Rightarrow B \in I$ which is a contradiction.
Therefore,
there exists $A \in \mu - \{ \phi \}$ such that $B = A$[mod I].
b) Let $A \in \mu - \{ \phi \}$ such that $B = A$[mod I].
Then $A = (B - J) \cup I$, where $J = B - A$, $I = A - B \in I$.
If $B \in I$, then $A \in I$ by heredity and additivity, which contradict to $\mu \cap I = \phi$.
Therefore,

$\Rightarrow B \in B_r(X, \mu, I) - I$

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